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A MODEL FOR QUANTIFYING FLOW AUGMENTATION BENEFITS

E. E. Pyatt, et al

Flordia University
Gainesville, Florida

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A MODEL FOR QUANTIFYING FLOW AUGMENTATION BENEFITS

by

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With increasing quantitative and qualitative demands being placed upon national water resources, improved management practices are vitally needed. In principle, one powerful management tool is systems analysis, wherein mathematical optimizing techniques are employed to effect rational tradeoffs between competing demands for water use, but this tool, in turn, rests upon the availability of methodologies for quantifying the benefits (economic value) of each water-use category. That is, systems analysis, before it can be employed comprehensively, demands a knowledge of the functional value of irrigation, flood control, municipal water supply, etc.

Little is known of the economic implications of low flow augmentation, one of the important water-use categories. Beginning with the premise that the value of low flow augmentation is measured by sewage treatment costs avoided, a hydrologic flow simulator and a water quality linear programming model were interfaced to develop a procedure for determining "willingness to pay" for augmentation. This generalized approach can be applied by others to their specific water pollution control situations.

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A MODEL FOR QUANTIFYING FLOW AUGMENTATION BENEFITS

FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
~~DEPARTMENT~~ OF THE INTERIOR

~~BY~~

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AUGUST, 1969

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ABSTRACT

With increasing quantitative and qualitative demands being placed upon national water resources, improved management practices are vitally needed. In principle, one powerful management tool is systems analysis, wherein mathematical optimizing techniques are employed to effect rational tradeoffs between competing demands for water use, but this tool, in turn, rests upon the availability of methodologies for quantifying the benefits (economic value) of each water-use category. That is, systems analysis, before it can be employed comprehensively, demands a knowledge of the functional value of irrigation, flood control, municipal water supply, etc.

Little is known of the economic implications of low flow augmentation, one of the important water-use categories. Beginning with the premise that the value of low flow augmentation is measured by sewage treatment costs avoided, a hydrologic flow simulator and a water quality linear programming model were interfaced to develop a procedure for determining "willingness to pay" for augmentation. This generalized approach can be applied by others to their specific water pollution control situations.

This report was submitted in fulfillment of Grant No. 16090 DRM between the Federal Water Pollution Control Administration and the University of Florida.

23) Key Words: Flow Augmentation, Water Quality Control, River Basins, Systems Analysis, Reservoirs and Impoundments, Benefit - Cost Analysis.

SECTION 1
SUMMARY

1.1 Conclusions

The overall objective of this study has been to develop a generalized methodology for quantifying the benefits of low flow augmentation for water quality management in a complex river system. The benefits of low flow augmentation have been defined as waste treatment costs avoided. This objective has been accomplished by integrated utilization of computer based simulation and optimization methods, visualized as a closed loop information feedback system. The models are structured in modular form, so that any watershed can be analyzed by simply selecting the appropriate number of modules.

The simulation models developed in this study provide a broad-based capability for analyzing water quality in complex river systems. Specific use of this capability can be made to select test conditions for the optimization model wherein it is desired to analyze the subset of the watershed in which significant water quality interdependencies exist during a selected time period. Given this set of conditions, the optimization model determines the combination of wastewater treatment plants which minimizes the total cost of meeting pre-specified water quality standards for a given amount of augmented flow. Analysis of the results permits the quantification of low flow augmentation benefits. Then, the analysis may either terminate or the procedure may be repeated for a different set of assumed conditions.

Analysis of the cost of wastewater treatment plants and storage facilities was undertaken within the scope of this study. Available cost information on primary, secondary and tertiary treatment was reviewed and cost functions were developed for these various treatment levels. The cost of storage facilities was estimated using statistical techniques to identify the more important variables which determine the desired reservoir volume. This work was combined with reservoir cost curves, expressed as a function of volume, to obtain a direct expression for reservoir costs as a function of the selected variables.

A significant contribution of the simulation effort was the synthesis of research in hydrology, hydraulics, stream pollution, and other areas into a comprehensive simulation capability that can serve a wide variety of purposes. Recent developments in these specialized research areas were incorporated into the programs. Partitioning the watershed into reaches, numbering the reaches and predicting river temperatures were effectuated using commonly accepted procedures. However, extensive contributions were made in hydrologic studies wherein synthetic hydrologic traces were developed at multiple places in the watershed. This work included procedures for aggregating all historical hydrologic gages into a subset of independent "basis" gages, analysis of flow regulation and its effect on the hydrologic regime, and other techniques which are described in the report. Also, recently devised extensions of water quality analysis which incorporate error terms for the deoxygenation and reaeration coefficients were incorporated into the methodology. Finally, procedures for sensitivity analysis were included to demonstrate the system response to changes in assumed conditions.

A separable convex programming model was developed in a generalized network format which permits efficient formulation and solution of optimization problems. The model includes the capability of simultaneously analyzing wastewater treatment and low flow augmentation with water of varying quality. Equations of continuity were developed in terms of quantities of the water, biochemical oxygen demand and dissolved oxygen resources to facilitate the interpretation of the results in terms of resource allocation. Emphasis was placed upon providing a meaningful interpretation of the results. Examining the dual problem permitted rigorous definition of local and regional "market areas" for waste management. Shadow prices impute the marginal value (measured in terms of waste treatment costs avoided) of low flow augmentation. The types of regional situations which may occur were categorized and a procedure for calculating equivalent prices for upstream BOD was presented. Low flow augmentation benefit functions for single or multiple sources of augmented flow were developed. Also, the effect of water quality of the augmented flow on the benefit function was analyzed.

1.2 Recommendations

1. The methodology developed in this research and thoroughly documented in this report, although subject to further refinement, is sufficiently sound in concept and execution that it should be embodied into extant multi-purpose project planning and evaluation procedures.
2. In using basic data from the Farmington River Basin (Connecticut and Massachusetts) to construct a generalized hydrologic simulation model, specific programs have been developed which should have considerable utility (quite apart from flow augmentation) for the various water-oriented agencies in the Hartford Metropolitan Area.

These agencies, particularly the Metropolitan District Commission (MDC), should familiarize themselves with potential applications. In general, these applications will relate to streamflows, reservoir contents, diversions for municipal water supply and reservoir releases for riparian commitments.

3. Further research is needed to quantify the net benefits of flow augmentation in a river system with competing uses for the water. Such work would require a methodology to determine the scarcity value of water as determined by examining all legitimate water uses simultaneously. This then could permit the determination of the total flow augmentation cost function, which is needed to determine the flow at which net benefits are maximized.
4. An explicit executive procedure is needed to automatically select the desired critical period and critical region from the simulation models.
5. The optimization model fruitfully could be extended from the deterministic case to the stochastic case using chance-constrained programming or a related method.
6. The option of by-pass piping could be incorporated into the analysis with relatively little additional effort and therefore should be a part of future studies.
7. Analysis of integrated use of storage facilities to provide hydropower, regulated flow for nuclear or conventional steam plants and low flow augmentation should be undertaken because of the growing importance of thermal pollution. These investigations should include the possibility of modifying power production scheduling of mixed hydro and steam systems to reduce the deleterious environmental effects.

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SECTION 2
INTRODUCTION

Planning and management of the water resources of major river basins is neither a recent concept nor a recent practice. Most notably, the Corps of Engineers, for some decades, has been charged by The Congress with performing engineering investigations, designing, constructing and, in some cases, operating physical facilities for the enhancement of navigation. Likewise, the Bureau of Reclamation, with jurisdiction in the seventeen western states, has, under the Reclamation Act of 1902, promoted the development of large-scale irrigation projects.

Historically, these water management schemes, although often vast in scope and, therefore, in cost, have not been comprehensive with respect to the full array of beneficial purposes. Development of the Columbia River Basin, for example, has been oriented primarily toward achieving economic efficiency in hydroelectric power generation and this achievement has been realized at the expense of foregoing opportunities to realize other beneficial purposes.

More recently, the federal government has not only encouraged, but has made mandatory, the inclusion of consideration of beneficial purposes other than those traditionally associated with agency missions. We find, then, that the two most prominent federal, water-oriented construction agencies - the Corps of Engineers and the Bureau of Reclamation - now are required by law to take a comprehensive view of water resources planning. The complexity of their in-house project evaluation procedures, as well as the evaluation procedures of coordinating bodies such as the Bureau of the Budget and the Water Resources Council, has magnified accordingly.

Benefit-cost analysis, or that is project evaluation, is, by definition, contingent, *inter alia,* upon the availability of techniques for quantifying the expected value of each of the separate beneficial purposes proposed for inclusion in a given project. Some beneficial purposes, such as flood control and irrigation, are inherently susceptible to attempts at quantification. Flood control benefits, for example, are calculated as average annual flood damages expected to be averted by the proposed project while irrigation benefits are, quite simply, computed by reference to the market value of incremental crop yield. These procedures have been codified through a series of government documents beginning with the "Green Book" of 1950 (1) and progressing to Bureau of the Budget Circular A-47 of 1952 (2), the revised "Green Book" of 1958 (3), the Report of the Panel of Consultants to the Bureau of the Budget of 1961 (4) and Senate Document #97 of 1962 (5).

However, other beneficial purposes, although given brief mention in the "Green Book", are elusive and very difficult to quantify. Examples of beneficial purposes for which, to date, acceptable quantification methodologies have not been developed are: (1) municipal water supply, and (2) low flow augmentation for the assimilation of organic loads discharged downstream to a watercourse. This is not to say that no methodological approaches exist conceptually nor that such approaches have not been applied. Municipal water supply benefits may be equilibrated to the cheapest alternative mode of supply or they may be computed by a consideration of the sum of the economic value of municipal water to the public health and to industrial output (6,7). Low flow augmentation (or water pollution control) is discussed in the "Green Book" (3, p. 45). It is stated that:

> "While pollution abatement may contribute significant economic returns to society and individuals, under prevailing practices relatively few of the benefits of pollution control are measured directly in monetary terms. ... In the absence of market determined values to serve in the measurement of water pollution control benefits, economic indicators of the worth of pollution abatement must be sought in derived measures of value. ... There is also need for extending the scope of measurement practices by devising simulated market conditions ... to establish a value for pollution abatement comparable to that obtained for other project purposes."

The research reported upon in subsequent sections addresses itself to precisely that need - extending the scope of measurement practices for low flow augmentation. It is important to recognize that the research sought to develop a generalized methodology for quantifying the benefits of flow augmentation. All references to specific watersheds - mainly the Farmington River Basin in Connecticut and Massachusetts - are made solely for the purpose of validating the generalized methodology.

Central to the development of the methodology was the construction of a conceptual model for quantifying low flow benefits. Our model was quite straightforward and, we think, realistic. The basic premise was this: THE VALUE OF FLOW AUGMENTATION FOR THE PURPOSE OF WATER POLLUTION CONTROL MOST APPROPRIATELY IS MEASURED BY THE DOWNSTREAM, COLLECTIVE SEWAGE TREATMENT COSTS WHICH ARE AVOIDED WHEN SPECIFIED WATER QUALITY CRITERIA MUST BE MET. More specifically, for stated permissible levels of dissolved oxygen in a river basin and for known raw BOD loads which ultimately must be discharged to the river, how much will regionally coordinated sewage treatment cost with flow augmentation upstream and how much will it cost without flow augmentation upstream? The difference in these sewage treatment costs is defined as the economic value of flow augmentation.

The authors recognize that flow augmentation, by law, may not be employed "in lieu of adequate treatment" downstream. The conceptual model, then, should not be construed as advocacy to the contrary. In general, and especially as water quality problems become increasingly acute, the downstream treatment costs avoided will be tertiary treatment costs. In any case, the methodology presented in this report does not depend upon and does not constitute a recommendation for violating the commonly accepted definition of "adequate treatment".

The authors also recognize that in any particular situation benefits other than pollution control benefits may accrue to the provision of flow augmentation. These may include enhanced recreational opportunities, enhanced fish and wildlife, increased dependable water supply downstream, improved esthetics and, in the case of estuaries, repulsion of salt water intrusion. No attempt has been made to quantify these aspects of flow augmentation. We do not regard this as a serious deficiency since most, if not all, flow augmentation projects* will be initiated because of a need for the assimilation and dilution of pollutants.

The conceptual quantification model, stated above, was incorporated into a working, hypothetical (but realistic) river basin model which could be programmed for computer simulation. In this manner, input flows to the region could be treated as stochastic variables - as they are in nature - and other variables such as streamflow temperature, reservoir size and location, reservoir release rules, hydrographic parameters, location of cities, levels of BOD loading and water quality standards all could be generalized.

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* A "low flow project" usually can be expected to be part of a multi-purpose project.

It may be helpful to the reader's understanding of our overall methodological approach to enumerate the steps which we proposed to follow at the time the study was initiated:

1. Construct a hypothetical river basin model - hypothetical, but nonetheless representative of an Eastern U. S. basin - having a dendritic tributary pattern.

2. Selecting a typical value for long-term mean discharge at the mouth, synthesize hydrologic data for the main stem and its tributaries. Employ tables of random numbers in the generation of such data.

3. Assign physical (length, width, and depth) and hydraulic (roughness) characteristics to the river system.

4. Assign (or generate) thermal values to runoff, as a function of basin location and season of the year.

5. Locate approximately five to ten major cities, at random, within the watershed. The wastes from each city would be disposed to the watercourse.

6. Develop arbitrary population, sewage flow and raw sewage strength projections for each community for the next 50 years. Assign thermal values to waste loads.

7. Estimate cost, including capital cost, of treating a unit volume of sewage by contemporary primary, secondary and tertiary techniques.

8. Estimate storage costs allocated to low flow augmentation by utilizing generalized cost curves of Corps of Engineers.

9. Employ simulation to determine the system response (D.O. levels) to various degrees of waste treatment at each city. Input flows would be stochastic and unregulated.

10. Employ simulation to determine the system response when waste treatment and low flow augmentation are employed conjunctively.

11. With the economic value of low flow augmentation defined as waste treatment costs avoided, determine that economic value by difference, (for similar D.O. levels) using items 9 and 10, above.

12. Perform sensitivity analysis.

13. Apply the methodology to a real river basin.

These steps in fact were not followed seriatim nor were they followed without considerable modification.* However, they suffice to convey an overview of the spirit which guided our research.

The thirteenth and final step, calls for applying the generalized methodology to a specific, real situation. Any one of a number of watersheds might have been chosen, provided that two criteria were met: (1) the size of the drainage basin had to be large enough to encompass a range of physiographic and engineering complexities, research budget, and (2) basic data had to be readily available. Because one of the authors (EEP) was formerly associated with the Travelers Research Center, Inc., Hartford, Connecticut at the time (1965) a comprehensive water resources study of the Farmington River Basin was performed (8), and because he therefore had access to voluminous file data both at his office and in the offices of various federal, state and local agencies in the Hartford metropolitan area, it was decided, early in the course of the research, that the Farmington Basin would serve as out test case.

The reader should keep in mind that from the viewpoint of the objectives of the research project, our sole interest in the Farmington Basin was to illustrate that the general methodology will yield acceptable results in a practical situation. In the formal sense, then, we were not seeking to develop an operational strategy for the Farmington nor were we seeking recommendations to the Hartford area agencies who so kindly provided us with basic data. On the other hand, it was our hope throughout the study that, more by accident than by design, some insights would be gleaned from the test case which could be capitalized upon by agencies such as the Metropolitan District** and the Farmington River Watershed

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* Sections 7 and 9 represent significant modifications.

** The Metropolitan District, also known as the Metropolitan District Commission (MDC), supplies water to the Hartford area from the Farmington Basin.

Association*. Indeed, on 18th and 19th September 1969, Mr. Michael Long, Research Engineer for MDC, and Mr. Harold S. Peters, Executive Director of FRWA, met with the authors in Gainesville to determine how the study might best be used to their advantage. We believe that the principal spin-off of the study is the capability of simulating the hydrology of the Farmington (see Section 8).

During the third, and final, year of the research project, FWPCA requested that we perform some special studies of our comprehensive model (9):

> "In addition to general testing of the comprehensive model with the Farmington River data, hypothetical model runs should be made to examine the following:
>
> 1. Effects of varying policies
>
>   a. Maximum use of flow augmentation permitted to minimize waste treatment necessary to meet stream standards.
>
>   b. Flow augmentation permitted only during summer months.
>
>   c. Flow augmentation permitted only when natural flow drops below some prescribed low flow, e.g., 2-year 7-day low flow.
>
>   Fixed downstream flow rate must be maintained.
>
>   Reservoir outflow must be equal to or greater than inflow during low flow periods.
>
> 2. Effects of variation in quality of reservoir releases for flow augmentation on resulting downstream quality
>
>   a. Release is from hypolimnion with zero dissolved oxygen.
>
>   Release has high concentration of nutrients and/or BOD.
>
>   Release is warm water from epiliminion.

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* FRWA is a non-profit, citizens' conservation group with an avid interest in the water resources of the Farmington Basin.

3. Special attention should be given to the decrease in relative benefits of flow augmentation due to the essentially fixed maximum waste assimilation capacity of a particular basin compared to the ever-increasing waste loads."

These studies, essentially under the rubric of our step 12 (sensitivity analysis), are reported upon in Section 8.6.

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SECTION 3
OVERVIEW

This section contains a brief description of the contents of the remainder of the report. The overall methodology for quantifying flow augmentation benefits consists of two main components: a simulation model and an optimization model. From the simulation model, hydrologic characteristics of the watershed are obtained and water quality parameters determine where and when standards are violated. This information is then utilized in determining a critical period and delineating a subset of the watershed to be analyzed. The optimization model then performs the task of determining the combination of wastewater treatment facilities which meet the water quality goals at the least cost to the region for a specified amount of flow augmentation.

The scope of this study includes analysis of the cost of waste treatment and storage facilities. A cost analysis of primary, secondary, and tertiary waste treatment and the cost of storage facilities are presented in Section 5.

The major theoretical developments associated with this study are outlined in Sections 6 and 7 which describe the simulation and optimization models, respectively. Simulation as a method for analysis of complex problems is discussed pro and con and the reasons for its selection for this problem are set forth in Section 6. The mathematical model components and the connecting logic are developed into a complete model for the computer simulation of the stream flow and water quality at points along a stream system. The methodology and the theoretical formulation of the optimization model, presented in Section 7, leads to the development of a separable convex programming model. The objective is to determine the combination of wastewater treatment plants which minimizes the total regional cost of meeting prespecified water quality standards for a given quantity of augmented flow. Approaches for analyzing specific regional situations are presented to illustrate the interpretation of the model.

The remaining sections deal with application of these methods to selected problems. The simulation model was tested using the Farmington River Basin as the study area. The results are described in Section 8. The preparation of the basic data for use in the model is described and the resulting equations are presented. The results of the sensitivity of the system to the various variables are given. Special studies designed to show the effects of imposed conditions are described and the results are reported. The optimization model was applied to a hypothetical, but complex, region to demonstrate its operation for the purposes of this study. The results are contained in Section 9. The input data for the model are presented along with changes made in the data when flow augmentation is considered. The application of the optimization model illustrates how the model is structured and operated. The latter part of the section demonstrates the value of the model for quantifying the benefits of low flow augmentation.

The appendices of the report contain the computer programs of the models along with user's instructions on how to operate these programs. Definitions of terms used in this study are contained in Appendix A1. Two auxiliary programs which can be used to fit a mathematical formula to a set of data points are described in Appendix A2. A compilation of all of the basic data, excepting the actual historical gage data, used in the application of the simulation model to the Farmington River Basin, is found in Appendix A3. A detailed set of instructions for the application of the simulation and optimization models is presented in Appendix A4. The purpose of the various main and auxiliary programs is described in detail. Program input and output formats and data are listed for easy reference. A diagram of program logic and a dictionary of variables are included for each main and auxiliary program. Lastly, a sample interfacing of the simulation and optimization models is presented in Appendix A5.

SECTION 4
LITERATURE SURVEY

A survey of the literature in the areas of endeavor associated with water quality, water resources and water resource management is a formidable task, indeed. The problems of the best use of this most necessary of the natural resources are solved only through interactions between engineers, economists, politicians, industrialists and conservationists; the literature reflects this diversity. And because each of these groups is vitally concerned about water, the volume of written matter is increasing to a substantial enormity. The present work implies an interest limited to the engineering-economic dimension and the associated analytical methods employed. Even with these limitations, the literature is extensive.

4.1 Development of the Dissolved Oxygen Model

The first dissolved oxygen model for predicting oxygen balance in a flowing stream was presented by Streeter and Phelps in 1925 (1). Despite remarkable advances in the development of equipment, techniques and methods, this classic work over the years has withstood the tests of many investigations. The formulas, which are based upon two velocity constant parameters, describe the oxygen balance in the stream as a function of distance (or time) from a waste load discharge point. The first parameter, K_1, is the deoxygenation velocity constant and the second, K_2, is the reaeration velocity constant. These parameters describe the action in a gross way, with the effects of several known interacting factors considered as being included in K_1 and K_2. Several investigators have proposed modifications which separate out known factors and thus more accurately represent the effects and improve the reproductibility of the stream conditions by the model.

In a later work, Phelps (2) and Velz (3) describe the effects of immediate (chemical) oxygen demand, sludge deposits, biological extraction of pollutants by bottom growths and, aquatic plants as a source of dissolved oxygen. Neither presented any quantitative data or methods for accounting for these factors. Thomas (4) introduced a third rate constant, K_3, to account for the loss or gain of biochemical oxygen demand (BOD) due to sludge deposits. It was assumed to be equal to K_3L, where L is the BOD concentration in the stream. Dobbins (5) considered, in addition to Thomas K_3, the addition of a constant BOD loading along the stream, the addition of oxygen by photosynthesis, the removal of oxygen by diffusion into the benthal layer and a longitudinal dispersion factor. Dobbins concluded that the effect of longitudinal dispersion on the oxygen balance is negligible in most fresh water streams.

O'Connell and N. A. Thomas (6) studying the effects of benthic algae concluded that "oxygen produced by benthic algae and other attached plants has little beneficial effect on the oxygen balance of streams; on the contrary nighttime respiratory requirements can cause seriously low daily minimum DO concentrations."

Camp (7) described the findings of a study made on the Merrimack River, Massachusetts. He stated that atmospheric reaeration as a source of oxygen is relatively insignificant compared to photosynthesis by algae and in this connection, recommended the light and dark bottle technique for meaningful evaluation of this effect. Camp also stated that the removal of BOD through settling is large compared to the removal by oxidation of suspended BOD. Some parameter values for the Merrimack were given.

Although much work has been done on the factors which can be separated from the basic biochemical deoxygenation and atmospheric reaeration, the literature still does not contain a usable method or usable values of the factors. This is not to say that it is impossible to select values of K_1 and K_2 for a given reach of stream and expect the computed sag curve to be substantiated by observations. Considerable work also is being done to allow better prediction of the oxygen conditions where only the basic factors dominate. The original assumption that the deoxygenation velocity constant K_1 is dependent only upon the waste discharge is subject to question. Gannon (8) demonstrated that the K_1 determined by the usual laboratory method (9) is substantially lower than the river deoxygenation constant determined by sampling in a river where time of flow between sample points is known. The implication was that K_1 is dependent not only on the character of the waste but also on the condition in the stream. It was concluded that extraction and storing of organic materials by biological growths, mixing in the river and the influence of nitrification in the bottle-incubated samples all contributed to the wide difference. That mixing makes a difference in the deoxygenation rate was shown by Lordi and Heukelekian (10).

In an experiment designed to show the difference in the value of K_1 determined in the BOD bottle and in a simulated stream environment, Isaacs and Gaudy (11) concluded that the values were not different, provided that the seed concentration in the bottle was the same as in the stream and that a multiple phase, or higher order, expression is used to describe the exertion of the BOD. The implication is, then, that in real environments, there are mechanisms which, if they exist in the stream, can materially affect the apparent rate at which deoxygenation proceeds. Additional studies of the type reported by Saunders (12) will lead to a better understanding of the role of attached stream bacteria but may not be of great help in quantifying K_1 until some method is worked out to determine the extent of the growths in a stream.

The evaluation of the reoxygenation velocity constant, K_2, has recently received considerable attention. The process of atmospheric reaeration is physical in nature and is more amenable to analysis than is biochemical oxidation. O'Connor and Dobbins (13) worked out the relationships between fluid turbulence and the fundamental physical laws of reaeration and presented formulas for reaeration under turbulent conditions. They found that K_2 is proportional to the coefficient of molecular diffusion and velocity of flow and inversely related to the depth raised to a power greater than unity. This theoretical approach was also used by Krenkel and Orlob (14). Another similar formula has been proposed by Thackston and Krenkel (15).

A group at TVA (16) made use of the opportunity afforded them by discharges from stratified reservoirs relatively free of pollution, yet nearly devoid of dissolved oxygen. Field measurements were taken and, using dimensional analysis and multiple regression techniques, the observed reaeration rates were related to the hydraulic properties of the river channels. The resulting predictive formulas for K_2 were not materially different from those developed by the more rigorous methods.

An empirical formula obtained by plotting available laboratory and field data has been presented by Langbein and Durum (17). This formula relates K_2 directly to the velocity and inversely to the 1.33 power of average depth. A coefficient in the formula appears to vary with geographical location, implying that slope of the river bed has some linear effect on the value of K_2. Isaacs and Gaudy (18) found a similar relationship between K_2, velocity and depth, using a simulated stream.

In an interesting treatment of the problem, Thayer and Krutchkoff (19) considered the oxygen balance relationship in a stream as a stochastic process wherein BOD and DO are increased (or decreased) by increments over a short interval of time. This treatment led to a joint density function for both variables and affords a measure of variance from the mean values. Predicted downstream DO values thus have associated probabilities of occurrence. Kothandaraman (20) treated K_1 and K_2 as random variables with the effects of other mechanisms in the BOD-DO relationship (sedimentation, photosynthesis, etc.) included in the variability of these two factors. He states "The most probable values for the dissolved oxygen deficits predicted by the probabilistic model ... are found to be better estimates than the values predicted by the conventional deterministic approaches."

A similar approach was taken by Moreau and Pyatt (21) who considered that K_1 and K_2 were deterministic in the conventional manner but added an "ignorance or error" term to each. The error terms likewise contain the variability attributed to the indeterminable and intermittant effects. Nicholson (22) applied the technique of both Kothandaraman and Moreau and Pyatt in the development of a method for selecting among water quality alternatives.

These recent efforts to obtain relationships which allow prediction of values for DO for known river channel conditions have strengthened the methodology of Streeter and Phelps.

Attempts have been made to circumvent the problems of the Streeter-Phelps method. LeBosquet and Tsivoglou (23) proposed an "abbreviation of the Streeter-Phelps procedure" in which use is made of the linear relationships of BOD and rate of flow, and DO deficit and rate of flow, to obtain proportionality constants. Stream survey data were used to evaluate the constants which were used subsequently to predict quality conditions for selected stream conditions and loads. Churchill and Buckingham (24) used multiple regression techniques on observed data to obtain similar predictor equations. Okum, Lamb and Wells (25) used a comparable method to develop regression equations in an industrial pollution problem.

A different approach was described by Thomann (26), who used a systems analysis technique wherein the input is transformed by the system to result in the output. In this application, the input is the BOD loading at the point of discharge, the system is the river which incorporates biochemical decay, reaeration, flow time and other factors which occur in streams, and the output is the DO level. The system transfer functions may be mathematical expressions akin to the Streeter-Phelps equations (in differential form) or, if stream data are available, the transfer functions may be constants determined as the ratio of output to input. Note a similarity to the LeBosquet and Tsivoglou method.

It is universally recognized that water temperature is a significant factor in both deoxygenation and reaeration in a stream. Although there is still controversy over the relation between water temperature and the values of K_1 and K_2 (27), experimenters are converging upon values for temperature coefficients. The disparity in values causes errors of minor consequence in comparison with those due to the imprecision in K_1 and K_2 values. Thus, when temperatures are known, methods for correcting for its effects are readily applied.

However, a problem appears when temperatures vary with distance along the stream, such as occurs when cold water is released from a stratified reservoir or warm water is discharged from a power plant. Temperature variations are related to the heat budget in a stream. Much of the methodology in heat budget studies was developed for and during the Lake Hefner evaporation investigations (28). These relationships have been applied to stream flows by Raphael (29), Delay and Seaders (30) and Brown (31). In general, the technique used is to estimate the gains and losses of energy in a body of water to determine the change in stored energy. The change in stored energy per unit volume of water is converted to temperature change. The recent concern about thermal pollution is sure to result in the development of more precise methods for determining temperature changes in streams.

4.2 Systems Techniques in Water Resources

The methods of operations research, systems analysis and modern mathematical analysis have been applied in increasing intensity in the past few years. The methods admit, in many cases, to direct application in water resources management and, although the number of people in the water resources field having an ability in the systems field is relatively small, these few have made substantial progress. The literature attests to this.

The first major effort to apply operations research and systems analysis in water resources studies was by the Harvard Water Program, which was started in 1956 and culminated in 1962 with the publication of a textbook-like report of the objectives, concepts, methods and techniques (32). The program produced disciples who continue to be active. Fiering (33, 34, 35), Hufschmidt and Fiering (36), Thomas (37) and Matalas (38) are among the many works of the group. The subjects are all concerned with operations research, systems and simulation techniques in the solution of water resources design problems.

Another sprinkling of papers came from Northwestern University where Charnes has influenced the use of systems techniques. Lynn, Logan and Charnes (39); Logan, et al. (40); Lynn (41); Deininger (42) and Heaney (43) are examples of work from this source. The Lynn influence has since spread to Cornell University where such works as Liebman and Lynn (44), Loucks and Lynn (45), and Loucks, ReVelle and Lynn (46) have resulted.

In a U. S. Public Health Service study of the Delaware River Basin, extensive use of these methods of analysis was reported by Thomann (26,47), Thomann and Sobel (48) and Sobel (49).

The portion of the Delaware Basin that is estuarine was included in the study and methods were presented that allowed predictions of oxygen levels. The pollution problems in estuaries are usually more difficult of analysis than those of rivers.

The review is not intended to be complete - there are several other groups, notably at Oregon State University, Texas A & M University and the University of Florida, using systems techniques in water resources studies. Many other individuals also are contributing to the literature.

Although the basic concepts of sensitivity are not new, one of the bonuses that has accrued from the application of modern mathematical methods and computers to decision-making is the rediscovery of its value. In many instances, computers make its computation feasible whereas without the computer, consideration of sensitivity is impractical. Sensitivity is the change in a system caused by a small change in one or more of the system variables. Until relatively recently, the designers and decision-makers relied upon intuition and planning experience but now sensitivity analysis can be employed to obtain this valuable information.

The use of sensitivity analysis in water resources planning has been reported in several instances. McBeath and Eliassen (50) employed sensitivity analysis to identify the parameters most critical to the design of activated sludge treatment plants. Young, Schrecongost and Fitch (51) studied the sensitivity of variables in the design of reservoirs. The application of sensitivity analysis to hydrologic research was reported by Vemuri, et al. (53). This latter work outlined some of the basic theory. In a sensitivity study on the relative importance of variables in water resources planning, James, Bower and Matalas (53) concluded that the importance of the four variables studied was, in descending order; (1) projected economic development, (2) water quality objective, (3) dissolved oxygen modeling, and (4) hydrology. Kothandaraman (20) and Nicholson (22) both reported the sensitivity of K_1 and K_2 in dissolved oxygen modeling.

4.3 Operational Hydrology

Operational hydrology is the application of modern mathematical methods to generate synthetic streamflow data. It is not a design technique - more, it is a technique for generating design data (35). Several such data generators have been developed. Fiering (33) described, in considerable detail, the development of the multivariate Markovian model which provided design data for the Harvard studies. Fiering (35) later expanded the description in book form. Matalas (38) made a detailed mathematical and statistical analysis of the method and showed that bias is introduced when one sample of historical data is used to develop parameters which are used, in turn, to generate many samples of data. Benson and Matalas (54) proposed to correct these deficiencies using parameters derived from generalized multiple-regression relations with physical and climatic characteristics of the basin. No detailed methodology has appeared. The Harvard model generated monthly flow data.

In a modification of the Harvard model, Harms and Campbell (55) observed annual flows were normally distributed while monthly flows have a skewed distribution that is log-normal. They generated annual and monthly flows separately and adjust the monthly generated flows to correspond with the annual generated flow. No indication was given as to increase in precision, if any, afforded by this modification.

Beard (56) developed a monthly stream flow generator based upon multiple regression using as independent variables: (1) the flow at the same station for the preceding month, (2) the flow for the current month at each upstream station, and (3) the sum of flows for all stations for the second through the seventh antecedent months. Flow values were all log-transformed. A random component was added. Regression coefficients are beta coefficients relating standard deviates, so no regression constant is required.

Quimpo (57) used spectral analysis techniques to generate daily flow data at a single station. Detectable trends first were removed to provide stationarity. Then autocovariances for daily lags up to 1,095 (3 years) were computed and a spectral analysis was performed. A Fourier representation of the deterministic component was then made using the annual cycle, which usually is evident in the spectral density-frequency representation, and five subharmonics as bases. A random component was included. The process can be given a Markovian character by making the current random component a linear combination of antecedent random components.

The Harvard synthetic hydrology generator also was modified by Young and Pisano (58). The analysis was made using residuals instead of flow data. Residuals are essentially standardized flow data computed by subtracting the mean from the data value and dividing by the standard deviation. Data were transformed to normalize. Monthly synthetic data were generated. This method is almost identical to the method used in FLASH, the generator developed for this work, the most notable difference being that FLASH generates weekly data. The development of the method by Young and Pisano was independent of that of FLASH.

Daily stream flows also were generated by Payne, Neuman and Kerri (59). Their model generated multiple-station daily data to simulate historical flow sequence having frequency characteristics similar to those of the historical data. The model is similar to that of Beard, excepting that the time of occurrence of the flows is rearranged so that peak values for each month (year, or season) occur on the same day each month (year or season). After the data are rearranged, Beards' regression technique was used. The program was written in DYNAMO.

The short-comings of operational hydrology methods are beginning to appear in the literature. Mandelbrot and Wallis (60) pointed out that "use of a Gauss-Markov process implies fitting high-frequency effects first and worrying about the low-frequency effects later." Or, too little attention has been paid to extreme events which affect designs but which do not appear when historical data not containing extreme events are used to generate many years of data. They feel the methods in use are "first approximation" and that even "first approximation" should endeavor to represent extreme events.

4.4 Models and Simulation

Examples of simulation abound - technical literature in every field describes applications of simulation techniques for solving complex practical problems. Simulation is important to the studies of hydrology and water quality, as is reflected by the extent of its use. Examples of this use are reviewed here.

The hydrology of a watershed, including those factors related to groundwater, was simulated by Crawford and Linsley (61) in the work widely known as the Stanford Watershed Model. James (62) has used the Stanford Model in a computer simulation study of the effects of urbanization on flood peaks. Goodman and Dobbins (63) attempted to model the physical, economic and administrative interrelationships in water pollution control programs in watersheds where water is used for municipal and industrial supply, disposal of wastewaters and recreation. Studies of estuarine water quality by simulation have been reported by Thomann (26), Thomann and Sobel (48) and Dornhelm and Woolhiser (64). Simulation methods at present are the only practical way to represent such a complex environment. Analog simulation techniques have been applied recently in the design and operation of activated sludge systems for wastewater treatment where the system functions prove to be difficult to solve by analytical methods (65).

Simulation as a technique also is being applied to complex theoretical problems. Two examples in the hydraulics field are Streeter's work in the solution of complicated water hammer problems (66) and the work of Baltzer and Lai (67) on unsteady flow in waterways. Of simulation, Anderson (68) has said "simulation is a very rewarding exercise. --- increasingly frequent use will be made of simulation in hydrology for it offers the only economical way of 'experimenting' with large areas and long periods of time."

4.5 Flow Regulation - Reservoir Water Quality

Modern mathematics and operations research techniques also have been applied to flow regulation and reservoir storage aspects of watershed management. An early use was by Langbein (69) who applied queuing theory to determine the amount of holdover storage for regulating streamflow. Stall (70) used a non-sequential series of low-flow events to improve the mass-curve analysis and interpretation of low-flow characteristics on a recurrence interval basis.

Optimality of design of a multiple-purpose reservoir was investigated by Hall (71). A method was proposed in which releases from a reservoir were first allocated, by means of dynamic programming, to each purpose assuming no compatible uses. Having allocated water optimally under this assumption, this restriction was removed and the size of the reservoir reduced, allowing for multiple use of the same volume of water. The problem of optimal release sequences for multiple reservoirs on a watershed was investigated by Worley (72) in a study of the use of releases for water quality management.

The operation of reservoirs has received considerable attention. Loucks (73) developed computer and stochastic linear programming models for defining alternative policies for regulating reservoirs. The IBM/Mathematical Programming System was used. James (74) applied economic criteria to the derivation of rules for operating reservoirs and claims the greatest potential value of this technique is a realization of greater benefit from existing facilities.

The effects of reservoirs and reservoir operation on the quality of the stored, and released, water also is of considerable importance in water management. Early considerations were reported by Churchill (75) and, in more detail, by Churchill and Nicolas (76). These studies are typical of those from TVA in that they are well conceived, authoritative and amply supported by data obtained by direct observation of existing prototypes. Water quality variations in impoundments also were studied by Krenkel, Thackston and Parker (77). This paper is a relatively complete discussion and presentation of the current (1969) state of the art.

An interesting method for improving impounded water quality has been studied by Symons, et al. (78, 79). Water in an impoundment is mixed using pumps, which raise water from the depths and discharge at the surface. This practice increases the dissolved oxygen in the lower levels and increases the overall temperature. The practice is economically feasible under certain circumstances.

The use of reservoirs for two purposes, flood control and low flow augmentation for water quality control, has received attention by the Federal Water Pollution Control Administration. Many existing projects are for flood control use only and, with the increasing need of water for pollution control, a study of the benefits accruing to a trade-off between uses has been made (80). As the value of water for low flow augmentation for water quality control increases, it is probable that a re-evaluation of benefits will dictate changes in operating rules.

4.6 Regional Water Quality Management Models

Mathematical models of regional wastewater systems have been developed by several investigators. Following the initial work of Deininger (42), Loucks, Revelle, and Lynn (46), Liebman (44), Kerri (81), Thomann (82), Sobel (49), and Clough and Bayer (83) have addressed themselves to the problem of finding the optimal (least cost in this case) combination of wastewater treatment plants subject to satisfying prespecified water quality constraints.

The rationale behind regional wastewater management is to take advantage of the economies of scale that are known to exist. The output from these models has demonstrated clearly that significant savings _could_ be realized _if_ a regional management system existed. They typically do not. An alternate to wastewater treatment is to pipe the wastewater elsewhere in the system so as to better utilize the waste assimilative capacity of the system. Graves, Hatfield, and Whinston (84) have examined this alternative. However the usefulness of their analysis is delimited by their assumption that transfer coefficients are independent of flow. Lastly, low flow augmentation has been analyzed as another alternative and results from this analysis are described in this report.

Given that there are potential savings from coordinated waste management, the problem still remains of how to implement such systems. This has been considered beyond the scope of the work to date. For example, Clough and Bayer (83) state, "The model makes sense only if we postulate the existence of an agency that has the legal authority to manage water resources (for example, storage and stream flows) on a entire river system, and if we postulate the existence of a regulatory agency that has the legal authority to specify and codify stream water quality standards." It is reasonable to assume that it would be difficult to promote regional waste management solely on the basis of the overall possible cost reduction. Many questions remain. Why was this "region" selected in the first place? Could we still set up a management scheme if the i^{th} decision-making unit in our region refused to participate? How should the cost be apportioned among the participants?

Relatively little work has been done to answer these questions which need to be resolved if implementation of regional wastewater management is to be achieved. Upton has recently proposed a model to determine an optimal system of taxes on water pollution (85). However his description of the physical system is highly restrictive and therefore the analysis does not permit the type of generalizations being sought here.

Lastly, in the more general case, water quality management should be viewed as only one of many uses. Thus, the analysis should include the shadow price of water as a measure of its value in an alternative use.

A mathematical programming model has been developed in a multicommodity network format for this purpose. The model includes the capability of simultaneously analyzing wastewater treatment, and low flow augmentation with water of varying quality. Equations of continuity are developed in terms of quantities of the water, BOD, and dissolved oxygen resources to facilitate the interpretation of the results in terms of resource allocation. This approach is felt to be clearer than the use of the concentration of BOD and the dissolved oxygen deficit. Examining the dual problem permits rigorous definition of local and regional "market areas" for waste management.

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SECTION 5
COST ANALYSIS

5.1 Introduction

An analysis of the cost of waste treatment and the cost of storage facilities is presented in this section. Waste treatment costs are determined for primary, secondary and tertiary facilities. Reservoir storage cost evaluations include a regression analysis relating required reservoir storage and a set of independent variables.

5.2 Cost of Waste Treatment

An evaluation of low flow augmentation as a means of maintaining water quality entails some knowledge of the costs of waste treatment. When combined with the costs of providing varying levels of flow augmentation, these may be compared with the benefits derivable from maintaining various standards of water quality. Ideally this comparison leads to optimal levels of water quality, flow augmentation and waste treatment. It is the purpose of this section to estimate the total annual costs of waste treatment to achieve various levels of efficiency of BOD removal. Primary, secondary and tertiary treatment processes are considered, utilizing both field and hypothetical costs for treatment plants of differing capacities. A more complete description for this phase of the investigation is contained in (1).

5.2.1 Primary and Secondary Treatment Costs

Three field studies of sewage treatment plant construction costs by Rowan *et al*. (2), Logan *et al*. (3) and the Public Health Service (4) were selected and compared. The Public Health Service construction cost index, based on 1957-59 costs/mgd, was used to adjust construction costs from all three studies to the same year, 1968. Similarly, operation and maintenance cost studies by Rowan, Jenkins and Howells (5) and Logan *et al*. (3) were compared after adjustment to 1968 dollars by means of the Bureau of Labor Statistics Common Labor Index. Updated construction costs for each study were weighted by the sample size from which the costs were derived and a least-squares equation was developed for the average construction cost/mgd for primary, trickling filtration and activated sludge treatment. A similar equation was developed for operation and maintenance costs. It was not possible to calculate a coefficient of variation for the final regression curves since the original data were not readily available. However, results from the individual studies were similar in most cases.

Application of the equations to this study was limited by a number of factors. First, the dependence upon indices for converting cost data in various parts of the country to a single base introduces error due to regional differences, primary among which are construction methods.

Second, specific local problems are not accounted for. Third, definitive conclusions concerning the precision of the final construction cost equations are not possible. Fourth, and most significantly, no determination could be made of the sensitivity of treatment costs and efficiencies to variations in unit process design or operation. For these reasons more detailed cost data were deemed necessary.

Logan et al. (3) designed a series of treatment plants composed of unit processes for which construction, operation and maintenance costs were calculated. Total costs were obtained by adding the individual unit process costs. Logan's approach was adopted and expanded upon in this study. Construction costs at Kansas City for the various sizes of unit processes were presented in terms of 1960 dollars. These were adjusted to 1968 dollars for the average of the 20 cities comprising the PHS Index, under the assumption that labor and management productivity does not vary among the cities. Assuming that variations in unit process parameters are minor, annual operation and maintenance costs also were adjusted to 1968 dollars. Hypothetical treatment plants utilizing a large number of treatment process combinations were analyzed to determine total annual costs of operation. A design period of 20 years was chosen. Construction costs were increased by 25 percent to allow for legal and engineering fees, and the total capital cost was then amortized at an interest rate of four percent. The latter was chosen to represent interest rates on municipal revenue bonds.

The efficiency of BOD removal was determined for each of the process combinations and curves were drawn for each hypothetical treatment plant capacity relating total annual cost to percent removal of BOD. Capacities considered were 0.25, 1.0, and 10.0 MGD. These curves are shown in Figure 5-1 and they are compared in Figure 5-2 to curves derived from the field cost studies. The hypothetical curves are presented as "envelopes" indicating the range of costs to be expected for any effluent quality desired. The existence of a range reflects the variation in total annual costs attributable to sludge disposal costs, which are not included in this study. Sludge disposal costs will determine the expenditure necessary for sludge treatment and handling, which in turn will determine the location within the envelope. The bounds of each "envelope" may therefore be interpreted as representing high or low sludge handling costs. With the exception of the 0.25 MGD plant capacity, comparison of the hypothetical cost curves with those calculated from the field costs indicates close similarity between the two cost curves, as shown in Figure 5-2. The more detailed cost curves will be used in this study to determine the effect on water quality of incremental changes in primary and secondary waste treatment.

5.2.2 Tertiary Treatment Costs

As a parameter of waste treatment efficiency, BOD removal is not of primary significance when tertiary treatment is under consideration.

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5.2.2 Tertiary Treatment Costs

As a parameter of waste treatment efficiency, BOD removal is not of primary significance when tertiary treatment is under consideration.

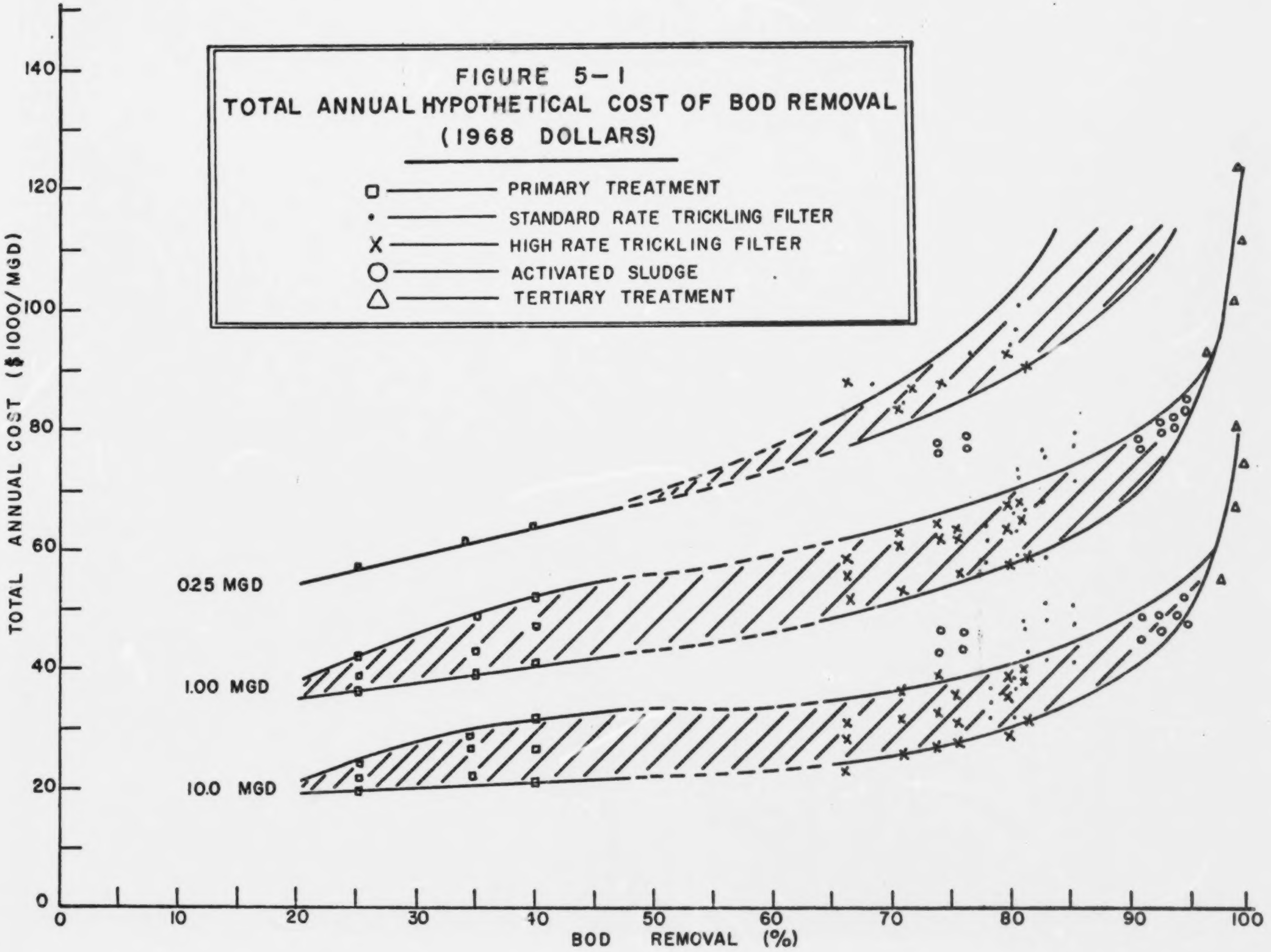

FIGURE 5-1
TOTAL ANNUAL HYPOTHETICAL COST OF BOD REMOVAL
(1968 DOLLARS)

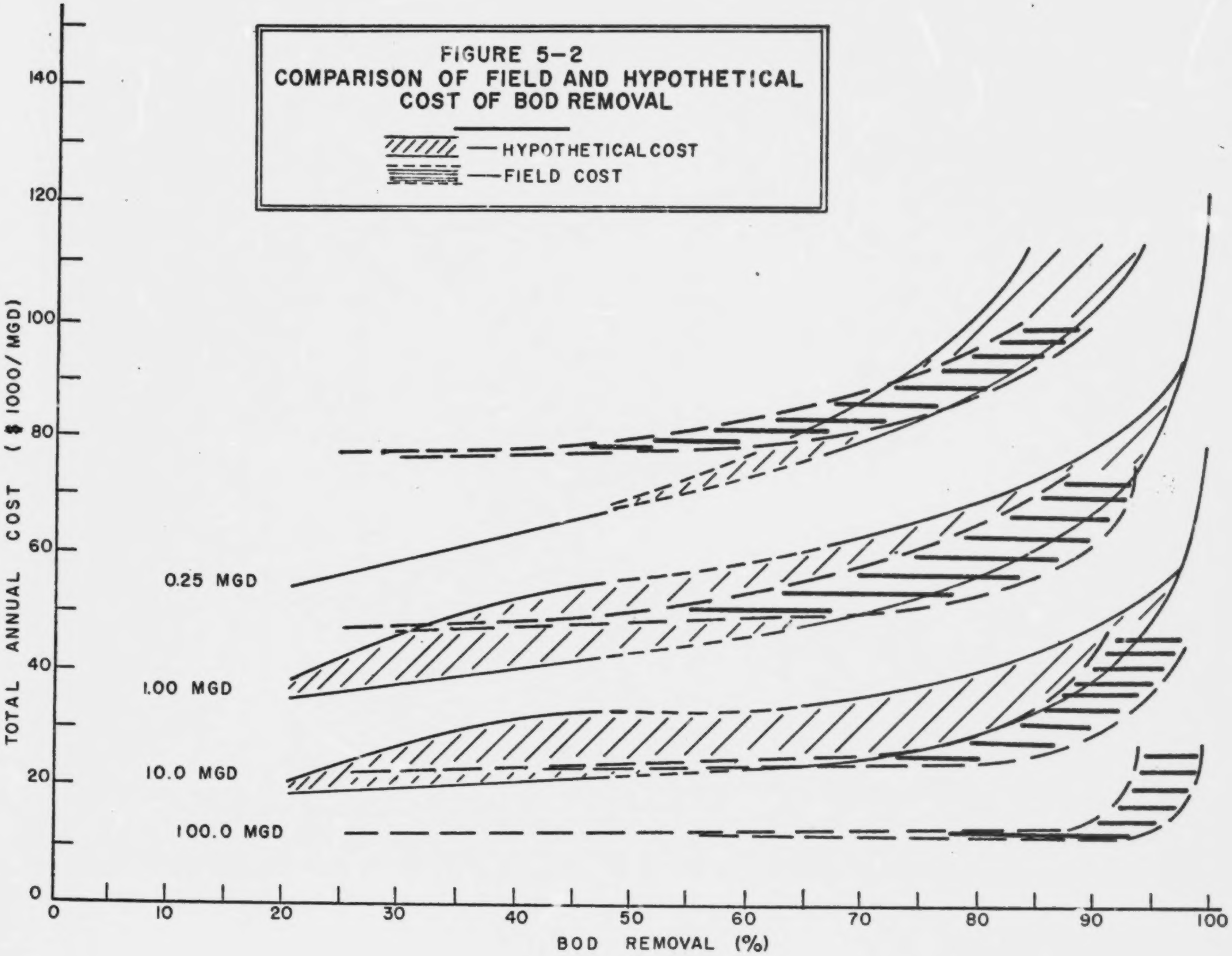

FIGURE 5-2
COMPARISON OF FIELD AND HYPOTHETICAL COST OF BOD REMOVAL

Other parameters become important, among which are total dissolved solids (TDS), ammonia, nitrogen, nitrate nitrogen, chemical oxygen demand (COD), suspended solids, phosphate, and bacterial count (MPN). However, BOD is removed by many of the tertiary unit processes, thereby enabling calculation of annual costs of BOD removal up to nearly 100 percent.

The four tertiary treatment processes considered are combinations of unit processes that are assumed to receive secondary effluent from an activated sludge unit that has already removed 94 percent of the BOD. The processes and their incremental efficiencies of BOD removal are listed in Table 5-1.

TABLE 5-1
INCREMENTAL BOD REMOVAL WITH TERTIARY TREATMENT

Process	Incremental BOD Removal (%)
1. Polyelectrolyte Coagulation + Filtration	95
2. Polyelectrolyte Coagulation + Filtration + Absorption	100
3. Foam Separation	30
4. Polyelectrolyte Coagulation + Filtration + PO_4 Removal + NH_3 Removal	95

Data determined in experimental studies at Lake Tahoe were used to calculate capital and operating costs for the first two processes (6). The design period was assumed to be 20 years and the interest rate was maintained at four percent. The tertiary plant was assumed to be operating during the 120 day period of lowest flows, while remaining inoperational for the remainder of the year. Cost data or foam separation are limited with the result that the calculated annual costs of this form of treatment are of limited accuracy. The last process was included for purposes of comparison. It is identical to the first process except that the removal of two other parameters normally associated with tertiary treatment are included. It therefore reflects more accurately the probable required expenditure associated with tertiary treatment. Capital costs for the first two processes were increased by 25 percent to cover engineering and legal fees; however, it was not possible to determine whether such an addition had already been made to the last two process costs, so the increase was omitted. All costs are considered to be slightly conservative.

5.2.3 Annual Cost Equations

For the purpose of this study, it was necessary to present the annual costs of waste treatment in two forms. The first, relating annual cost to percent removal of BOD for various flow rates, has been shown. It was also desired to relate annual cost of waste treatment to flow for various efficiencies of BOD removal. Equations of the form

$$Y = aQ^b \text{ for } 1 \leq Q \leq 100 \quad . \; . \; . \; . \; . \quad \text{[Eq. 5.1]}$$

where:

Y = total annual waste treatment costs,
a,b = coefficients, and
Q = design capacity of treatment plant,

were determined at seven points of BOD removal, thereby dividing the convex curves of Figure 5-2 into linear segments for incorporating into a linear programming model. The coefficients a and b for specified levels of BOD removal are shown in Table 5-2. These equations are valid within a range of treatment plant capacity from one to one hundred MGD.

TABLE 5-2
COEFFICIENTS FOR EQUATION [5.1]

Segment	a	b	BOD Removal (%)
1	47,000	-0.31535	30
2	49,600	-0.31740	50
3	54,500	-0.33175	70
4	60,000	-0.34065	80
5	66,000	-0.34460	85
6	74,000	-0.34655	90
7	110,000	-0.37020	95

5.3 Cost of Impoundments

While waste treatment reduces the sources of pollution, low flow augmentation can be used to enhance the assimilative capacity in the receiving waters in two ways. First, it brings about an increased degree of dilution of the pollutants. Secondly, it displaces the demands placed upon the dissolved oxygen resource in a stream to points downstream of the waste sources. Although their mechanisms are different, waste treatment and flow augmentation can be used jointly to meet a set of water quality standards in a stream. The fact that marginal costs increase sharply at high levels of waste treatment suggests that consideration be given to an economic tradeoff between these two methods. This requires data on low flow augmentation costs.

In 1960, Cook (7) developed a set of regional reservoir cost curves for the United States. These curves, which express cost per unit volume as a function of total volume, indicate definite economies of scale. Economic factors indicate that it is unlikely that a reservoir might be built exclusively for low flow augmentation. Rather it is a potential single use of a multiple-purpose reservoir. In this multi-purpose framework, the problem of evaluating the costs of low flow augmentation is quite elusive. Each purpose exhibits a unique pattern of water demands and claims on reservoir storage space over time. As a result, both conflicting and complementary situations arise, which must be resolved and accommodated by means of operating rules.

Therefore, in a multiple-purpose project, a series of analyses must be performed before the cost of low flow augmentation can be meaningfully evaluated. The process may be described in two basic steps. The first step concerns the reservoir as an entity which serves a multitude of purposes, but which nonetheless has one overall cost associated with it. This cost can be linked directly, by means of curves similar to the ones Cook derived, to the reservoir volume. It follows that the cost of a reservoir is determined ultimately by the seasonal patterns for the different purposes. Given the inflows, an overall operating rule, and the seasonal target demand pattern for the combination of uses, the reservoir capacity required over a certain length of time can be calculated. In this investigation, such a process was carried out, under varying hydrologic conditions and policies. The result was a regression relationship between required capacity and the variables which largely defined the operation of the model. At this point the breakdown of the overall demand by purpose had not yet been considered.

The second step examines the internal situation of the reservoir, namely how low flow augmentation and their demands interact to produce claims on storage space. If the value of releases from the reservoir can be measured in comparable units, it is feasible to derive a set of operating schemes which would maximize overall net benefits. This approach permits the investigation of complementarity and competition among the different purposes.

5.3.1 Summary of Methodology

A functional relationship between required reservoir storage and a set of pertinent variables was developed (8). These variables reflect the nature of the inflows and overall demands, and the probability that the reservoir would not become empty. This information was then translated into cost data by using the regional reservoir cost curves developed by Cook.

A small-scale synthetic streamflow generator was used to provide inflows to the reservoir for the desired time period. The variables in the generator were: average monthly flow, monthly standard deviation, and monthly serial correlation coefficients,based on a one-month lag.

The results of curve-fitting experiments conducted on hydrologic data were relied upon for simplification purposes. These results indicate that the variations of the above variables with respect to time exhibit a definite and consistent cyclic pattern. All three parameters show approximately sinusoidal patterns, of a different nature in each case. The average monthly flow varies according to a sine curve of one cycle. The average monthly coefficient of variation, which is a surrogate for the standard deviation, also exhibits a sinusoidal pattern of one cycle, but is shifted 180 degrees with respect to the above. The monthly serial correlation coefficient is found to approximate a sine wave of two cycles. The likeness of each of the first two patterns to its trigonometric counterpart is enhanced by means of a logarithmic transformation.

In this program, the upper and lower limits to the range of these sinusoidal variations are specified, and thereby determine the remaining monthly values of the parameters. Several runs of the reservoir operation model were conducted, using different sets of these three upper and lower limits, to observe varying responses in storage requirements for certain demands. The monthly streamflow values obtained from the generator were in all cases subjected to a logarithmic transformation.

The link between reservoir inflows and outflows is provided by a set of operating rules. Essentially, these rules set the amount of water to be released at any time, given the storage level, the inflow rate, the outflow rate, and the season of the year. In this phase of the study, no breakdown of water demand by user was made. Instead, consideration was given to the combined effect of withdrawals for different purposes, by means of time-varying release rates.

The operating rule used in the program is a modified version of that developed by Moran (9). Basically, the operating rule states that if the amount of water available is less than a preset target release, the entire amount present will be released. On the other hand, if the amount of water available is greater than a certain specified value, releases in excess of the target rate will be made to avoid overflow. The modifications introduced into the model consist of the use of actual inflows, storage volumes, and outflows, instead of their distributions. Also the capability for handling time-varying target release rates is incorporated.

For simplification purposes, the target release pattern was represented by a sine curve of a yearly cycle. The parameters which determined the monthly values were an upper limit, a lower limit, and a phase-shift value. These three parameters were also assigned varying values to measure their relative impact on the behavior of the system.

The assumed sinusoidal pattern provides a useful and simple parametric tool. In a specific instance where the target release pattern is known, it might be expressed more accurately in terms of another function or superposition of functions. In that case, the use of deterministic parameters, although more complex, would still be desirable from the standpoint of regression analysis.

Next, a method was devised to examine the relationship between storage capacity and the probability that the reservoir may become empty in any given year. This probability was obtained by operating reservoirs of different sizes for a certain number of years and calculating the ratio of the number of years in which there was deficiency to the total number of years. A computer version of the mass diagram analysis calculated the storage volume required for no deficiencies, using a relatively short time period. The trial reservoir capacities were values higher and lower than the above, in order to provide a wide testing range. These reservoirs were operated in the main program to determine their deficiency ratios.

Prior to formulation of a method for operating the model and drawing statistical conclusions, it is worthwhile to review the variables involved:

1. QMAX: maximum average monthly streamflow, in MGD/square mile.
2. QMIN: minimum average monthly streamflow, in MGD/square mile.
3. CMAX: maximum average monthly coefficient of variation of streamflows.
4. CMIN: minimum average monthly coefficient of variation of streamflows.
5. SMAX: maximum average monthly coefficient of serial correlation of streamflows.
6. SMIN: minimum average monthly coefficient of serial correlation of streamflows.
7. DMAX: maximum monthly demand rate, in MGD/square mile.
8. DMIN: minimum monthly demand rate, in MGD/square mile.
9. DLAG: the time period elapsed between the largest flow and the largest demand, in months.
10. PROB: the probability that the reservoir will not become empty in any given year.

The square of each of the variables was included to permit investigating non-linear effects.

In the main program, variables 1 through 9 define the monthly inflows and outflows for each of the different trial reservoir capacities. Variable 10 is a part of the output from this program. Subsequently, a multivariate regression analysis is performed. In this program, variables 1 through 10 are the independent variables, and the response of interest, V, is the storage volume required for a certain probability level.

A statistical framework well-suited to this case is the factorial experiment. As its name implies, this experiment permits the simultaneous examination of the effects of varying two or more factors. This technique assigns certain specific and constant values, or levels, to each factor and then proceeds to examine subsequent responses. In the analysis of the results, the effect of each factor can be established with the same degree of accuracy as if only one factor had been varied at a time; the interaction effects between the factors can also be determined.

If two levels, high and low, are assigned to each of the nine factors (not including the probability terms, which are dependent upon these), the total possible number of combinations of factors would be 2^9, or 512. This number is more than is needed to estimate the parameters in the desired functional relation, so a complete factorial experiment was not employed. Instead, the statistical device used was a fractional factorial experiment, in which only a fraction of all the possible factor combinations was investigated. Because there were only 9 parameters to be estimated in the functional relation of these factors, it seemed that the minimum number of experiments consistent with a fractional design was 16. Thus, a 1/32 fractional factorial design was selected.

The application of this technique permits evaluation of the main effects, i.e., those resulting solely from the influence of a single given factor. However, a penalty is paid in terms of the decreased accuracy in the evaluation of those effects because they are confounded with other interaction effects which involve a large number of factors.

Table 5-3 shows the upper and lower values of the parameters used in the simulation. Table 5-4 indicates the combinations of upper and lower levels in each of the 16 "loops" in the main program.

5.3.2 Results

The relationships investigated were of two types. One consisted of an additive relationship of the form:

$$Y = a + b_1 x_1 + b_2 x_2 + \dots + b_n x_n \quad \dots\dots\dots \text{[Eq. 5.2]}$$

the other was expressed in the multiplicative form

$$Y = a\, x_1^{b1}\, x_2^{b2} \dots x_n^{bn} \quad \dots\dots\dots\dots\dots \text{[Eq. 5.3]}$$

where the x's represent the independent variables and Y the estimate of the dependent variable. Eight different transformations of the independent variables in the two forms were considered. Numerous criteria could be used to determine the best regression equation. In this study, the equation chosen is that which produces the largest cumulative multiple correlation coefficient. The square of this number indicates the proportion

TABLE 5
VALUES OF THE PARAMETERS USED IN THE FACTORIAL EXPERIMENT

Parameter (1)	Upper Level (2)	Lower Level (3)
1. Maximum average flow	0.9 MGD/square mile	0.7 MGD/square mile
2. Minimum average flow	0.7 MGD/square mile	0.5 MGD/square mile
3. Maximum average coefficient of variation	1.0	0.5
4. Minimum average coefficient of variation	0.5	0.25
5. Maximum serial correlation coefficient	0.75	0.5
6. Minimum serial correlation coefficient	0.5	0.25
7. Ratio of maximum demand to average flow	0.8	0.4
8. Ratio of minimum demand to average flow	0.4	0.2
9. Month in which peak demand occurs	9	6

TABLE 5-4
PARAMETER LEVELS USED IN EACH FACTORIAL COMBINATION[a]

Combination Number	Parameter Number[b]								
	1	2	3	4	5	6	7	8	9
1	L	L	L	L	L	L	L	L	L
2	U	U	U	L	U	U	L	L	L
3	U	U	L	U	U	L	U	L	L
4	L	L	U	U	L	U	U	L	L
5	U	L	U	U	U	L	L	U	L
6	L	U	L	U	L	U	L	U	L
7	L	U	U	L	L	L	U	U	L
8	U	L	L	L	U	U	U	U	L
9	L	U	U	U	U	L	L	L	U
10	U	L	L	U	L	U	L	L	U
11	U	L	U	L	L	L	U	L	U
12	L	U	L	L	U	U	U	L	U
13	U	U	L	L	L	L	L	U	U
14	L	L	U	L	U	U	L	U	U
15	L	L	L	U	U	L	U	U	U
16	U	U	U	U	L	U	U	U	U

---

[a] U denotes upper level; L denotes lower

[b] Enumerated on Table 5-3.

of the total variability of the dependent variable which is accounted for by the independent variables.

The final relationship chosen was:

$$\begin{aligned} v = {} & -194 + 305(\mathrm{DMAX}) + 251\,(\mathrm{SQPR}) + 71.2\,(\mathrm{CMAX}) \\ & + 5.05\,(\mathrm{DLAG}) + 118\,(\mathrm{DMIN}) - 88.9(\mathrm{WMIN}) + 46.7\,(\mathrm{CMIN}) \\ & - 143(\mathrm{PROB}) + 41.2(\mathrm{SMIN}) - 24.1\,(\mathrm{SMAX}) - 30.3(\mathrm{QMAX}) \end{aligned} \quad \ldots\ldots \text{[Eq. 5.4]}$$

where v is the required storage volume per square mile of drainage area of the watershed, expressed as acre feet/square mile.

Actual demands and streamflows are usually expressed in units of cubic feet per second rather than on a drainage area basis using acre feet/square mile. Also, the generalized cost curves developed by Cook express reservoir costs in terms of total storage, in acre feet. Equation [5.4] may be modified to express total storage volume as follows:

$$\begin{aligned} V = A\{ & [-194 + 251(\mathrm{SQPR}) + 71.2(\mathrm{CMAX}) + 5.05(\mathrm{DLAG}) \\ & + 46.7(\mathrm{CMIN}) - 143(\mathrm{PROB}) + 41.2(\mathrm{SMIN}) - 24.1(\mathrm{SMAX})] \\ & + 197(Z_1) + 76.8(Z_2) - 57.2(Z_3) - 19.6(Z_4)]\} \end{aligned} \quad .\ . \text{[Eq. 5.5]}$$

where V is the total storage volume in acre feet, A is the number of square miles in the drainage area, and Z_1 through Z_4 represent DMAX, DMIN, QMIN and QMAX, respectively, expressed in cubic feet per second.

This expression for required storage volume was incorporated into a general relationship for the cost of storage, as a function of the factors that affect storage cost. The cost of a reservoir of a given storage volume can be estimated from Cook's regional cost curves shown in Figure 5-3. The northwestern United States, designated as region B, was selected to illustrate the use of the cost equation.

The unit cost equation for region B is:

$$c = aV^b = 1800\ V^{-0.2} \quad \ldots\ldots\ldots\ldots \text{[Eq. 5.6]}$$

where V is the volume required. The total capital cost of the reservoir, C, is expressed as:

$$C = 1800\ V^{0.8} \quad \ldots\ldots\ldots\ldots \text{[Eq. 5.7]}$$

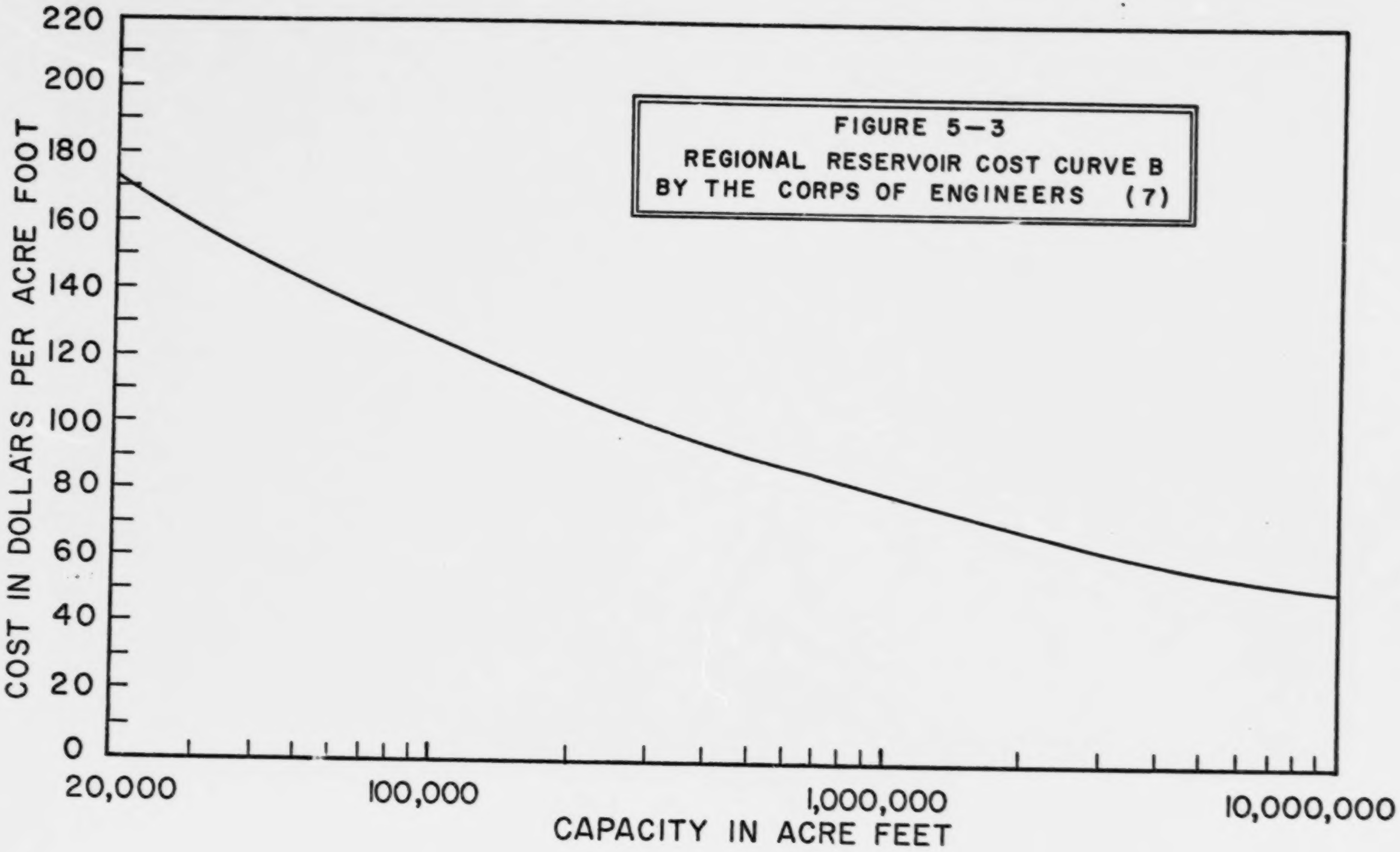

FIGURE 5—3
REGIONAL RESERVOIR COST CURVE B
BY THE CORPS OF ENGINEERS (7)

Substitution of equation [5.5] into equation [5.7] gives the generalized cost relation for region B.

$$\begin{aligned} C = 1800\{&A[-194 + 251(\mathrm{SQPR}) + 71.2(\mathrm{CMAX}) + 5.05(\mathrm{DLAG}) \\ &+ 46.7(\mathrm{CMIN}) - 143(\mathrm{PROB}) + 41.2(\mathrm{SMIN}) - 24.1(\mathrm{SMAX})] \\ &+ [197(Z_1) + 76.8(Z_2) - 57.2(Z_3) - 19.6(Z_4)]\}^{0.8} \qquad \text{[Eq. 5.8]} \end{aligned}$$

The results of this study permit the estimation of the required reservoir size in terms of the desired deficiency level and statistical parameters describing streamflows and demands. The cost of storage may then be obtained from existing generalized regional cost curves.

The regression equation accounts for roughly 80 percent of the variability of the response. The single most significant variable is the maximum monthly demand. This parameter, along with the square of the probability of no deficiency, and the maximum monthly correlation coefficient, account for the majority of the variability in the required storage.

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SECTION 6
THEORETICAL DEVELOPMENT OF SIMULATION MODEL

6.1 Introduction

In this chapter, the rationale, concepts and mathematical models for the simulation of water quality in a river system are developed. Emphasis is placed upon the theoretical aspects. The more practical aspects, such as how the problems are formulated for the computer and what input and output formats are used, are covered in Appendix A4, entitled "Users Instructions." Although the "Users Instructions" are a complete set of instructions and can be used independently, the reader should review this chapter carefully to gain the necessary insight to the overall simulation methods. Then, when the simulation model is being set up and used, this chapter may serve as a reference for and adjunct to the "Users Instructions."

The simulation model is made up of a group of mathematical models which are linked together by a programmed logic. The purpose of the model is to generate a reasonably accurate representation of the stream flow and oxygen balance in a river system. The value of the simulation model depends to a great extent upon the use of the high-speed digital computer for fast and accurate computations and application of logic. The speed of the computer not only provides the results within an acceptable time but also reduces materially the costs to obtain the results. In short, the computer is a necessary appurtenance in the use of the simulation model. For this reason, the development of models, techniques and logic is made for direct application of computer methods.

This chapter briefly discusses simulation as a method for analyzing the response of intricate systems and shows that simulation is, for the problem at hand, the method that is most likely to succeed in acceptable system representation. A portion of the chapter is devoted to the description of river system data needed for simulation and its preparation for use in the programs of the simulation model. Following this, the method for simulating river flows, flow regulation and water quality parameters is developed. Finally, brief discussions are presented on the subjects of sensitivity of variables and transfer functions, the latter for use as a rapid means of predicting the system response to an imposed set of conditions.

6.2 Simulation

In this age when designs are complex and involve many millions of dollars, it becomes necessary to make the best use of all available design methods. Two of the methods used in planning and testing are the techniques of modeling and simulation. Modeling is the representation of a system, or part of a system, in a mathematical or physical form to demonstrate the behavior of the system. Simulation involves development of the model and subjection of the model to various environmental situations to explore the nature of the results which are equivalent to, or in some way representative of, the results it is desired to investigate in the system.(1)

If it is possible to develop a mathematical model of a system that is both a reasonable representation of the system and is amenable to analytical solution, the mathematical model as a method is usually more precise, less costly and quicker than simulation. When a system is too complex to be described by a single, manageable mathematical model, the technique of simulation often can be used. Simulation involves the construction of an overall model which can be described by a number of interdependent mathematical models, each having behavior that can be determined analytically, and operating the overall model in such a manner that the interdependencies are recognized and accounted for so that it is representative of the system. Thus, a large complex system is represented by a series of interrelated mathematical models which are used in a natural order or sequence as directed by logic or operating rules to obtain the overall effect of the interaction of the individual models.(2)

The use of simulation as a tool for planning, construction and operation of complex systems has increased in geometric proportions in the past two decades. It is suggested that the real reason is attributable to the development of the digital computer. Simulation, no doubt, has led to the development of many complex systems, but also it has allowed better planning and analysis of systems that would have been built anyway. Additionally, it allows the analysis of existing natural systems to work out control devices and operating procedures for the best use of the natural system. Simulation is ideally suited to provide the information needed in the analysis of the economics of water use where multiple uses are in competition for available water. This is the use of simulation in this work.

The role of the digital computer in simulation is extremely important. Aside from the fact that the computer makes mathematical computations with unbelievable speed and accuracy, it is able to store and recall mathematical formulas and data in copious quantities. It is possible to formulate the coding so that parameter values can be changed during the simulation "run." A principal advantage in the use of the computer is its ability to simulate in "fast time"; i.e., the simulation by the computer proceeds at a much faster pace than the real system would operate. This advantage is obvious when it is the purpose of the simulation to generate hundreds of years of data, which the computer does in minutes. The whole program coding and data are contained in a manageable "deck" of cards or magnetic tape and in a matter of minutes simulation can proceed. Also, with proper attention to input data and parameters, the program may be used to simulate in a different environment.

Simulation is not a panacea for all analysts of complex systems. It has limitations. The method is similar to experimentation as a means of determining cause and effect. It takes a series of trials wherein the input parameters are varied to be able to predict the optimum solution. It is not possible to determine analytically the maximizing stationary point nor is it possible to look at the dual solution to determine the parameter sensitivity. To have arrived at the optimum solution in the first simulation run would be blind luck or would imply a relatively precise knowledge of the system. Even if the analyst is lucky on the first run, he probably would not recognize the results as being optimum until other runs show that it is. Thus, simulation is used as a replacement for experimenting with the real system because the latter is too inconvenient, too time consuming or too costly, or because it is not physically possible to create the test conditions in the real system.

As implied above, simulation is, compared to mathematical modeling, a relatively imprecise technique.(3) It provides data for statistical analysis rather than exact results and compares alternatives rather than generating the optimum. As a consequence, simulation is relatively expensive to use, particularly when the purpose is to determine the sensitivity of the values of the parameters which govern the model results.

Although the limitations of simulation might, at this point, seem to discount its usefulness, it is still a most powerful tool, having many advantages over alternative methods, if indeed there happens to be an alternate method. Goode and Machol(4) state that, "it is safe to say that no large-scale system would be constructed today without some simulation, and in a well-executed system design, simulation would be used continually."

The steps in a simulation study are: (1) development of the model, (2) preparation of the computer program and (3) design and performance of experiments and analysis thereof. The development of the model is covered in the balance of this chapter while the preparation of the computer program is described in Appendix A4 entitled "User's Instructions." The design and performance of experiments and analysis of results is the subject of Section 8, entitled "Application of Simulation Model."

6.3 Preparation for Simulation

The analysis of a system by simulation requires that the analyst be thoroughly familiar with the operating realities of the system and with the objectives of the study.(5) The analyst begins by establishing the overall real system. He then begins to break down the real system into components, being careful to maintain the continuity of flow with each step. The components are, in turn, broken down until each one can be expressed by a mathematical model which reasonably represents its real system counterpart. The linkage is maintained throughout this process. Operating rules must then be established to govern the functioning of the mathematical models. For example, if condition A exists in the simulated system, the model does operation C, but if condition B occurs, the model then shifts to operation D.

When all models, linkages and operating rules are ready, they must be tested to assure first that they operate, and secondly that the results are correct. Usually it is advisable to set up the various parts as subroutines. Testing begins on the individual subroutine and continues as subroutine after subroutine is added until the overall model is in satisfactory operation. Often it is necessary to use a desk calculator to check the results of one "pass" of the simulation to be assured that they are, in fact, correct. If there is available any information about the behavior of the real system, it is advisable to input the corresponding conditions and see if the model reproduces the behavior within acceptable limits.

In the formation of the simulation model, it is well to be aware of the admonishment of Hillier and Lieberman that "...the simulation model need not be a completely realistic representation of the real system. In fact, it appears that most simulation models err on the side of being overly realistic rather than overly idealized. With the former approach, the model easily degenerates into a mass of trivia and meandering details, so that a great deal of programming and computer time is required to obtain a small amount of information.

Furthermore, failing to strip away trivial factors to get down to the core of the system may obscure the significance of those results that are obtained."(5)

In the balance of the written work which follows, the thrust is toward the application of simulation to a watershed to create an acceptable approximation of river flow and water quality in that watershed. Accordingly, the topics will be: (1) simulation of the physical watershed, (2) simulation of stream flows in the watershed and (3) simulation of water quality in the watershed.

6.3.1 Simulation of the Physical Watershed.

A basic need for development of a watershed model is a topographic map. The U. S. Geological Survey topographic maps, called Quadrangles, covering 7 1/2 minutes of latitude and longitude on a 1:24,000 scale are admirably suited for this purpose. For large watersheds, i.e. several thousand square miles in area, the 15 minute Geological Survey maps may be better suited, however.

The first step is to determine the drainage areas of the watershed, the tributaries and upstream of each gage location. To do this, the ridge lines around the watershed and tributaries are drawn by interpretation of the contour lines on the map. The areas are then determined with a planimeter. Ordinarily, it is not necessary to outline and measure each branch. In a dendritic stream pattern, it usually is not necessary to separate out the drainage areas for more than the highest three orders of streams excepting where a lower order stream contains a reservoir of significant importance, in which case the areas up to and including the reservoir should be separated and measured. Areas tributary to gage locations are required, regardless of stream order.

The order of streams by Horton's classification (6) is determined by the following criteria. A stream having no tributaries is a first order stream. Where two first order streams meet, they form a second order stream. Where two second order streams meet a third order stream is formed. This system of classification is carried on for the watershed for which a simulation model is needed. The separation of more than the highest three orders of streams is tending toward over-realization and will result in little value gained for much additional detail.

The second step is to discretize the stream pattern by dividing it into "reaches." The stream system is continuously changing from its headwaters to its mouth; the flows increase, the slopes usually get flatter, the width and depth increase and the velocities generally increase. The changes in the stream are most apparent at the points where tributaries discharge and if the minor changes between tributaries are neglected, the stream is discretized. This establishes one of the criteria for establishing "reaches"; i.e., a reach is a portion of the stream between significant tributaries.

The reason for discretization of the stream into reaches is that it is necessary to assume the stream is made up of a number of connected segments in which each segment has constant properties. Reaches are therefore established so that at every major or significant change in the stream there is a new reach. Significant changes are, in addition to the confluence of a tributary, a definite change in stream bed slope, the discharge of a waste load and, because of its major affect on the stream, each reservoir must be considered a reach. A long section of stream having acceptable uniformity of properties should not be divided into two or more reaches because of its length. It is not necessary that reach points be established at gage locations.

Obviously, the selection of reaches is a place in the formulation of the watershed model where the tendency toward over-realization would be great. There is no real reason for limiting the number of reaches, excepting perhaps to meet the constraint of available machine storage, but more than 50 reaches is likely to be cumbersome to operate and analyze. The program coding is limited to a watershed having a maximum of 50 reaches.

An important step is the numbering of the reaches. Experience with the test watershed has indicated that it is best to number the reaches in the following manner. Beginning at the downstream end of the main stem, number consecutively upstream on the main stem. The selection of the upstream tributary which is the main stem is arbitrary but one, preferably the longest, should be "main stem." Following the consecutive numbering of the main stem reaches, begin at the downstream-most tributary and, without skipping any numbers, number the reaches of this tributary. Work upstream, tributary by tributary, until all reaches are numbered. The reach numbers should be a complete set of 1, 2, 3, ..., n where n is the total number of reaches.

The next step is to determine the hydraulic characteristics of each reach of the system. The rate of flow, Q, in the reach will have been determined by the hydrology simulation. It is necessary to determine the mean velocity so that the time of flow, needed in the quality simulation, can be determined. Additionally, if values of the reaeration velocity constant are not otherwise available, they may be computed knowing the mean velocity and mean depth of flow in the reach. The length of the reach and its average slope can be obtained from the map.

There are several ways to determine the needed velocity and depth. One way would be to select an average cross section in the reach and using a current meter, develop a rating curve in the manner described by Linsley, Kohler, and Paulhus. (7) The data for the rating curve can be used to formulate mathematical relationships between velocity and discharge and depth discharge so they may be concisely included in the program coding. It should be noted that the velocity-discharge and the depth-discharge relations, when plotted on semi-logarithmic paper, are straight lines. Theoretically, a minimum of two measurements at different flows would be enough to define the relationships.

Another method, involving a small amount of field work, utilizes any of the well-known open channel flow formulas. One of them is the Manning formula:

$$Q = AV = A \frac{1.486}{n} R^{2/3} S^{1/2} \quad . \; . \; . \; [\text{Eq. } 6.1]$$

where,

Q = The discharge rate, in cubic feet per second, (cfs.)

A = The area of the cross section, in square feet, (ft.^{2})

V = The mean velocity, feet per second (ft./sec.)

n = The roughness factor

R = The hydraulic radius = area of cross section/wet perimeter, in feet (ft.)

S = The slope of the hydraulic grade line = average slope in reach.

The cross sectional area is the product of the average width and the average depth, A=wd. For most river cross-sections, $R \cong A/w \cong d$. Equation 6.1 becomes:

$$Q = AV \cong wd\ (\frac{1.486}{n}\ d^{2/3}\ S^{1/2}). \ . \ . \ . \ . [\text{Eq. } 6.2]$$

and solving for d:

$$d \cong \left(\frac{Qn}{1.486\ w\ S^{1/2}}\right)^{3/5} . \ . \ . \ . \ . \ . \ . \ . \ . \quad [\text{Eq. } 6.3]$$

In this form, w, n and d are not known. If the shape of the cross section is such that w is approximately constant for a wide range of Q and if n can be considered constant, then:

$$d \cong K \left(\frac{Q}{S^{1/2}}\right)^{3/5} . \ . \ . \ . \ . \quad [\text{Eq. } 6.4]$$

where,

$$K = \left(\frac{n}{1.486\ w}\right)^{3/5} . \ . \ . \ [\text{Eq. } 6.5]$$

The value of w can be obtained by field measurements, or by scaling from the map if the river is large, to determine an average w for the reach. The value of n may be selected from compilations of photographs and descriptions for stream channels in which roughness coefficients have been determined.(8)

To obtain a feeling for the significance of an error by the assumption that w is constant, take the example Q = 2256 cfs., $S^{1/2}$ = 0.0292, A = 618 ft.^{2}, d = 6.18 ft., w = 100 ft. If w = 90 ft. instead of 100 ft., a 10 percent error, the value of d for the given values of Q and S, is d = 6.6 ft., which is 6.8% in error. The value for velocity is 3.65 ft./sec. for the given example. If the value of w is 90 then V = 3.81 ft./sec. which is 4.7% in error.

If the selected value of n is 10 percent too high, the computed value for d will be about 5.5 percent too high and for n 10 percent low, the value for d will be about 6.1% too low. The value of v will be 1/0.9= 11.1% high for n 10 percent too low and 9.1% low for n 10 per cent too high.

This method is presented without recommendation. It affords considerable savings in field work, which is traded for a possible loss in precision if the assumptions made are in error.

A third method, which involves no field work on the part of the investigator, can be used. It makes use of the studies of geomorphology carried out by the U. S. Geological Survey and reported by Leopold and Maddock(9). It was found that if the frequency of occurrences of discharge is the same at all points along a stream, then the relationships between width and discharge, depth and discharge, and velocity and discharge are constant for the stream. These relationships vary as some power of the discharge such that:

$$w = aQ^b \quad \text{[Eq. 6.6]}$$

$$d = xQ^f \quad \text{[Eq. 6.7]}$$

$$v = kQ^m \quad \text{[Eq. 6.8]}$$

where a, b, c, f, k and m are constants, w=width, d = mean depth and v = mean velocity. These relationships, which plot as straight lines on semi-log paper, hold for all ranges of discharge up to bankfull stage. It was also shown that because:

$$Q = \text{area x velocity} = wdv \quad \text{[Eq. 6.9]}$$

$$Q = aQ^b \times cQ^f \times kQ^m = ack\ Q^{b+f+m} \quad \text{[Eq. 6.10]}$$

it follows that:

$$b + f + m = 1.0 \quad \text{[Eq. 6.11]}$$

and

$$a \times c \times k = 1.0 \quad \text{[Eq. 6.12]}$$

In these equations, the constants b, f, and m are the slopes of the lines on the semi-log plot and a, c and k are the intercepts.

The values of the constants are determined at the locations of the gaging stations by obtaining the rating curve and cross section at each station from the U. S. Geological Survey. The depth-discharge relation is obtained by plotting the rating curve on semi-log paper. The width is obtained from the cross section, with width and discharge being related by corresponding depth values. The discharge divided by the product of width and depth gives the velocity.

The above description of methods for obtaining the hydraulic characteristics of the reaches of the watershed is not intended to be exhaustive. Use should be made of available data in the best manner possible. Resort to the latter method should be made only if no other supporting data are available. Keep in mind that the assumption is made that the discharge frequencies at all points are equal. For a large watershed, where the geographic features affect the distribution of rainfall and where base flows are significant and different in the tributaries, the use of these relationships may result in unacceptable error.

6.3.2 Simulation of Hydrology

An important factor in the simulation of water quality, and the effect augmentation of the flow rates has on it, is an accurate determination of how much water will be available and what frequency distribution it will have. Designers and planners need a reliable estimator of the mean flow and a knowledge of the variations that can occur in this mean flow. Designs of water regulating and water quality control structures are made to span economic time horizons of forty to fifty years or more. Therefore, the flow estimator and its variations must be known for at least an equal period.

Water which appears in streams originates from rainfall, often with little lag in time. The many conditions which must be met to cause rainfall make the occurrence of rainfall a stochastic process and, understandably, the occurrence of stream flow is also stochastic. Accurate numerical description of these stochastic processes is given in terms of statistical parameters determined by the analysis of replicate random samples. Therefore, to be able to describe stream flow accurately, the statistical properties must be determined from a number of random samples.

Few streamflow records span forty years, particularly in a developing watershed. Therefore, the available data barely provide one sample where eight to ten are needed to allow even small sample statistical procedures. The development of the digital computer, and the mathematical methods which extend its utility, provide a tractable means to generate additional samples. The assumption is made that the statistical parameters of the historical data are unbiased of the population of data. These parameters are statistically the same as the sample of historical data. The generated data are simulations of the historical data. The beauty and utility of the method lie in the fact that the computer can simulate hundreds of years of data in a matter of minutes and samples as good as the historical data may be selected from the output. The method does not increase the precision or confidence in the mean but surely it extends the confidence that can be placed in the variability of the data.

The stream-gage flow generator is described in a following section. A description of the preparation for its use follows below.

The primary source of streamflow data is from the U. S. Geological Survey which maintains many recording gages on streams and tributaries throughout the country. These data are made available as average daily flows, in cubic feet per second, in published form or, more appropriately for the use here, they are obtainable on magnetic tape.

A magnetic tape will contain all the available data for each of several stations and represents an attractive savings in the logistics of securing and preparing of these data for use.

Sometimes it is possible to obtain streamflow data from other sources. Some of the states maintain gaging stations and make the data available to the public. Additionally, industries, notably the power companies and others which use large quantities of water, maintain a few gaging stations.

To use the historical data as a basis for generating synthetic data by the method used in this work, it is necessary that there be a complete trace of data for each station, and that the span of years of the data be the same for each station. Also, it is necessary that the data be transformed to a normal frequency distribution, if they are not already normal. Data preparation therefore involves editing the raw data traces and filling any gaps, selecting the gages to be used to give the maximum length of data, while still using gage locations that adequately describe flows in the region, and selecting a normalizing transform. Three computer programs have been prepared to accomplish these tasks. The bases for these programs are discussed below. The computer programs and their use are described in greater detail in Appendix A4.

6.3.2.1 Edit and Fill

The raw data must be checked to determine that they are for the proper station, that the data for all stations have the same beginning time and ending time and that all data are in the proper time sequence. Data on cards usually will have the station number, year, month and a number 1, 2, 3 or 4, there being four cards to store one month's daily flows. A single station having 50 years of data will have 2600 cards. Data on tape must be printed out for editing and filling. It is obvious that this preparation is a formidable task. A computer program is included which reads the data from tape or cards, checks for the proper sequence of data, determines which data points are missing, fills them and outputs the information on tape, on cards or on the printed page for use. The program also computes average weekly gage readings, or average monthly gage readings, if instructed to do so by a control statement. The program is called "CHKDATA."

The procedure in filling missing data points is statistical in nature. When a data point is found missing, note is made of the day, month and station. The computer finds the mean and standard deviation of all other data for that day, month and station and computes the missing data points by the formula:

$$Q_{i,j,k} = \mu_{i,j} + \sigma_{i,j} r_k \quad . . . . \text{[Eq. 6.13]}$$

where:

$Q_{i,j,k}$ = the computed value of the k^{th} missing point for day i of the year and j^{th} station.

$\mu_{i,j}$ = the mean of all other data for day i of the year and j^{th} station.

$\sigma_{i,j}$ = the standard deviation of all other data for day i of the year and j^{th} station.

r_k = the random variable associated with the k^{th} missing data point.

6.3.2.2 Normalize.

A study of the frequency of occurrence of the daily average values of stream flow at any gaging station usually shows that the majority of the data values is less than the mean of all values. This is caused by a small number of very large values which increase the value of the mean to a value greater than the median. Where the mean is greater than the median, the distribution is said to be skewed right.

To generate gage flow data that are statistically identical to the historical data would require that the three statistical parameters needed to describe the distribution be duplicated in the generated data. Although a technique has been developed to consider skewness, it is generally not used in favor of transforming to normalize skewed data, then using a simpler equation which generates normally distributed data. Inverting the transformation then returns the skewness to the distribution of the generated flow.

To illustrate the difference in the two generating equations, the normal generating equation, in simplified form for one station, is:

$$(x_{i+1} - \mu_x) = \rho_x(1)\,(x_i - \mu_x) + \sigma_x \varepsilon_{i+1}(1-\rho_x{}^2(1))^{1/2} \qquad .\text{[Eq. 6.14]}$$

where:

x_{i+1}, x_i = flow at time points i+1 and i.

μ_x = mean flow.

σ_x = standard deviation of flow.

$\rho_x(1)$ = the lag-one serial correlation coefficient.

ε_{i+1} = a standard normal random deviate, (0,1).

Note that two parameters, mean and standard deviation, along with the serial correlation coefficient are needed to generate serially correlated normal data. To include the effect of skewness, the standard normal random deviate, ε_{i+1} must be replaced by:

$$\xi_{i+1} = \frac{2}{\gamma_\xi}\left(1 + \frac{\gamma_\xi \eta_{i+1}}{6} - \frac{\gamma_\xi{}^2}{36}\right)^3 - \frac{2}{\gamma_\xi} \quad . . . .\text{[Eq. 6.15]}$$

where:

$$\gamma_{\xi} = \frac{[1 - \rho_x^{3}(1)]\ \gamma_x}{[1 - \rho_x^{2}\ (1)^{3/2}]} \quad . . . . [\text{Eq. } 6.16]$$

where:

ξ_{i+1} = the new random component which is approximately gamma distributed (preserves skewness).

γ_{ξ} = the skewness of ξ.

γ_x = the skewness of the historical data.

η_{i+1} = standard normal random deviate.

$\rho_x(1)$= the lag-one serial correlation coefficient of the historical data.(10)

If the attraction of the simpler equation is not enough to direct the user to the transformation - normal equation - inverted transformation route, the requirement for normal data in order to apply the multivariate techniques used in the multi-station generator most certainly does. It is necessary, then, to consider normalizing transformations.

The normal distribution function is defined by:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)}{2\sigma^2}} \quad . . . \quad [\text{Eq. } 6.17]$$

where:

μ is the mean and σ^2 is the variance, parameters of the distribution. The probability that a variable is less than or equal to x is:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad . . [\text{Eq. } 6.18]$$

where t is a dummy variable.

If (x,u) are data points of a skewed distribution, then u = f(x) and the cumulative distribution function is:

$$P(x) = \Phi\ (f(x)) \quad . . . . [\text{Eq. } 6.19]$$

from whence:

$$dP(x) = p(x) = \Phi(f(x))\ df(x) \quad . . . . [\text{Eq. } 6.20]$$

and

$$P(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(f(x))^2}{2}} f'(x) \quad . . . . [\text{Eq. } 6.21]$$

The variable x is not normally distributed but the function f(x) is normally distributed. Theoretically, it is possible to find the function f(x) which transforms the skewed distribution into a normal one.(11)

The problem is to find a new representation of $u = f(x)$ which will result in u being normally distributed. In general, the function will be of the type:

$$f(x) = g(x) - g\ (u) \quad . . . . [\text{Eq. } 6.22]$$

where g(x) does not contain unknown parameters. The most likely candidate is:

$$g(x) = \log x \quad . . . . [\text{Eq. } 6.23]$$

which has been widely used in transformations of this type. Chow writes "It is generally found that hydrologic data of many kinds are lognormally distributed."(12) A log-normal distribution is defined as one in which the logs of the variable are normally distributed.

Other transformations which change the scale of the variable are possible. A few of them are:

$$\begin{aligned} g_1(x) &= (x)^n \\ g_2(x) &= \frac{1}{x} \\ g_3(x) &= \frac{1}{x+\alpha} \quad . . . . [\text{Eq. } 6.24] \\ g_4(x) &= \log\ (x+\alpha) \\ g_5(x) &= (x+\alpha)^n \end{aligned}$$

In general, it is better not to introduce additional parameters so that the distribution is described fully by the mean and variance.

In this work, six transformations have been selected which give a range of scale changes in an effort to obtain, by trial, the transform that normalizes. The transforms are in two forms:

$$g_1(x) = (x)^n \quad . . . . [\text{Eq. } 6.25]$$

where $n = 0.25$, $n = 0.5$, and $n = 0.75$;
and,

$$g_4(x) = \log(x+\alpha) \quad . . . . . . [\text{Eq. } 6.26]$$

where $\alpha = 0$, $\alpha = 0.25$, μ and $d = 0.5\mu$.

The next problem is to work out a technique to determine if the transformed data are indeed normally distributed and, if more than one transform normalizes, which is the best. This is a problem in nonparametric statistics and involves determining how well a set of data approximates a known distribution. The classical test is the χ^2 test for goodness of fit. An alternate test, the Kolmogorov-Smirnov test, is said (13) to be more powerful than the χ^2 test. The latter test is used here.

The Kolmogorov-Smirnov test proceeds by letting F(x) be the cumulative distribution function (cdf) of a completely specified theoretical cdf, the normal cdf in this case. Let $S_n(x)$ be the cdf of the sample based upon n observations. Form the difference:

$$D(x) = \max\ (F(x) - S_n(x)) \; . \; . \; . \; . \; . \; . [\text{Eq. } 6.27]$$

where D is the maximum difference between the distributions, which occurs at x. This maximum difference, D(x) is compared against tabulated $^{(14)}$ values for various levels of significance. For values of n greater than 35, the critical value for level of significance, $\alpha = 0.05$, is given by:

$$d_a^{(n)} = 1.36/\sqrt{n} \; . \; . \; . \; . \; . \; . \; . \; . \; . [\text{Eq. } 6.28]$$

which is compared to D(x) in terms of relative frequency. If the maximum difference in relative frequencies is less than $d_a^{(n)}$, the conclusion is made that the data are normal.

If more than one of the transformations is shown by the test to be normal, it has been assumed that the one having the smaller value of D(x) is the "better" transform and should be used.

6.3.2.3 Selecting Basis Gages.

A basis gage is one for which historical data are used to develop the parameters for generating synthetic gage data, and the synthetic gage data of which is used to generate the stream flow. A gage selected for one use automatically becomes used for the other. Where there is the possibility of selection, i.e. where the watershed contains more than five to seven gaging stations, the selection of basis gages is of importance. Obviously, where gages in the watershed are few and the records are short, all available data should be used and the problem of selection is non-existent.

The need for selection of basis gages is most apparent when the watershed contains more than ten gages. The programming in this work limits the number to ten. A less obvious reason is that a significant portion of the simulation program length results from storage of gage parameters and from their computation and use. The elimination of unneeded gages requires less machine storage and would shorten computation time, both of which may be desired. Of course, the savings in storage can be realized only if the proper DIMENSION statements are modified to show the true number of gages used. Gages should be eliminated only if the simulation program is constrained by limits on machine storage and computation funds and only if it can be demonstrated, as described below, that considerable interdependence exists between gages.

A brief insight to the method of computing stream flows illustrates the rationale used in the reduction of the number of basis gages. The computation of stream flows at reach points is a process of interpolation and extrapolation of the synthesized gage data. The assumptions are made that: (1) the flow at a gaging station is derived from the watershed upstream with each unit of area upstream contributing to the flow; and (2) the flow at any reach point is a linear combination of the gage flows for gages nearest that reach. The weight factors in the linear combination are functions of the watershed areas upstream of the gages and of the reach points. A gage located well upstream on a long tributary leads to a considerable extrapolation of the gage data in computation of stream flows for the mid-reaches of that tributary. A spring near the gage may result in a flow at the gage having little variation which, when extrapolated downstream, would exhibit a steadiness of flow that is not realistic. This gage should not be used as a basis gage when the gage configuration is such that data of this gage are extensively extrapolated in computing flows in reaches some distance downstream.

With only a few modifications, the routine which develops the transformation matrix used to compute reach point flows from gage data can be used to aid in the selection of basis gages. The process is essentially trial and error and careful thought and analysis can lead to the proper selection with only a few trials. The method involves using the historical data available for each gage, selecting several gages as basis gages and the others as "estimate" gages. The routine uses the basis gage data to compute the flow at the estimate gage location and then compares the computed data with historical data for that estimate gage. The correlation between the historical data and the estimate data for a gage indicates the degree of interdependence of the two. If the correlation is high, it can be concluded that the estimate gage would provide nearly the same information as the basis gage and therefore that estimate gage can be eliminated.

In general, basis gages should be selected from among those gages having the longest record and the largest upstream area provided, of course, that the likely candidate is not materially affected by upstream regulation.

Computational methods and details of the computer routine used in the selection of basis gages are described in Appendix A4.3.

6.3.2.4 Transforming Synthetic Gage Data.

The stream flow at any reach point in the watershed is a linear combination of the generated gage flow data and weighting coefficients which are functions of areas upstream of the gages and reach points. The assumptions made are set forth in the sub-section above. The computation is programmed and the only preparation needed is to determine the upstream areas corresponding to all reaches and gages. These reach numbers, gage numbers and areas are inputs to subroutine TRAN which computes the transformation matrix, the elements of which are the weight coefficients.

The method of computation for the five possible gage-reach location combinations is set forth in detail in Appendix A4.5.

6.3.2.5 Formulating Parameter Inputs

There are three ways to handle parameter and data inputs to the simulation program. The first, and preferable way, is to formulate the relationships in mathematical terms and equations and control their use by a set of programmed operating rules. The second way is to enter the relationships in array form and control the selection of the appropriate value by a set of programmed operating rules. This second way requires more machine storage. The third way is used where the data inputs cannot be represented by programmed relationships, in which case the data are placed on punched cards and read in as required.

Two programs have been written to fit mathematical relationships to data. One program is for data which show periodicity, while the other fits either a polynomial of degree up to 7 or exponentials of the type:

$$f(x) = Ax^B \quad . . . . \quad \text{[Eq. 6.29]}$$

$$f(x) = Ae^{Bx} \quad . . . . \quad \text{[Eq. 6.30]}$$

These programs are described in detail in Appendix A2.

6.3.2.6 Regulating Flows

Regulation, as referred to in this work, is the change in natural stream flow due to the operations of man. Regulation therefore includes the change in flow caused by increasing or decreasing the storage in a reservoir or lake, evaporation of water from the surface of a reservoir or lake, diversion of water out of the river basin or to another reach not immediately downstream from the point of regulation, irrigation withdrawals from the river and any other withdrawals or discharges for municipal and industrial use.

The regulation effects of a reservoir are governed by the operating rules which translate reservoir or flow conditions into releases. When the release rules are well defined and inclusive of all conditions, it is not difficult to program them for simulation. The principal variable affecting the choice of the appropriate operating rule is the current elevation of the water surface. For instance, in a multi-purpose reservoir, if the water surface elevation is above the lower level of the flood control pool, the rule may specify both the release of the surplus water and its rate of release.

To simulate the river system, it is necessary to maintain an inventory of water stored in each reservoir. The change in storage is equal to the volume flowing in, less the volume flowing out, and less any losses during the time frame. The change is added to the total volume stored in the previous time frame to obtain the new volume. A change in storage affects the level of the water surface which, in turn, affects the releases. The relationship between volume of water stored and the corresponding depth must be known. Ordinarily, this relationship is obtainable from the owner of the reservoir and is often furnished either as a curve or in tabular form. The relationship should be converted to a polynomial or exponential form for programming.

An inevitable loss from a reservoir is that due to evaporation. The magnitude of this loss is a function of the evaporation rate and the surface area of the reservoir. For a given reservoir, the surface area is a function of the volume of water stored and it is necessary to determine this relationship. Surface area-depth curves or tables often are available from the reservoir owner or can be determined from maps containing water depth information.

The relationship also can be converted to a polynomial or exponential form.

The evaporation rate is dependent upon the heat budget in the reservoir and generally is periodic in nature, with an annual primary frequency. Often monthly or weekly evaporation rates, expressed in inches per unit of time, are available from the reservoir owner, U. S. Geological Survey, Soil Conservation Service, Forest Service or from local water utilities. These data can be fitted by a trigonometric series to obtain the parameters needed for mathematical expression of the evaporation rate - time of year relationship. The program FITCRV (see Appendix A2) can be used to determine the necessary parameter values. Usually, only one, or possibly 2 or 3, harmonics will be necessary for a good fit of evaporation data.

Diversion of water is another factor in the regulation of streamflows. Diverted water usually is obtained from a reservoir or impoundment. The loss to the watershed is accounted for by direct subtraction of the diverted water from the flow into the reservoir. It is necessary to obtain data on the demand for diverted water. These data usually are amenable to fitting by a periodic equation or, sometimes, by a polynomial. Long term trends usually can be represented by a polynomial of low degree which changes the mean term in the periodic equation each primary period during simulation.

Where water is stored for use in power generation, either for hydroelectric power or for cooling water in steam plant generation, there is regulation of the stream flow that must be considered. The relationship between rate of use and time should be determined and a mathematical model set up for ease in programming for simulation. The variety of possible conditions precludes making any specific statements or recommendations.

Similarly, withdrawals from the river system for irrigation and other uses, and/or discharge to the river, are peculiar to a given system and must be programmed the best way possible. Consideration should be given to neglecting all except the very significant changes in flow rate due to withdrawals and discharges.

6.6.3 Preparation for Simulation - Water Quality

The simulation of water quality in this work is limited to those parameters having to do with oxygen balance in the stream. Other quality parameters, conservative or non-conservative, can be simulated by the same techniques used here. Only the mathematical models describing the interactions need be changed.

The oxygen balance in the stream is dependent upon the biochemical degradation of waste loads discharged to the stream, physical factors, such as sedimentation of the solid fraction of the waste load and reoxygenation to replenish oxygen used, and biological factors such as the production of oxygen by the plant and animal life in the stream. Almost every physical, biological and chemical relationship in the stream is temperature dependent. It therefore is necessary to account for temperature and its time dependent variations so that those factors which are significantly affected by temperature are properly represented.

6.3.3.1 Temperature

For many rivers, stream temperature data are not routinely collected in a systematic program, as are stream flow records. For this reason, it may be difficult to find adequate temperature records for streams in a developing river basin. Temperature data are important to the pollution control agencies, fisheries bureaus and the electric utilities' stream generating plants and therefore these agencies may be a source or records.

Although water temperature variability is less than that of air, there are many factors which affect water temperature and, indeed, considerable variability has been observed. It has been reported (15) that considerable variation in stream temperature occurs due to the exposure of the stream to the sunlight. Stream sections exposed to direct sunlight several hours per day are heated more than those which are shaded much of the day. Shallow, fast-flowing streams heat up more quickly than slower, deeper streams when the air temperature is higher than the water and cool more quickly when the air is cooler than the water. Reservoirs, particularly those which stratify, have a considerable moderating effect on river temperatures and, in general, tend to result in cooler temperatures when long term averages are determined (16).

Ideally, a stream quality simulation project would be provided with several years of temperature records for several locations in the watershed. These records could be fitted with time-dependent mathematical expressions and reach-indexed to account for the variation with time and along the stream. If the temperature record is lesser in extent than this ideal, a combination of techniques and assumptions will be needed to make the best use of available data. The use, for instance, of a time frame, or averaging interval, of one week instead of one day precludes refinements to account for variations having periods of less than one week.

Ward (16) found that the annual variation in stream temperature could be described by the simple sine function:

$$T(L) = a \sin(bx+c) + \bar{T} \quad . \; . \; . \; . \quad [\text{Eq. } 6.31]$$

where T(L) is the temperature corresponding to the time angle, b; a is a constant representing the maximum amplitude of the variation; c is the phase angle to account for the occurrence of the peak at time other than $bx = \pi$ and $\bar{T}$ is the mean annual temperature. Ward also found that the average annual temperature does not vary appreciably from year to year and further, the analysis of the twelve monthly average temperatures yields almost the same equation as the analysis of the 365 daily temperature readings. There are, however, variations in temperature (16) and these variations can be simulated by use of the formula:

$$T_i = T(L) + \sigma_T R_i \quad . \; . \; . \; . \; . \; . \quad [\text{Eq. } 6.32]$$

where T_i is the estimated temperature in the i^{th} time interval, T(L) is the temperature for the L^{th} week of the year corresponding to the i^{th} time interval, σ_T is the standard deviation of the historical temperature data and R_i is a standard normal random deviate.

If the simple sine curve, [Eq. 6.31], does not adequately represent the temperature-time relationship in the stream, it may be possible to fit a periodic curve having several harmonics to the historical data. The method is described in Appendix A2 "Curve Fitting Techniques." The technique described in Appendix A2 fits periodic data having a shape which deviates from that of the classical sine curve. For instance, using this technique, a mathematical expression can be developed to fit historical data which exhibit several months of constant temperature, say 0°C as would be the case in the northern streams, with otherwise typical variations for the balance of the year.

An important temperature-dependent factor is the limit of solubility of atmosphere oxygen in water. Solubility also is dependent upon the concentration of dissolved solids (17). The solubility of oxygen decreases slightly with an increase in dissolved solids, but this decrease is neglected in this work. A mathematical relationship has been developed for the solubility of oxygen in distilled water (18):

$$\text{D.O.} = 14.652 - 0.41022T + 0.0079910T^2 - 0.000077774T^3 \; . \; [\text{Eq. } 6.33]$$

where D.O. is the saturation concentration in mg/l of dissolved oxygen at temperature, T, in degrees centigrade. This formula has been "rounded off" to:

$$D.O. = 14.65 - 0.41T + 0.008T^2 - 0.00008T^3 \quad \ldots \quad [Eq.\ 6.34]$$

for computations in this work.

6.3.3.2 Velocity Constants

Two processes operating simultaneously produce a balance in the concentration of dissolved oxygen when organic pollutional material is introduced into a river. The deoxygenation process uses available atmospheric oxygen which has been dissolved in the river water while the reoxygenation process replenishes this dissolved oxygen supply from the atmosphere.

The deoxygenation process is biochemical in that microscopic organisms utilize the organic material for their life processes and in so doing oxygen is used and organic material is stabilized. The rate at which oxygen is used is dependent upon several factors: temperature, availability of the organic material as food for the organisms, mixing, presence of rocks upon which growth is localized and many others. It is seen that these factors are properties both of the nature of the organic material and of the stream. The rate of deoxygenation is characterized by the velocity constant, K_1, defined as the ratio of oxygen demand satisfied in a unit of time to the oxygen demand present at the beginning of that time. The reaction is assumed to be first order; that is, the rate of satisfaction of oxygen demand is proportional to the remaining oxygen demand.

The reoxygenation process is physical in that only physical processes are involved in the solution of oxygen from air at the water surface and the transport of this oxygen into the water volume. Henry's Law, "The weight of a given gas that dissolves in a fixed quantity of a given liquid, at constant temperature, is found to be directly proportional to the partial pressure of the gas above the solution"(19) governs the solution. As soon as the oxygen concentration in the water decreases, oxygen is obtained from the air, a supply that is relatively constant. The oxygen dissolved at the surface is carried into the water by advection, convection and/or molecular diffusion. The rate of reoxygenation therefore must be a function of the rate of solution and the rate of transport of the dissolved gas. The overall rate of reoxygenation is characterized by its velocity constant, K_2, which is defined as the constant of proportionality in the relationship:

$$\frac{dc}{dt} = K_2 (c_s - c_t) \quad \ldots \quad [Eq.\ 6.35]$$

in which the rate of change in concentration of dissolved oxygen, dc/dt, at any time, t, is proportional to the degree of undersaturation, $c_s - c_t$, at that time.

The concentration of dissolved oxygen (DO) in a stream carrying an organic pollutional loading is dependent on the interaction of these two processes. The DO concentration in a river system is not only dependent upon the amount of organic material and the amount of water in the stream but also upon the rates which the oxygen demand is exerted and the oxygen is replenished. Most of the mathematical formulations of the oxygen concentration as a function of time (also distance in a flowing stream) utilize the velocity constants, K_1 and K_2, as parameters. The exception are those formulations based upon regression of "cause and effect" data (20,21) which require a considerable supply of data. It is necessary, then, to determine values of K_1 and K_2 to model the oxygen balance in a river system.

The traditional method for determining K_1 (22) is to determine the biochemical oxygen demand (BOD) of the waste in the standard method (23) excepting that several BOD-time pairs are determined to obtain the BOD-time relationship. Analyses of relationships to find the value of K_1 can be done in several ways (24 25, 26). There is considerable evidence (27, 28, 29, 30), however, that the value of K_1 determined under quiescent conditions in the laboratory is not representative of the K_1 that would result in streamflow conditions even if corrected for the temperature difference. Quantitative results that allow evaluation of the factors affecting stream values of K_1, however, are not available. The procedure used in this work is to evaluate K_1 in the traditional manner, i.e. using the BOD bottle technique, and to add an error term, r, which is described below.

The state of the art of evaluation of K_2 is much the same as the evaluation of K_1. The physical nature of reaeration makes the problem a little more amenable to analysis and there has been considerable work done recently, hopefully leading to a better evaluation of K_2 (31, 32, 33, 34, 35). In the latest of these, Thackston and Krenkel propose the predictive formula:

$$K_2 = 0.000125 \left(1 + \left(\frac{\bar{u}}{\sqrt{gh}}\right)^{1/2}\right) \frac{S_e g}{h}^{1/2} \quad . . \text{[Eq. 6.36]}$$

where $\bar{u}$ is the mean velocity, g is the gravitational constant, h is the mean depth and S_e is the slope of the energy grade line. Other formulas are of the general form:

$$K_2 = \frac{a\bar{u}}{h^b} \qquad \text{[Eq. 6.37]}$$

The formula of Langbein and Durum (33) was shown to fit the river and laboratory data of many of the previous investigations. This formula is:

$$K_2 = \frac{3.3\,\bar{u}}{h^{1.33}} \qquad \text{[Eq. 6.38]}$$

Previously, the method used to estimate K_2 was to determine the deoxygenation constant, K_1, usually by laboratory methods, and using this value of K_1, fit a predicted oxygen sag curve to an observed sag curve by varying K_2 until a satisfactory fit is obtained. This method requires a stream survey to obtain an observed sag curve. It would appear, with the advances being made in estimating K_2, a reverse procedure, i.e. estimating K_1 from an observed sag curve and K_2 value, might be more appropriate.

In a system as complex and variable as a river, it can be expected that other discernible factors affect the overall oxygen balance. Factors which have received attention (36, 37, 38,39) are the removal of solid BOD fraction from the water of the stream by sedimentation, the addition of BOD to the water by the bottom sludges, the removal of BOD by slimes or growth on rocks and by rooted plants and the alternate addition and use of DO by aquatic plants in the photosynthetic process. It is common practice to refer to the rate of removal by sedimentation using K_3, a third rate constant. While there is considerable evidence that these other factors are present, there have been few data reported which allow use of the relationships to obtain numerical results. A few data for a specific river system are presented by Camp (38).

Although the original oxygen balance relationships set forth by Streeter and Phelps (22) considered only deoxygenation by organic material in solution and suspension and reoxygenation only from the atmosphere, these investigators recognized that other factors were present. They chose to include the effects of these other factors into the overall deoxygenation and reoxygenation constants. A similar method recently was proposed by Moreau and Pyatt (40).

They write "In lieu of accounting for additional factors individually, as proposed by Dobbins and others, with attendant difficulties of parameter estimation, for purposes of this study, it is sufficient to lump added factors for BOD and DO into 'ignorance' or 'error' terms that can be described as random variables." They add a factor r, called deoxygenation error term, to the general deoxygenation equation (see section 6.6 below) to get:

$$\frac{dL}{dt} = -K_1L + r \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.39]}$$

where L is the BOD concentration and t is time. Similarly, they add a factor s, called the reoxygenation error term, to the general oxygen sag (DO deficit) equation to get:

$$\frac{dD}{dt} = K_1L - K_2D + s \quad \ldots\ldots\ldots \text{[Eq. 6.40]}$$

where D is the DO deficit. If r and s are considered to be constant over the integrating time interval, then integration of equations [6.39] and [6.40] gives, respectively:

$$L = (L_o - \frac{r}{K_1})\, e^{-K_1t} + \frac{r}{K_1} \quad \ldots\ldots\ldots \text{[Eq. 6.41]}$$

and,

$$D = \left(\frac{K_1L_o - r}{K_2-K_1}\right)(e^{-K_1t}-e^{-K_2t}) + \left(\frac{r+s}{K_2}\right)(1-e^{-K_2t}) + D_oe^{-K_2t} \quad \text{[Eq. 6.42]}$$

where L is the BOD at time of flow t after the BOD was L_o, D is the DO deficit at time of flow t after the initial conditions of L_o and D_o and other variables are as previously defined. These equations are based upon steady-state, uniform flow and unvarying "constants" for the integrating time interval and their use describes the river conditions in a series of discrete steps to approximate continuously changing (with respect to time and distance) river conditions.

If stream quality survey results are available, it is possible, using equations [6.41] and [6.42], to determine values of r and s and find their means and standard deviations. Then the values of r and s to be used in equations [6.41] and [6.42] for predicting stream conditions would be r and s where:

$$\hat{r} = \bar{r} + \sigma_r R \ldots\ldots\ldots\ldots \text{[Eq. 6.43]}$$

$$\hat{s} = \bar{s} + \sigma_s R \ldots\ldots\ldots\ldots \text{[Eq. 6.44]}$$

where $\bar{r}$ and $\bar{s}$ are observed mean values of r and s, σ_r and σ_s are the standard deviations of the observed values of r and s, and R is a standard normal random deviate.

Mention also should be made of the work of Kothandaraman (41) who considered K_1 and K_2 to be random variables. The work of Kothandaraman and Moreau and Pyatt was extended by Nicolson (42) who presented equations for expected values and variances for L and D with K_1, K_2, r and s considered as random variables.

It is not intended in this work to present an exhaustive and authoritative discussion of the evaluation of K_1, K_2 and other factors, with specific recommendations as to methods to use. The simulation model is set up to receive parameter inputs for K_1, r and s which must be estimated or determined by the user for the specific application. The values of K_2 are computed using the Langbein and Durum formula (Eq. 6.38] for each reach. The values of K_1 are waste load-indexed while r and s are reach-indexed. It is a simple matter to convert K_1 to reach-indexed if desired.

6.3.3.3 Waste Loads.
The quantitative aspects of the water quality simulation are obtained by placing waste loads on the system. Waste discharges constitute a significant change in the stream conditions and therefore they enter the upper end of a reach by reason of the fact that reaches are delineated by the significant changes in the river system. The assumption is made that waste discharges become completely mixed at the point of entry to the stream.

The waste load data needed for the simulation model are the reach location of the discharge, the rate of discharge, and the BOD and DO concentration of the waste. The reach location is the geographical location of the waste discharge point in terms of the reach index system described previously. Where two or more discharges are made close together and the stream conditions do not otherwise change enough to warrant division into reaches, the discharges can be considered to occur at one point. The rates of discharge are added and the concentrations are adjusted to their weighted means.

Waste load data usually are computed from predictions of future population, from planning reports, and from projections of future activity by industries. Typically, the data are extrapolations of recorded data, or possibly an enthusiastic guess of planners, and are often later proven to be not very accurate. One of the beauties of simulation is that quickly, and relatively inexpensively, several levels of growth and expansion can be tried and a feeling for the sensitivity of the resulting stream conditions to population change can be obtained.

Waste loads are usually variable, having hourly, daily, weekly and monthly fluctuations, and, because of growth, a long term trend. The fluctuations having periods less than the averaging interval, as described below, need not be considered. The fluctuations having periods greater than the averaging interval can be described by fitting a mathematical equation or by time-indexing the data. Long term trends usually can be expressed by a polynomial of low degree or by programming increments to be added in the appropriate "do loop."

6.3.4 Preparation for Simulation - Time Scale

There are two considerations with respect to time that must be dealt with in preparing a model for water quality simulation. First, it must be decided how the time scale is to be represented during the simulation run and,secondly, it must be decided how many years of simulation are needed to obtain the data required to substantiate the decisions that will be made.

6.3.4.1 Representation of Time Scale

Simulation as a technique for analysis may be either continuous with respect to time or sequence of events, or it may be step-oriented with respect to time or sequence of events. Usually, continuous simulation complicates the mathematical representation by the necessary inclusion of the time variable. The simulation of a continuous system can be approximated by a series of discrete simulations assuming steady-state conditions over a time interval. Naturally, the shorter the time interval, the closer the approximation is to the continuous simulation. Digital computers are unable to handle the continuous problem, except as an approximation using discrete steps with accuracy achieved by taking small step (time) increments.

River systems are, in the strict sense, continuous and unsteady with respect to time in all aspects; flow, quality, temperature, biological life, to name a few. Because river systems are so complex, mathematical representations are usually made in terms of steady-state conditions and changes are considered as step changes. Such a method is required when the system is simulated using a digital computer.

The success and usefulness of the simulation of river systems depends considerably upon the length of the time step selected. Although a short time step or averaging interval will more nearly approximate system conditions, it imposes the burden of providing the mathematical model and data needed to develop such precision. A short averaging interval also produces more data to analyze; for example, a weekly averaging interval produces one-seventh as much data as a daily averaging interval.

The selection of the averaging interval should be governed by consideration of what time interval will allow indication of the significant variations in system response while entailing a minimum of detail. If the objective of the simulation is to determine if the low flow criterion of the minimum flow for seven consecutive days at a frequency of once in 10 years, as has been adopted by many states (43), is met under the simulated conditions, then there is little to be gained by using an averaging interval shorter than seven days. Further, if to make an operations change in a treatment plant or a regulating structure to obtain reasonably steady conditions at the new level of operation requires a lead time of several days, the averaging interval should be not less than the lead time required. Also, shorter averaging intervals require more computer storage to program and increase the cost of simulation runs.

The computer program set up in this work employs an averaging interval of one-forty eighth of a year, which is 7.6 days. A monthly averaging interval may possibly miss a 7 day period of low flow and attendant critical conditions, whereas a daily averaging interval would increase the detail needed for data, the cost of a computer run, the program storage required, and the amount of output to analyze. The averaging interval of 1/48 of a year, rather than 1/52 normally defined as one week, offers nominal advantages in computing periodic functions and in programming for the computer.

6.3.4.2 Duration of Simulation

The selection of the length of time for simulation depends upon the use to which the results are to be put. Simulation of water quality in a stream may be time-varying or time-independent when considered from the standpoint of duration. If one of the purposes of simulation is to establish the time when an event will occur, or a change is needed, then it is necessary to input time-varying data to represent the growth and/or changes in the system. If the purpose is to determine the state of the system at a given time in the future, the inputs will be time-constant and of the level that are estimated to exist at that time.

The time-varying simulation allows the determination of how time affects the river system and when changes are needed to maintain water quality goals. It is necessary to input waste load data that reflect the increased discharge rates and organic loadings that occur as a result of population growth and industrial development. Anticipated significant changes in water use and regulation also must be considered. These changes must be programmed into the simulation model. Each simulation run then will be one sample of river conditions for the selected period, probably thirty to fifty years. It will be necessary to make ten or more such simulation runs to obtain the information needed to establish the relationship between the degree of change and the time when the change should be made.

If it is desired only to determine the state of the river system at some given future time, the simulation need is much more simple. The input data and information must be those estimated to exist at that future time and therefore it is not necessary to program changes into the simulation run. In this case, each year of simulation is a sample of the result,and the conclusion to be drawn as to the state of the system at that future time can be obtained by statistical analysis of the samples. Thirty or more samples, i.e. years of simulation, would be appropriate.

6.4 Synthesis of Gage Data

Simulation of water quality in a river system requires that data be available from which stream flows at all reach points can be determined. These data must faithfully represent the stream flows that can be expected to occur at the times and places encompassed in the simulation.

The lack of adequate stream flow data is one of the major reasons for resorting to simulation for studies in the areas of water regulation and water quality control. Methods have been developed to generate or synthesize stream flow data which are statistically identical to the available recorded flows and are therefore considered to be estimates obtained from the population of stream flows. Further, methods have been developed to simulate stream flow gage data that are not only serially correlated for faithful representation of the observed serial correlation at a single gaging station and cross correlated between stations for faithful representation of basin-wide observed conditions, but also cross and serial correlated for faithful representation of basin-wide conditions in multiple time lags.

The method used in this work is classified as a multivariate Markovian gage data generator. The basic method was developed elsewhere, as reported in the literature (44, 45, 46), but has been modified and extended in this work.

Multivariate indicates that more than one trace of data is generated and Markovian indicates that the value of each variate at a given time is dependent upon, or correlated with, the value of that variate in the first preceding time interval but is independent of any values previous thereto. It is assumed that the interval in which serial correlation occurs is one month, to correspond to the observed natural phenomenon that serial correlation of daily and weekly average flows exists for periods of up to one month but are unreliable for longer periods.

The method for synthesizing gage data is described in detail below. The programming, inputs and outputs and program coding for the generating process by digital computer are contained in the Appendix A4.

It should be understood at the outset that the goal in the generation of synthetic streamflow data is to develop sequences of data for each stream gaging station for which historical data are used as a basis. In this work a gage, the historical data from which are used to compute the parameters for generating flow data, is defined as a "basis gage." Although stream flow data at all points along the river system are needed for the water quality simulation, the first step is to generate the gage data. The generated gage data are then transformed to stream flow data.

As stated above, the generated gage data are developed so as to be statistically identical to the historical data. The development in this section assumes that the data are normally distributed or that they are transformed so as to be normally distributed. See 6.3.2, above. The model therefore needs only to preserve the mean and standard deviation of the historical data in order to preserve the statistical identity.

The historical data are analyzed statistically to determine the mean, standard deviation, and the multiple-lag correlation coefficients for each basis gage and for each time period, j; the period being equal to the averaging interval.

The historical data can be shown in matrix form as:

$$\left[\begin{array}{ccccc} X_{1,1} & X_{1,2} & \cdots & \cdots & X_{1,m} \\ X_{2,1} & X_{2,2} & \cdots & \cdots & X_{2,m} \\ \vdots & \vdots & & & \vdots \\ X_{j,1} & X_{j,2} & \cdots & \cdots & X_{j,m} \\ \vdots & \vdots & & & \vdots \\ X_{48,1} & X_{48,2} & \cdots & \cdots & X_{48,m} \\ \vdots & \vdots & & & \vdots \\ X_{p+1,1} & X_{p+1,2} & \cdots & \cdots & X_{p+1,m} \\ X_{p+2,1} & X_{p+2,2} & \cdots & \cdots & X_{p+2,m} \\ \vdots & \vdots & & & \vdots \\ X_{p+j,1} & X_{p+j,2} & \cdots & \cdots & X_{p+j,m} \\ \vdots & \vdots & & & \vdots \\ X_{p+48,1} & X_{p+48,2} & \cdots & \cdots & X_{p+48,m} \end{array}\right] \begin{array}{l} \overline{} \ \ldots \ [\text{Eq. } 6.45] \\ \\ \\ \text{Year 1} \\ \\ \underline{} \\ \\ \overline{} \\ \\ \\ \text{Year K} \\ \\ \underline{} \end{array}$$

where p = 48K and m is the number of stations. The value of $\mu_{i,j}$, the mean flow at station i for the j^{th} "week" of the year, is given by summing all the elements having a j index in the column corresponding to the i^{th} station. Similarly, the values of $\sigma_{i,j}$ and $\sigma_{i,j}$ can be computed. σ_{ij} is the standard deviation of the data for the i^{th} station and j^{th} week and ρ_{ij} is the serial correlation, for station i, between the j^{th} week and, say, the (j-1)st week; i.e., the lag-one serial correlation.

If a single station is considered first, the generating model is the recursive equation:

$$x_{p+j+1} = \mu_{j+1} + \beta_j(x_{p+j}-\mu_j) + R_{p+j}\,\sigma_{j+1}(1-\rho_j{}^2)^{1/2} \quad . . \text{[Eq. 6.46]}$$

where the data value, x_{p+j+1}, is generated from the last one generated, x_{p+j}. β_j is the regression coefficient for estimating the value of any j+1 indexed value from the j indexed value, R_{j+1} is a standard normal random deviate, σ_{j+1} is the standard deviation of the j+1 indexed values, and ρ_j is the serial correlation coefficient between the j^{th} and (j+1)st values. p equals 48 times the number of years of data already generated. This model preserves the mean, variance and the serial correlation of the historical data and, because the x's are normally distributed, provides a complete statistical description of the data.

This single station model, [Eq. 6.46], can be extended to handle multiple stations and multiple time lags. In this case, the mean, standard deviation and serial correlations for each station remain the same, but it is necessary to compute cross correlations between stations and cross-serial correlations between stations for multiple time lags. The model becomes a matrix equation:

$$\underline{x}_{kj+1} = \underline{\mu}_{j+1} + \underline{\beta}_j(\underline{x}_{kj}-\underline{\mu}_j) + \underline{R}_{kj}\underline{\sigma}_{j+1}(1-\rho_j{}^2)^{1/2} \quad . . \text{[Eq. 6.47]}$$

where $\underline{x}$ denotes an mxl matrix, m is the number of stations (variables), $\underline{\mu}$, $\underline{\sigma}$ and $\underline{R}$ are also mxl matrices, while $\underline{\beta}$ is a pm x pm matrix, p is the number of lag periods considered, and $\underline{\rho}$ is also a pm x pm matrix.

Note that the order of the square matrices for $\underline{\beta}$ and $\underline{\rho}$ is (pm x pm), where p equals the number of lag periods and m is the number of stations. It makes no difference in the development of the correlation matrix whether the correlations are cross (between stations) or serial (between different time lags for the same station). That is, it is immaterial whether the a_{13} element, for instance, in the correlation matrix is the correlation between the station 1 - station 3 data at zero time lag or is the correlation between station 1, time t data and station 1, time t-2 data. Thus pm = n equals the number of variates under consideration. In this work, the number of lag periods is taken as four, corresponding to the assumption that weekly average flows exhibit dependence upon preceding flows for one month.

The correlation matrix is then (n x n) and is of the form shown below for three gaging stations and four weekly lag intervals:

$$
\begin{array}{ll|cccc|cccc|cccc}
 & & \multicolumn{4}{c|}{\text{Station 1}} & \multicolumn{4}{c|}{\text{Station 2}} & \multicolumn{4}{c}{\text{Station 3}} \\
 & & \text{lag 0} & \text{lag 1} & \text{lag 2} & \text{lag 3} & \text{lag 0} & \text{lag 1} & \text{lag 2} & \text{lag 3} & \text{lag 0} & \text{lag 1} & \text{lag 2} & \text{lag 3} \\
\text{Station 1} & \text{lag 0} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{110} & a_{111} & a_{112} \\
 & \text{lag 1} & a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & & & & & \vdots & \vdots & \vdots \\
 & \text{lag 2} & a_{31} & a_{32} & a_{33} & a_{34} & & & & & & & & \\
 & \text{lag 3} & a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & & & & & & & \\
\text{Station 2} & \text{lag 0} & a_{51} & a_{52} & \vdots & & & & & & & & & \\
 & \text{lag 1} & a_{61} & \cdots & & & & & & & & & & \\
 & \text{lag 2} & a_{71} & \cdots & & & & & & & & & & \\
 & \text{lag 3} & a_{81} & \cdots & & & & & & & & & & \\
\text{Station 3} & \text{lag 0} & a_{91} & \cdots & & & & & & & & & & \\
 & \text{lag 1} & a_{101} & \cdots & & & & & & & & & & \\
 & \text{lag 2} & a_{111} & \cdots & & & & & & & & & & \vdots \\
 & \text{lag 3} & a_{121} & \cdots & & & & & & & & & \cdots & a_{1212}
\end{array}
$$

[Eq. 6.48]

The element a_{34}, for example, is the correlation between the flow at station 1, lag 2, and station 1, lag 3, and is equal to a_{43}, while $a_{25} = a_{52}$ is the correlation between station 2, time t, and station 1, time t-1.

The elements in the correlation matrix are computed by the equation:

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.49]$$

where x and y are deviations from the mean and r is the correlation coefficient.

If the cross correlation coefficients are all equal to zero, indicating that the gage data for each station and time are independent of the data for every other station and time, the problem reverts to a series of single station models. It is highly unlikely that the cross correlations will be zero, so a multivariate process is needed to handle the cross correlations.

The basis for the following development is contained in textbooks on multivariate statistical analysis (47). Consider, in vector notation, the random vector X which can be partitioned into two sets:

$$X^{(1)} = \begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ \cdot \\ X_q \end{bmatrix} \quad \text{and} \quad X^{(2)} = \begin{bmatrix} X_{q+1} \\ \cdot \\ \cdot \\ \cdot \\ X_p \end{bmatrix} \quad . \; . \; . \; . \; . \; [\text{Eq. } 6.50]$$

where:

$$X = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{eq. } 6.51]$$

Assume the p variables have a joint normal distribution with means:

$$E\,[X^{(1)}] = \mu^{(1)} \text{ and } E\,[X^{(2)}] = \mu^{(2)} \quad . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.52]$$

and covariances:

$$E\,[(X^{(1)}-\mu^{(1)})\,(X^{(1)}-\mu^{(1)})'] = S_{11} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.53]$$

$$E\,[(X^{(2)}-\mu^{(2)})\,(X^{(2)}-\mu^{(2)})'] = S_{22} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.54]$$

$$E\,[(X^{(1)}-\mu^{(1)})(X^{(2)}-\mu^{(2)})'] = S_{12} = S_{21} \quad . \; . \; . \; . \; [\text{Eq. } 6.55]$$

The mean vector has been partitioned into:

$$\mu = \begin{bmatrix} \mu^{(1)} \\ \mu^{(2)} \end{bmatrix} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.56]$$

and the covariance matrix has been partitioned into:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 6.57]$$

Note that if $X^{(1)}$ and $X^{(2)}$ are independent, $S_{12} = S_{21} = 0$.

Reference (47) continues, developing expressions for the joint density, $f(x^{(1)}, x^{(2)})$, and the marginal density of $X^{(2)}$ at $x^{(2)}$ and from these an expression for the conditional density, $f(x^{(1)}|x^{(2)})$. By reason that conditional distributions derived from joint normal distributions are normal, the density $f(x^{(1)}|x^{(2)})$ is normal and consists of q variates. Further, this conditional density has a mean:

$$E[X^{(1)}|x^{(2)}] = \mu^{(1)} + S_{12}S_{22}^{-1}(x^{(2)} - \mu^{(2)}) \quad \ldots\ldots \quad [\text{Eq. } 6.58]$$

and the covariance matrix is:

$$E[X^{(1)} - E(X^{(1)}|x^{(2)})][X^{(1)} - E(X^{(1)}|x^{(2)})'] = S_{11} - S_{12}S_{22}^{-1}S_{21}$$

$$\ldots \quad [\text{Eq. } 6.59]$$

The matrix $S_{12}S_{22}^{-1}$ is the matrix of regression coefficents of $X^{(1)}$ on $x^{(2)}$. The vector [Eq. 6.58] is called the regression function. Note that the first two terms of the generating model are the regression function where:

$$\beta_j = S_{12}S_{22}^{-1} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 6.60]$$

is the regression coefficient.

For simplicity, consider only two variates and the conditional distribution of X_1, given $X_2 = x_2$. In this case, $S_{11} = \sigma_1^2$, $S_{22} = \sigma_2^2$ and $S_{12} = \sigma_1\sigma_2\rho$, where ρ is the correlation coefficient between X_1 and X_2. The regression coefficient is:

$$S_{12}S_{22}^{-1} = \frac{\sigma_1\rho}{\sigma_2} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 6.61]$$

and the covariance is:

$$S_{11} - S_{12}S_{22}^{-1}S_{21} = \sigma_1^2 - \frac{\sigma_1^2\sigma_2^2\rho^2}{\sigma_2^2} = \sigma_1^2(1-\rho^2) \quad \ldots \quad [\text{Eq. } 6.62]$$

These relations can be extended in matrix form, but the principle is illustrated by this bivariate example.

Note that the square-roots of the elements of the covariance matrix are the coefficients of the third term of the generating equation [Eq. 6.47]. The product of this coefficient and a standard normal random deviate, R, preserves the variance of the historical data in the generated data, as described below.

The covariance matrix:

$$S_c = S_{11} - S_{22}^{-1} S_{21} \quad \text{. . . . . [Eq. 6.63]}$$

is a (q x q) symmetric matrix having real elements. These properties in a matrix assure real, positive eigenvalues, orthogonal eigenvectors corresponding to the eigenvalues, and that the matrix can be diagonalized. Further, the elements of the diagonal matrix are the eigenvalues (48).

The characteristic values of the covariance matrix are principal components which are linear combinations of random variables having special properties in terms of variances (49). Anderson (49) proves that a q-component random vector, $\underline{X}$, having $E[X] = 0$ and $E[XX'] = \underline{S}$, has an orthogonal linear transformation:

$$\underline{U} = \underline{BX} \quad \text{. . . . . [Eq. 6.64]}$$

such that the covariance matrix of $\underline{U}$ is $E[UU'] = \underline{V}$ where:

$$\underline{V} = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & 0 \\ 0 & 0 & \cdot & \cdot\cdot & \lambda_q \end{bmatrix} \quad \text{. . . . . [Eq. 6.65]}$$

where, further, $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ . \ . \ . \geq \lambda_q \geq 0$ are the roots of the correlation matrix, $\underline{S}$. Further, the r^{th} component of $\underline{U}$ has the maximum variance of all normalized linear combinations uncorrelated with $U_1 \cdots U_{r-1}$. This is to say that the variance associated with the r^{th} eigenvalue is greater than that associated with any smaller eigenvalue.

Further, Anderson (49) shows that "The generalized variance of the vector of principal components is the generalized variance of the original vector, and the sum of the variances of the principal components is the sum of the variances of the original variates." Thus, the principal component analysis of the original covariance matrix preserves the variance of the historical data in the generated data.

The principal components are linear transforms of the original covariance matrix elements and are orthogonal - that is, independent. They are essentially correlations between fictitious stations each of which is unaffected by all others. Thus, all cross correlations equal zero and all correlations may be represented by serial correlation using the single station model, [Eq. 6.46]. After these independent correlations are made, the linear transformation is inverted and the resulting matrix elements are used appropriately to generate synthetic data by [Eq. 6.47].

In the programming (see Appendix A4.4), the regression coefficients are designated B_i and the variance coefficients are designated C_i. It is interesting to note that in test simulations using the Farmington River gage data approximately one-third of the values of C_i, corresponding to the smaller eigenvalues, are zero within the limits of accuracy carried. This indicates that the variance in the historical data is all accounted for by only part of the coefficients and that there is, in fact, a measure of dependence in the data.

Values of B_i and C_i are determined for each week of the year. These values apply only to the gage pattern corresponding to the basis gages used in their development and a change in basis gages requires recomputation of the B_i and C_i. The program is set up to output the B_i and C_i on magnetic tape so that in subsequent runs using the same basis gages, it will not be necessary to generate them again.

The multivariate generating equation [6.47] can be rewritten in the form:

$$Q_{t,1} = A_{\tau,i} + \frac{1}{4n} \sum_{i=1}^{n} \sum_{j=1}^{4} B_{\tau-j,i}(Q_{t-j,i} - A_{\tau-j,i}) + R_t C_{\tau-j,i} \quad . \; . \; \text{[Eq. 6.66]}$$

where:

$Q_{t,1}$ = the generated flow for the current week, t, for gage i, t = 1 . . . 48 N, where N = the number of years of generation.

$A_{\tau,1}$ = the deterministic component for gage i for the τth week of the year, corresponding to week of the year of t.

$B_{\tau-j,i}$ = the regression coefficient which relates the current gage flow being generated to the gage flows for other gages and times previously generated, j = 1,2,3,4 corresponding to the multiple lags.

R_t = a standard normal random deviate, t, is the time frame after start of generation.

$C_{\tau-j,i}$ = the coefficient which relates the variance of the generated data to the historical data.

τ = the week of year corresponding to t, $\tau = 1, \ldots, 48$, and

n = the number of basis gages.

This form more nearly describes the manner in which the data are generated by the program algorithm.

The values of $A_{\tau,i}$ are periodic in nature with an assumed principal period of one year. They are determined by a least squares fit of the historical data by a Fourier series of six harmonics. The program FLASH computes the $A_{\tau,i}$, $B_{\tau-j,i}$, $C_{\tau-j,i}$ and generates the gage data. The program and its use are described in Appendix A4.4.

6.5 Simulation of Regulated Flow

In the previous section, the method for generating synthetic gage data was described. Simulation of stream flow at any point in the river system requires that these gage data be transformed into stream flow data and then account for any flow regulation resulting from the withdrawals, discharges and changes in storage in the river system.

The transformation of gage data to streamflow data at any reach point is based upon the assumptions that: (1) the rate of flow at any point in the river system is proportional to the area of the total watershed upstream of that point, and (2) the rate of flow at any point in the river system is a linear combination of the gage flows of the nearest upstream and downstream gages only. Having obtained the watershed areas upstream of all basis gages and reach points, knowing the location of the gages with respect to the reach points and applying the above assumptions, it is a simple matter to compute the weight factors needed to make the transformation.

For example, if reach point 0 has an upstream area of DA0 and it is desired to compute Q0, the flow at reach point 0, find the nearest gages upstream and downstream from point 0. If the nearest upstream gage is S2 and the nearest downstream gage is S1 having areas DAS2 and DAS1, and flows QS2 and QS1, respectively, then:

$$\frac{Q0}{DA0} = \alpha_1 \frac{QS1}{DAS1} + \alpha_2 \frac{QS2}{DAS2} \quad \ldots\ldots\ldots \text{[Eq. 6.67]}$$

where α_1 and α_2 are functions of the areas upstream of 0, S1 and S2.

In this case:

$$\alpha_1 = \frac{DA0-DAS2}{DAS1-DAS2} \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.68]}$$

$$\alpha_2 = \frac{DAS1-DA0}{DAS1-DAS2} \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.69]}$$

where α_1 and α_2 are factors of a linear interpolation of the area upstream of point 0 between the upstream areas of S1 and S2.

The weight factors are given by:

$$\mathrm{wt}(.,1) = \frac{QO}{QS1} = \alpha_1 \frac{DAO}{DAS1} = \frac{DAO}{DAS1}\left(\frac{DAO-DAS2}{DAS1-DAS2}\right) \quad .\;.\;.\;.\;.\;[\text{Eq. } 6.70]$$

$$\mathrm{wt}(.,2) = \frac{QO}{QS2} = \alpha_2 \frac{DAO}{DAS2} = \frac{DAO}{DAS2}\left(\frac{DAS1-DAO}{DAS1-DAS2}\right) \quad .\;.\;.\;.\;.\;[\text{Eq. } 6.71]$$

Thus:

$$QO = \mathrm{wt}(.,1)QS1 + \mathrm{wt}(.,2)QS2 \quad .\;.\;.\;.\;.\;.\;.\;.\;[\text{Eq. } 6.72]$$

There are five possible combinations of gage-reach point locations and therefore there are five different formulas for computing the weight factors. These are described in detail in Appendix A4.3.

The program coding is arranged so that given only the various reach indices, the corresponding upstream areas, and the gage locations by reach index and their upstream areas, the weight factors are computed for all basis gages for all reaches and are placed in a matrix format. Then, for a given set of generated gage flows set up in vector form, the unregulated streamflows at all reach points are computed by matrix multiplication. The unregulated flows then are corrected for regulation to obtain simulated stream flows in each reach.

Regulation of stream flows, in the context used in this work, includes any man-made changes in the flow and any changes that result in the water mass balance due to man's activities. An example of the latter is the water lost to evaporation from a man-made reservoir.

There is nothing profound about accounting for the changes in flow due to regulation. The manner of regulation in each reach must be determined from a study of the watershed and any proposed plans for regulation. The regulation is programmed, in mathematical terms if possible, for computer simulation. The regulation in each reach and each time frame is computed. The regulated flow in any reach is assumed to be the unregulated flow in that reach plus the sum of the regulations upstream of that reach. If the time of flow in the river system is greater than the time averaging interval, it will be necessary to program the regulations to account for the difference in time. However, in a river system of such magnitude that the time of flow is greater than the averaging interval, it may be proper to consider two or more regions or possibly consider neglecting all except the major regulation effects. Appendix A4.5 contains typical reservoir operating rules programmed for simulation.

6.6 Simulation of Water Quality

The water quality considerations in this work are limited to oxygen balance relationships. As described briefly above, the oxygen balance equations formulated by Streeter and Phelps (22), modified by the deoxygenation error term, r, and reoxygenation error term, s, as proposed by Moreau and Pyatt (40), are the basis for the water quality simulation model. Two reach situations are modelled, one for the normal flowing river, and the other for a reservoir or impoundment. In the latter, it is assumed that the reservoir contents are completely mixed.

Although there have been a few studies and surveys made on water quality in reservoirs (49, 50, 51), no one yet (1969) has developed a satisfactory methodology for considering the fate of BOD in a reservoir and its effect on the DO concentration in the reservoir and effluent. This is probably due to the considerable variability of reservoirs and the considerable variability, with time, of conditions in a given reservoir. Fortunately, the majority of reservoirs are built in the upper reaches of the streams and on tributaries at locations upstream of significant BOD loads and the problem therefore may not arise. However, formulas are developed in 6.6.2, below, for complete mixing in reservoirs.

6.6.1 Flowing Streams.

The basic deoxygenation equation developed by Streeter and Phelps (22) is:

$$\frac{dL}{dt} = -K_1L \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\text{Eq. } 6.73]$$

in which the time rate of change of BOD, $\frac{dL}{dt}$, is proportional to the remaining BOD, L. As described previously, K_1 is the proportionality constant. Because of the variability of K_1, an error factor, r, is added to give:

$$\frac{dL}{dt} = -K_1L + r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\text{Eq. } 6.74]$$

To solve the differential equation, [Eq. 6.74], take a Laplace transformation and manipulate algebraically in the following steps:

$$sL(s)-L(o) = -K_1L(s) + \frac{r}{s}$$

$$L(s)[s+K\ ] = \frac{r}{s} + L(o)$$

to obtain:

$$L(s) = \frac{r}{s(s+K_1)} + \frac{L(o)}{s+K_1} = \frac{\frac{r}{K_1}}{s} - \frac{\frac{r}{K_1}}{s+K_1} + \frac{L(o)}{s+K_1} \quad . \; . \; . \quad [\text{Eq. } 6.75]$$

Noting that $L(o) = L_a$, the initial BOD, and making the inverse transformation:

$$L(t) = \frac{r}{K_1} - \frac{r}{K_1} \quad e^{-K_1 t} + L_a \quad e^{-K_1 t}$$

and rearranging:

$$L(t) = (La - \frac{r}{K_1})e^{-K_1 t} + \frac{r}{K_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.76]}$$

L(t) is the BOD t days after the initial BOD was L_a, and K_1 is the deoxygenation velocity constant.

The dissolved oxygen deficit equation, also developed by Streeter and Phelps (22) is:

$$\frac{dD}{dt} = K_1 L - K_2 D \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.77]}$$

or the time-rate change of the dissolved oxygen deficit is the sum of the deoxygenation and reoxygenation rates. K_2 is the reoxygenation velocity constant and D is the dissolved oxygen deficit. If the reoxygenation "error" term, s, (40) is added, equation [6.77] becomes:

$$\frac{dD}{dt} = K_1 L - K_2 D + s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.78]}$$

Let p = s, the reoxygenation "error" term, so as not to confuse the conventional s used for the transformed variable, and make the Laplace transformation to solve the equation:

$$sD(s) - D(o) = K_1 L(s) - K_2 D(s) + \frac{p}{s}$$

$$D(s)(s+K_2) = K_1 L(s) + D(o) + \frac{p}{s}$$

Substitute the value for L(s) given by [Eq. 6.75] and note that D(o) = D_a:

$$D(s) = \frac{1}{s+K_2}\left(\frac{r}{s} - \frac{r}{s+K_1} + \frac{K_1 L_a}{s+K_1}\right) + D_a + \frac{p}{s} \quad . \; . \; . \; . \quad \text{[Eq. 6.79]}$$

Using the partial fraction expansion, combining terms and making the inverse transformation gives:

$$D(t) = \left(\frac{K_1 L_a}{K_2 - K_1} - \frac{r}{K_2 - K_1}\right)(e^{-K_1 t} - e^{-K_2 t}) + \frac{1}{K_2}(r+s)(1 - e^{-K_2 t})$$

$$+ D_a e^{-K_2 t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.80]}$$

Equation [6.80] is the equation for the dissolved oxygen, D, at time t after the BOD was L_a and the DO deficit was D_a.

The critical deficit, defined as the maximum DO deficit (that is, where the DO in the stream is minimum) is of major interest in water quality studies. The critical deficit is found by taking the derivative of equation [6.80] with respect to t, equating this derivative to zero and solving for $t = t_c$:

$$\frac{dD(t)}{dt} = -K_1\left(\frac{K_1L_a}{K_2-K_1} - \frac{r}{K_2-K_1}\right)e^{-K_1t} + K_2\left(\frac{K_1L_a}{K_2-K_1} - \frac{r}{K_2-K_1}\right)e^{-K_2t}$$

$$+ (r+s)e^{-K_2t} - D_aK_2\,e^{-K_2t} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.81]$$

Set $\frac{dD(t)}{dt} = 0$ and divide by $K_1\left(\frac{K_1L_a}{K_2-K_1} - \frac{r}{K_2-K_1}\right)e^{-K_1t}$ to get:

$$0 = -1 + \frac{K_2e^{-K_2t}}{K_1e^{-K_1t}} - \frac{K_2D_a + (r+s)e^{-K_2t}}{K_1\left(\frac{K_1L_a}{K_2-K_1} - \frac{r}{K_2-K_1}\right)e^{-K_1t}} \quad . \; . \; [\text{Eq. } 6.82]$$

which, when rearranged, gives:

$$e^{(K_2-K_1)t_c} = \frac{K_2}{K_1} - \frac{[K_2D_a-(r+s)]\ (K_2-K_1)}{K_1(K_1L_a-r)} \quad . \; . \; . \; . \; [\text{Eq. } 6.83]$$

Take the logarithm of both sides and divide through by (K_2-K_1) to get:

$$t_c = \frac{1}{K_2-K_1}\ \ln\left(\frac{K_2}{K_1} - \frac{[K_2D_a-(r+s)](K_2-K_1)}{K_1(K_1L_a-r)}\right). \; . \; . \; . \; [\text{Eq. } 6.84]$$

If t_c is evaluated and substituted for t in equation [6.80], the resulting $D = D_c$, the critical deficit.

Note that in the event $K_1 = K_2$, equations [6.80] and [6.84] become indeterminate. In this case, the integrated form of equation [6.78] becomes:

$$D(t) = [K_1(L_a-\frac{r}{K_1})t + D_a + \frac{(r+s)}{K_1}]e^{-K_1t} - \frac{r+s}{K_1} \quad . \; . \; . \; [\text{Eq. } 6.85]$$

Further, the critical condition, obtained in the manner above, is given by:

$$t_c = \frac{1}{K_1} - \frac{D_a}{K_1L_a-r} + \frac{r+s}{K_1(K_1L_a-r)} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 6.86]$$

Again, substitution of t_c for t in equation [6.85] gives D_c, the critical deficit, for the condition when $K_1 = K_2$.

A principal use of the simulation model is to determine where and when there is a violation of a stream standard minimum DO level. The model computes the DO deficit at the upstream and downstream ends of each reach and checks to determine if a maximum, or critical point, occurred within the reach. It then selects the maximum value of the three DO deficit values; that is, at the upper end of the reach, the lower end of the reach or at a critical point in the reach (if one exists) and subtracts it from the DO saturation value to obtain the minimum DO concentration in the reach. This minimum value is checked against the minimum standard and if the value is less than the standard, the program indicates a violation, giving the minimum DO value and the reach and week in which it occurred.

The assumption is made that there is complete and instantaneous mixing at the reach points. Where a BOD load is discharged, at the upper end of a reach, the L_a, or initial BOD in the reach is given by:

$$L_a = BOD_{in} = \sum_{i=1}^{n} \frac{BODOUT_i \times Q_i + BOD_w + Q_w}{\sum_{i=1}^{n} Q_i + Q_w} \quad . . . \text{[Eq. 6.87]}$$

where BOD_{in} is the BOD concentration at the upstream end of the reach of interest, $BODOUT_i$ is the BOD concentration at the downstream end of the reach(es) immediately upstream, Q_i is the rate of flow of the reach(es) immediately upstream, BOD_w is the BOD concentration of the waste discharging at the upstream end of the reach of interest, and Q_w is the corresponding rate of waste discharge, and n is the number of tributaries at the upstream end of the reach. Similarly, the value of D_a is obtained for the initial DO deficit in the reach.

The assumption also is made that the K_1 value of the mixed waste and streamflow is obtained by the formula:

$$K_{1R} = \frac{\sum_{L=1}^{m} K_{1L}Q_L + K_1Q_w}{\sum_{L=1}^{m} Q_L^{\prime} + Q_w} \quad . . . . \text{[Eq. 6.88]}$$

where K_{1R} is the deoxygenation velocity constant in the reach of interest, K_{1L} is the velocity constant in the reach(es) immediately upstream, Q_L is the corresponding rate of flow, K_1 and Q_w are the velocity constants and discharge rate of the waste, and m is the number of tributaries immediately upstream which have received BOD loads upstream. Note that if an upstream reach is not subject to an upstream waste loading, its value of K_{1L} is zero and the $Q_L^{\prime}$ in the denominator applies only to those branches immediately upstream that are subject to upstream waste loading.

The simulation model starts at the upstream-most reach and progresses downstream computing the simulated water quality data for one time frame. The time is then incremented one time interval and the process of computing the data is repeated. The number of years of data to be simulated is a program input parameter.

6.6.2 Reservoirs.

As indicated previously, the assumption is made that the contents of a reservoir are completely mixed at all times. The program maintains a mass balance between the BOD and DO deficits in the incoming flows and the outgoing releases and diversions.

The rate of flow into the reservoir, Q_{in}, is given by:

$$QIN = \sum_{K} QREG(K) - QVAP \quad . . . . [Eq.\ 6.89]$$

where QREG is the regulated flow in the reach discharging into the reservoir and QVAP is the loss in the reservoir due to evaporation. K is the number of branches of the system immediately upstream discharging into the reservoir. The rate of flow out of the reservoir is:

$$Qout = RREL + DIV \quad . . . . [Eq.\ 6.90]$$

where RREL is the rate of release and DIV is the rate of diversion from the reservoir.

Let y(t) be the BOD concentration in the reservoir at time t. Let Y_{in} be the concentration of BOD entering the reservoir, computed by equation [6.87]. The differential equation for the rate of change of BOD concentration in the reservoir is:

$$\frac{dy}{dt} = -K_1 y + \frac{QIN}{V} Yin - \frac{Qout}{V} y - K_3 y \quad . . . . . [Eq.\ 6.91]$$

The factor K_3 is included to account for the loss in BOD in the reservoir contents due to settling to the bottom of the reservoir of solid matter having BOD. K_3 is assumed to vary linearly with the storage volume.

Equation [6.91] can be solved for Y(t) as follows. Let $Z = \frac{QIN}{V}$ and $W = \frac{Qout}{V}$. The Laplace transform of [Eq. 6.91] is:

$$sY(s) - Yo = -K_1Y(s) + \frac{ZYin}{s} - WY(s) - K_3Y(s) \quad . . . . [Eq.\ 6.92]$$

which leads to:

$$Y(s)[s+K_1+W+K_3] = Yo + \frac{ZYin}{s} \quad \ldots\ldots \text{[Eq. 6.93]}$$

Let

$$A = K_1 + W + K_3 \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.94]}$$

which when substituted into equation [6.93] gives:

$$Y(s) = \frac{Yo}{s+A} + \frac{ZYin}{s(s+A)} \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.95]}$$

Expanding by partial fractions gives:

$$Y(s) = \frac{Yo}{s+A} + \frac{ZYin/A}{s} - \frac{ZYin/A}{s+A} \quad \ldots\ldots\ldots \text{[Eq. 6.96]}$$

The inverse transformation of equation [6.96] gives:

$$Y(t) = Yoe^{-AT} + \frac{ZYin}{A}(1-e^{-AT}) \quad \ldots\ldots\ldots \text{[Eq. 6.97]}$$

Yo is the BOD concentration at t=o and T is the time since t=o. The value of Y(t) is the concentration of BOD in the reservoir at time t.

The average outflow BOD concentration, $\bar{Y}$ is computed by:

$$\bar{Y} = \frac{1}{T}\int_o^T Y(t)dt \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.98]}$$

This integral equation is also solved by using Laplace transformation as follows:

$$\bar{Y}(s) = \frac{1}{sT}\,Y(s) \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.99]}$$

Substitute the value shown above for Y(s) from equation [6.96] to get:

$$\bar{Y}(s) = \frac{Yo/t}{s(s+A)} + \frac{ZYin/AT}{s^2} - \frac{ZYin/AT}{s(s+A)}\,.$$

Using partial fraction expansion and taking the inverse transformation gives:

$$\bar{Y} = \frac{1}{AT}(Yo-\frac{ZYin}{A})(1-e^{-AT}) + \frac{ZYin}{A} \quad \ldots\ldots\ldots \text{[Eq. 6.100]}$$

$\bar{Y}$ is the average BOD concentration in the water leaving the reservoir during the time interval from 0 to T.

The dissolved oxygen deficit, C, in the reservoir and the average DO deficit, $\bar{C}$, in the water leaving the reservoir are similarly computed.

The differential equation for the time-rate of change of C is:

$$\frac{dC}{dt} = -K_2C + Cin\,\frac{Qin}{V} + \frac{CQout}{V} + K_1y \quad \ldots\ldots \text{[Eq. 6.101]}$$

Cin, the incoming dissolved oxygen deficit, is constant over the averaging interval. Making the Laplace transform of equation [6.101] gives:

$$sC(s) - Co = K_2C(s) + \frac{ZCin}{s} - WC(s) + K_1y(s) \ldots \text{[Eq. 6.102]}$$

Let

$$B = K_2 + W \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.103]}$$

and substituting into equation [6.102] gives:

$$C(s) = \frac{Co}{s+B} + \frac{ZCin}{s(s+B)} + \frac{K_1y(s)}{s+B} \quad \ldots\ldots \text{[Eq. 6.104]}$$

Substitute y(s) from equation [6.96] to get:

$$C(s) = \frac{Co}{s+B} + \frac{ZCin/B}{s} - \frac{ZCin/B}{s+B} + \frac{YoK_1}{(s+A)(s+B)} - \frac{K_1ZYin/A}{s(s+B)} - \frac{K_1ZYin/A}{(s+A)(s+B)} \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.105]}$$

Expand, using partial fractions, take the inverse Laplace transform and combine terms to obtain:

$$C(t) = \left(Co - \frac{ZCin}{B} + \frac{K_1}{A-B}\left(Yo - \frac{ZYin}{B}\right)\right)e^{-BT} + \left(\frac{K_1}{B-A}\left(Yo - \frac{ZYin}{A}\right)\right)e^{-AT} + \frac{Z}{B}\left(Cin + \frac{K_1Yin}{A}\right) \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.106]}$$

C(t) is the DO deficit concentration at time t given a deficit of Co and BOD of Yo at t=0 and Cin and Yin the incoming deficit and BOD, respectively.

The average deficit concentration in the reservoir is:

$$\bar{C} = \frac{1}{T}\int_0^T C(t)dt \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.107]}$$

This integral equation can be solved using Laplace transforms as follows:

$$\bar{C}(s) = \frac{1}{sT} C(s) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.108]}$$

which when the fully expanded value for C(s) from above is substituted gives:

$$\bar{C}(s) = \frac{1}{T}\left(\frac{Co}{s(s+B)} + \frac{ZCin/B}{s^2} - \frac{ZCin/B}{s(s+B)} + \frac{\frac{K_1Yo}{B-A}}{s(s+A)} + \frac{\frac{K_1Yo}{A-B}}{s(s+B)} + \frac{\frac{K_1ZYin}{AB}}{s^2} - \frac{\frac{K_1ZYin}{AB}}{s(s+B)} - \frac{\frac{K_1ZYin}{A(B-A)}}{s(s+A)} + \frac{\frac{K_1ZYin}{A(B-A)}}{s(s+B)} \right) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.109]}$$

Making a partial fraction expansion and taking the inverse Laplace transform leads to:

$$\bar{C} = \frac{1}{T}\left\{\left[\frac{1}{B}\left(Co - \frac{ZCin}{B}\right) + \frac{K_1}{AB}\left(Yo - \frac{ZYin}{AB}(A+B)\right)\right] + \frac{e^{-BT}}{B}\left[\frac{ZCin}{B} - Co + \frac{K_1}{A-B}\left(\frac{ZYin}{B} - Yo\right)\right] + \frac{ZT}{B}\left[Cin + \frac{K_1Yin}{A}\right] + \frac{K_1e^{-AT}}{}\left[\frac{ZYin}{A} - Yo\right]\right\} \; . \; . \; . \quad \text{[Eq. 6.110]}$$

$\bar{C}$ is the average DO deficit in waters leaving the reservoir in the interval encompassed by t=0 to t=T.

In the simulation, when the sequential computation of water quality data encounters a reservoir, the program switches to the subroutine RQUAL which computes the average BOD concentration, $\bar{Y}$, and the average DO deficit concentration, $\bar{C}$, in the waters leaving the reservoir. These values become the corresponding incoming values for the next downstream reach.

The factor K_3, sedimentation constant, has been included in this development as a means to account for any storage-related phenomenon that may be found to affect the basic oxygen balance relationships. It could account for, as inferred above, the settlement of a portion of the solid fraction of the BOD to the bottom and the subsequent satisfaction of a part of that BOD without drawing from the DO resources in the reservoir. It is suspected, without substantiating information, that K_3 is of small magnitude and for preliminary simulation runs, at least, K_3 should be set equal to zero.

6.7 Sensitivity of Variables.

When the simulation model is complete, it would be of advantage to have a means to determine the relative importance of the variables that have been included. If a small change in one of the variables makes a significant change in the simulated conditions, it may be necessary to be thorough and painstaking in determining the parameter or initial value of that variable. On the other hand, if a small, or even large, change in another variable results in little change in the result, its effect may be such that little effort and expense need be allotted to its evaluation. It may be found that the variable could be replaced by a constant or even could be eliminated. A sensitivity analysis affords the opportunity to look back at the model that has been assembled to learn of its characteristics and of the relative importance of its components.

Intuitively, the sensitivity of a system to a variable of the system is the change in the system caused by a small change in the value of the variable. If change is defined as the ratio of the differential variation of the function to the function itself, the change becomes dimensionless (52). The sensitivity function is defined as:

$$S_k^{F(k)} = \frac{\text{change in system response, } F(k)}{\text{change in parameter, } k} \quad . . . \text{[Eq. 6.111]}$$

and in the dimensionless form:

$$S_k^{F(k)} = \frac{dF(k)/F(k)}{dk/k} . \quad . . . . \text{[Eq. 6.112]}$$

If the change in the system response for a single system variable is considered, the problem is two-dimensional and the sensitivity is the slope of the system function with respect to the variable. If the system response to changes in two system variables is considered, the systen function becomes a response surface. Although more than three dimensions are difficult to visualize, this concept may be extended in a mathematical sense to n dimensions, or n variables of a system.

In classical mathematics, the sensitivity of a function to a variable is given by the partial derivative of the function with respect to the variable. Consider for simplicity, a function $\mu = f(x,y)$ of two variables. If it is desired to find the sensitivity of μ with respect to x, assign a fixed value to y, i.e., $y = y_o$. The resulting:

$$\mu = f(x, y_o) . . . . \text{[Eq. 6.113]}$$

is a function of a single variable, x. Equation [6.113] is the equation of the intersection of the surface of $\mu = f(x,y)$ and the plane $y = y_o$. The rate of change of μ at (x_o, y_o) is given by:

$$\left.\frac{d\mu}{dx}\right|_{x=x_o} = \lim_{h\to 0} \frac{f(x_o+h,y_o) - f(x_o,y_o)}{h} \quad . . . . \text{[Eq. 6.114]}$$

extending to n variables:

$$v = g(x_1, x_2, \ldots x_n) \quad \ldots\ldots\ldots \text{[Eq. 6.115]}$$

the rate of change of v for a small change in one of the independent variables, x_i, is given by:

$$\frac{\partial v}{\partial x_i} = \lim_{h \to 0} \frac{g(x_1, x_2, \ldots, x_i + h, \ldots x_n) - g(x_1, \ldots, x_i, \ldots x_n)}{h} \quad \ldots\ldots\ldots \text{[Eq. 6.116]}$$

In this case, the system response is v and the changing parameter is x_i.

In the case of unconstrained optimum conditions, $\frac{\partial v}{\partial x_i} = 0$, so for smooth functions in the immediate vicinity of the optimum point, v is insensitive to small changes in x_1. For the constrained optimum case, it can be shown (53) that:

$$\frac{\partial v}{\partial x_i} = -\lambda_i \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.117]}$$

where λ_i, sometimes called the sensitivity coefficient, is the Lagrangian multiplier. The value of $-\lambda_i$ at the constrained optimal condition is the sensitivity of the function v to small changes in the variable x_i. It can be shown (54) that the Lagrangian multipliers and the dual variables in a linear programming problem are identical and both then provide a measure of the sensitivity of a variable to the objective function of a linear optimization problem.

If the independent variables in the relationship:

$$v = g(x_1, x_2, \ldots, x_n) \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.115]}$$

are random, then v is a random variable. The sensitivity of v to change in one of the x_i can be approximated by determining that portion of the total variance of v that is attributable to x_i. Beginning with:

$$Y = g(x) \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.118]}$$

the expected value of Y is:

$$E[Y] = \int_{-\infty}^{+\infty} g(x)\ f(x)\ dx \quad \ldots\ldots\ldots \text{[Eq. 6.119]}$$

where f(x) is the density function of x. If g(x) is difficult to evaluate (55), expand [Eq. 6.118] in a Taylor's Series about point a:

$$Y = g(x)\Big|_a = g(a) + \frac{g'(a)(x-a)}{1!} + \frac{g''(a)(x-a)^2}{2!} + \ldots \qquad \text{[Eq. 6.120]}$$

or,

$$Y = \sum_{i=0}^{\infty} \frac{g^{(i)}(a)(x-a)^i}{i!} \quad \ldots\ldots\ldots \quad \text{[Eq. 6.121]}$$

Now let a = μ, the mean of g(x), and substitute into Eq. [6.121]:

$$g(x) = \sum_{i=0}^{\infty} \frac{g^{(i)}(\mu)(x-\mu)^i}{i!} \quad \ldots\ldots\ldots \quad \text{[Eq. 6.122]}$$

Write the first three terms of Eq. [6.122]:

$$g(x) \cong g(\mu) + g'(\mu)(x-\mu) + \frac{g''(\mu)(x-\mu)^2}{2} \quad \ldots\ldots \quad \text{[Eq. 6.123]}$$

Taking the expected values of each term:

$$E[g(x)] \cong g(\mu) + g'(\mu)E[x-\mu] + g''(\mu)E[(x-\mu)^2]. \;.\;.\; \text{[Eq. 6.124]}$$

But, $E[x-\mu] = 0$ and $E[x-\mu]^2 = V(x)$, the variance of x, so that:

$$E[g(x)] \cong g(\mu) + 1/2\, g''(\mu)V(x) \quad \ldots\ldots\ldots \quad \text{[Eq. 6.125]}$$

To find the approximate variance of Y = g(x), expand Eq. [6.122] in a Taylor's Series of two terms and take the variance of both sides:

$$Y \cong g(\mu) + g'(\mu)(x-\mu) \quad \ldots\ldots\ldots \quad \text{[Eq. 6.126]}$$

$$V(Y) \cong 0 + [g'(\mu)]\, E[(x-\mu)^2] \quad \ldots\ldots\ldots \quad \text{[Eq. 6.127]}$$

or:

$$V(Y) \cong [g'(\mu)]^2 V(x) \quad \ldots\ldots\ldots \quad \text{[Eq. 6.128]}$$

This development can be expanded for functions of several variables (55). The approximate mean is given by:

$$E[v] = g(\mu x_1, \mu x_2, \ldots \mu x_n) + 1/2\left[\frac{\partial^2 g}{\partial x_i^2} x_1^2 + \ldots \frac{\partial^2 g}{\partial x^2_n} x_n^2\right]. \;.\;.\;.\;.\; \text{[Eq. 6.129]}$$

and the approximate variance is given by:

$$V[v] = \frac{\partial^2 g}{\partial x_1 2}\sigma_{x_1}{}^2 + \frac{\partial^2 g}{\partial x_2 2}\sigma_{x2}{}^2 + \ldots + \frac{\partial^2 g}{\partial x_n 2}\sigma_{x_n}{}^2 \;.\;.\;.\; \text{[Eq. 6.130]}$$

From equation [6.130] it is possible to obtain the relation needed, that is, $\partial g/\partial x_i$, the rate of change of the function g with respect to x_i. The partial derivatives in equation [6.130] are evaluated using nominal values of the variables and the total variance is computed. The sensitivity of each variable is obtained by taking each right hand factor one at a time, for variable x_k for instance, and setting up the relation:

$$\frac{\partial^2 g}{\partial {x_k}^2} = \frac{V[v]}{\sigma {x^2}_k} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.131]}$$

Both V[v] and ${\sigma_x}^2{}_k$ are known and equation [6.131] can be solved for $\frac{\partial g}{\partial x_k}$, the sensitivity of the function g to change in variable x_k.

When the analysis of a system is carried out by simulation methods, the method for determining the sensitivity of the system to changes in a variable depends upon the nature of the variable. If the variable is deterministic, having a nominal or assigned value, sensitivity is determined by making two simulation runs and comparing the results. In one run, the nominal value of the variable of interest is used. The second run is exactly the same as the first excepting that a small change is made in the variable of interest. The sensitivity is computed using a modified form of equation [6.104]:

$$S_k^{F(k)} = \frac{\Delta F(k)/F(k)}{\Delta k/k} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. 6.132]}$$

where $\Delta F(k)$ and Δk indicate the observed function change and directed variable change, respectively.

If the variable is random, its sensitivity is determined using equation [6.131]. In this case, the simulation is made and the data so obtained are analyzed for the total variance, V[v] in equation [6.131], and the variance of each variable, $\sigma {x_k}^2$. Then $\frac{\partial g}{\partial x_k}$ is obtained.

An unsophisticated way to approximate the sensitivity of a random variable is to make a simulation run for each variable parameter in the system. Each run is made allowing the variable of interest to vary as usual but assigning to all other random variables their expected value. The sensitivity is then computed using equation [6.132]. A run using the nominal values of the variables gives the value F(k) and k is known. The change made is Δk and the simulation produces $\Delta F(k)$.

6.8 Transfer Functions.

The water quality simulation model can be used to develop a cause and effect relationship between waste loads and downstream water quality that, in turn, can be used to estimate or predict the effect of a waste load, or set of waste loads, on the downstream reaches. The idea is obtained from the field of systems engineering (56).

The river, between a point of waste discharge and a downstream point, is considered to be a system. A system is any mechanism that transforms an input variable into an output variable according to the system function. A waste load, the input variable, enters the river, the system, which operates on the waste load to transform it into a water quality condition, the output variable. In this case, the system function could be represented by the Streeter-Phelps equations (22). The upstream BOD and DO deficit concentrations L_a and D_a, respectively, are transformed into downstream BOD and DO deficit concentrations, L and D according to the relationships:

$$L = L_a e^{-K_1 t} \quad \text{. . . . . [Eq. 6.133]}$$

and,

$$D = \frac{K_1 L_a}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) + D_a e^{-K_2 t} \quad \text{. . . . . [Eq. 6.134]}$$

Following in the manner of Moreau and Pyatt (40), let:

$$a_i = e^{-K_1 t_i} \quad \text{. . . . . [Eq. 6.135]}$$

$$c_i = e^{-K_2 t_i} \quad \text{. . . . . [Eq. 6.136]}$$

and,

$$b_i = (a_i - c_i) \frac{K_1}{K_2 - K_1} . \quad \text{. . . . . [Eq. 6.137]}$$

Then equations [6.133] and [6.134] can be written:

$$L = a_i L_a \quad \text{. . . . . [Eq. 6.138]}$$

and,

$$D = b_i L_a + c_i D_a . \quad \text{. . . . . [Eq. 6.139]}$$

General equations can be developed for the system function or transfer function for BOD and DO deficit. The following development is based upon the reach numbering system that was previously proposed (see section 6.3.1).

For a waste load at the upper end of reach 1, equation [6.138] can be written:

$$L_{1L} = a_1 L_{1U} \quad \text{. . . . . [Eq. 6.140]}$$

where the index 1 indicates the reach number and the indices U and L indicate the upper and lower ends of the reach, respectively. For a load at reach 2 and reach 1:

$$L_{1L} = a_1(L_{1U} + r_{12}a_2L_{2U}) \quad \ldots\ldots\ldots \quad \text{[Eq. 6.141]}$$

where r_{12} is the ratio of the rate of flow in reach 2 to the rate of flow in reach 1, the inverse of the dilution ratio afforded the reach 2 waste load.

Similarly, for a waste load at reaches 3, 2 and 1, the BOD at the lower end of reach 1 is given by:

$$L_{1L} = a_1L_{1U} + a_1r_{12}a_2L_{2U} + a_1r_{12}a_2r_{23}a_3L_{3U} \quad \ldots \quad \text{[Eq. 6.142]}$$

If the following assignments are made:

$$\alpha_{31} = a_1\ r_{12}\ a_2\ r_{23}\ a_3, \quad \ldots\ldots \quad \text{[Eq. 6.143]}$$

$$A_{31} = a_1\ a_2\ a_3, \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 6.144]}$$

$$A_{ii} = a_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{[Eq. 6.145]}$$

and

$$r_{31} = r_{12}\ r_{23}, \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 6.146]}$$

then equation [6.142] becomes:

$$L_{1L} = A_{11}\ L_{1U} + A_{21}\ r_{21}\ L_{2U} + A_{31}\ r_{31}\ L_{3U}. \quad \ldots \quad \text{[Eq. 6.147]}$$

The general equation can then be written as:

$$L_{jL} = \sum_{i=1}^{n} A_{ij}\ r_{ij}\ L_{iU},\ i > j \quad \ldots\ldots \quad \text{[Eq. 6.148]}$$

where j is the number of the downstream reach (the lower end) for which the BOD is being estimated, the i's are the upstream reach numbers where loads L_{iU} are discharged and n is the number of such loads. A_{ij} and r_{ij} are as defined above.

The transfer function relationships for the DO deficit can be developed in a similar manner. Starting at the downstream reach, using equation [6.131], write:

$$D_{1L} = b_1\ L_{1U} + c_1\ D_{1U} \quad \ldots\ldots\ldots \quad \text{[Eq. 6.149]}$$

for the DO deficit at the downstream end of reach 1 for a BOD load of L_{1U} and DO deficit of D_{1U} at the upstream end of reach 1. Similarly, place a load L_{2U} and deficit D_{2U} at reach 2 and compute D_{1L}. For reach 2:

$$D_{2L} = b_2 L_{2U} + c_2 D_{2U} \quad \ldots\ldots\ldots \text{[Eq. 6.150]}$$

but,

$$D_{1U} = r_{21} D_{2L} \quad \ldots\ldots\ldots\ldots \text{[Eq. 6.151]}$$

and,

$$L_{1U} = r_{21} L_{2L} = r_{21} a_2 L_{2U} \quad \ldots\ldots\ldots \text{[Eq. 6.152]}$$

Using equation [6.149] and substituting knowns for D_{1U} and L_{1U}:

$$D_{1L} = r_{21} L_{2U} (b_1 a_2 + c_1 b_2) + r_{21} c_1 c_2 D_{2U} \quad \ldots \text{[Eq. 6.153]}$$

which gives the value of the DO deficit at the lower end of reach 1 for a load L_{2U} and deficit D_{2U} at the upstream end of reach 2.

Using the same approach, it can be shown that for a BOD load and DO deficit at the upper end of reach 3, D_{1L} is:

$$D_{1L} = r_{31} L_{3U}(b_1 a_2 a_3 + c_1 b_2 a_3 + c_1 c_2 b_3) + c_1 c_2 c_3 r_{31} D_{3U} \quad \ldots\ldots \text{[Eq. 6.154]}$$

For a load and deficit at reach 4:

$$D_{1L} = r_{41} L_{4U}(b_1 a_2 a_3 a_4 + c_1 b_2 a_3 a_4 + c_1 c_2 b_3 a_4 + c_1 c_2 c_3 b_4) + c_1 c_2 c_3 c_4 r_{41} D_{4U} \quad \ldots\ldots\ldots \text{[Eq. 6.155]}$$

For a load and deficit at reach 5:

$$D_{1L} = r_{51} L_{5U}(b_1 a_2 a_3 a_4 a_5 + c_1 b_2 a_3 a_4 a_5 + c_1 c_2 b_3 a_4 a_5 + c_1 c_2 c_3 b_4 a_5 + c_1 c_2 c_3 c_4 b_5) + c_{51} r_{51} D_{5U} \quad \ldots \text{[Eq. 6.156]}$$

Note the pattern of coefficients in equations [6.153] through [6.156]. The coefficient of D_{jU} is c_{j1} r_{j1}. The coefficient of L_{jU} is r_{j1} multiplied by the factor in parentheses. The keys to the pattern in parentheses are:

(1) There are j factors and j terms in each factor;
(2) The b_j term appears once in each term and in the position corresponding to its subscript;
(3) All terms to the right of b_j are a's with terms to fill out the factor and subscripts corresponding to the position of the term in the factor; and

(4) All terms to the left of b_j are c's with terms and subscripts in proper order.

If loads and deficits occur at more than one reach point, the effects of each load and deficit are computed separately and added.

The point in developing equations [6.145] through [6.148] is primarily to show that the BOD and DO deficits at any point downstream is a linear combination of the BODs and DO deficits at all upstream reaches.

It is obvious that for a large number of reaches, the use of formulas of the type developed above would become very involved. The simulation model offers a means to determine the coefficients, A_{ij}, in equation [6.148] and an overall coefficient, D_{ij}, to substitute for the relationships developed above. The method is to apply a unit BOD load and unit DO deficit at an upstream reach and simulate to determine the BOD and DO deficit at all downstream reaches. This is repeated, placing unit loads at different upstream reaches so that the system is determined for each quality parameter for each possible i-j combination.

Because of the variability in the stream system, the transfer function coefficients are considered to be random variables. They are determined by simulation of the system for unit loads and deficits for a number of years to determine their variability. The overall transfer functions determined in this manner contain the effects of variable flow rates which can be accounted for as indicated below.

If the overall transfer function is called α_{ij}, then:

$$\alpha_{ij} = a_{ij}\, r_{ij} \quad \ldots\ldots\ldots\ldots [\text{Eq. } 6.157]$$

where α_{ij} relates the BOD concentration at the lower end of reach j to the unit BOD concentration at the upper end of reach i, r_{ij} relates the flow rate at the lower end of reach j to the flow rate at the upper end of reach i and a_{ij} is defined as the ratio α_{ij}/r_{ij}. This leads to:

$$\alpha_{ij} = BOD_j/BOD_i, \quad \ldots\ldots\ldots\ldots [\text{Eq. } 6.158]$$

$$r_{ij} = Q_j/Q_i \quad \ldots\ldots\ldots\ldots [\text{Eq. } 6.159]$$

and,

$$a_{ij} = \frac{\alpha_{ij}}{r_{ij}} = \frac{BOD_j Q_i}{BOD_i Q_j} \quad \ldots\ldots\ldots\ldots [\text{Eq. } 6.160]$$

By simulating the system operation for a number of years, it is possible to obtain enough a_{ij}-r_{ij} pairs for each averaging time interval to develop a regression equation of the transfer function a_{ij} on the ratio of flow rates r_{ij}. Assuming the relationship is linear, the regression equation would have the form:

$$a_{ijl} = B_{ijl}\, r_{ijl} + A_{ijl} \quad \ldots\ldots\ldots \text{[Eq. 6.161]}$$

where a_{ijl} is the BOD transfer coefficient from the upper end of reach i to the lower end of reach j during the lth week of the year; B_{ijl} is the slope and A_{ijl} is the ordinate intercept of the linear regression equation for the transfer function i-j for week l and r_{ijl} is the flow rate ratio for week l.

Similarly, the DO deficit relationships can be developed to obtain:

$$d_{ijl} = C_{ijl}\, r_{ijl} + D_{ijl} \quad \ldots\ldots\ldots \text{[Eq. 6.162]}$$

Here the deficit transfer coefficient is d_{ijl} and the regression equation constants are C_{ijl} and D_{ijl}.

The reader is referred to Appendix A4.6 for more information about these transfer coefficients and their use.

Section 6

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SECTION 7
THEORETICAL DEVELOPMENT OF OPTIMIZATION MODEL

7.1 Introduction

The overall objective of this section is to develop a methodology to quantify the benefits of low flow augmentation in a complex region. These benefits are to be measured in terms of wastewater treatment costs avoided. A complex region is viewed as one with multiple reservoirs and/or waste sources arranged in a configuration such that significant interdependencies exist among these entities. It is assumed that preliminary delineation of the subset of a watershed, defined as a region, has been determined. This selected subset is then partitioned into reaches which provide a mutually exclusive and collectively exhaustive representation of the region.

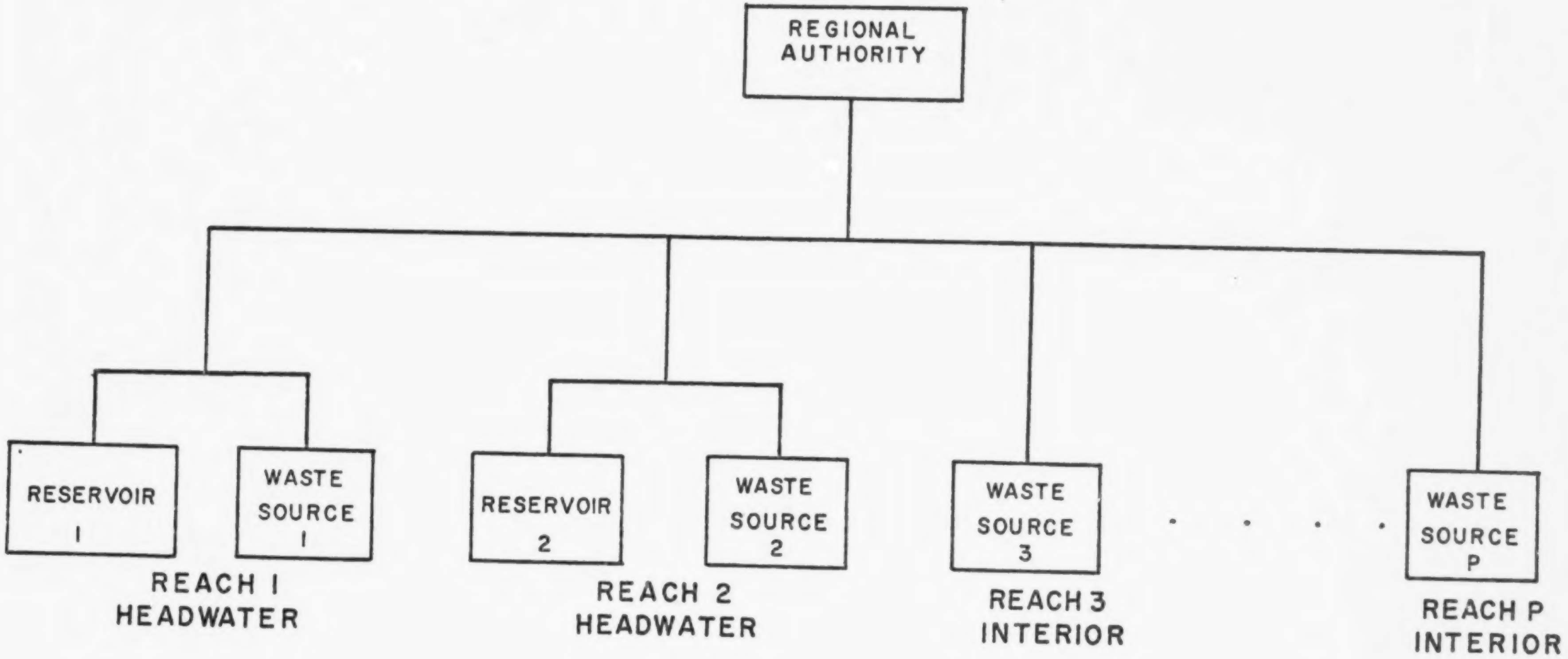
A separable convex programming model has been developed to determine the combination of wastewater treatment plants and low-flow augmentation reservoir releases which minimizes the total cost of meeting prespecified water quality standards. The optimal solution can be specified for any combination of decision-making units vis-a-vis analysis of the dual problem.

7.2 Regional Decision-Making Structure

The decision-making structure within this region is shown in Figure 7-1. A headwater reach may contain either one or two decision-making units - a reservoir or a waste discharge source. Headwater reaches are defined as those with no reaches upstream from them. Reservoirs can exist only on headwater reaches. Consequently, interior reaches contain only a single decision-making unit - a waste discharge source. The investment decision regarding wastewater treatment facilities or storage facilities is usually related to installing a facility capable of satisfying some extreme condition, e.g., 7 day low-flow which might be expected to occur every 10 years. Thus the primary focus for comparing alternatives will be on the critical period.

The regional authority seeks to act in the best interests of the entire region by inducing decision-making units to coordinate their activities. This authority transmits to the decision-making units a set of water quality standards for each reach. Each set of standards has been established after

FIGURE 7-1
ASSUMED REGIONAL DECISION-MAKING STRUCTURE



careful examination of the water quality needed to attain specified goals.

The regional authority provides the market mechanism for permitting any two or more individual decision-makers to make mutually advantageous transactions in order to reduce the total cost of satisfying a specified water quality standard. Previous investigators have restricted their analysis to cases where all of the intermediate decision-makers worked together. The more general case of specifying optimal decision rules for any subset of participating decision-makers will be included in the subsequent analyses.

Given a set of standards, the waste discharger in each reach is responsible for meeting that standard. The assumed objective of each waste discharger is to minimize the cost of meeting this standard. The waste discharger in a headwater reach has two alternatives available to him: construct wastewater treatment facilities, or obtain augmented flow from some upstream point. A waste discharger in an interior reach has a third alternative available to him: he can pay an upstream waste discharger to increase his treatment so as to reduce the waste inflow to the specified interior reach. Knowing his own wastewater treatment cost function, the waste discharger can estimate what he would be willing to pay for a unit reduction in BOD load entering his reach. However, in order to reduce the BOD load by one unit at that point it is necessary to remove more than one unit upstream due to BOD decay.

A reservoir may be considered to be a firm which supplies a product, water, according to the demand from potential users. The analogy of this system to the marketplace is evident if one views other suppliers of waste treatment and augmented flow as producing their product upstream and then transporting it to the market area (the reach) via the river system. The prospect of a reservoir adding low flow augmentation to its purposes may bring about a readjustment of operational decisions.

Two factors determine the usefulness of a parcel of water for low flow augmentation. The first factor is dissolved oxygen concentration, which varies according to the action of oxygen sources and sinks. The second factor is the rate of change in oxygen availability which can be expressed in terms of a reaeration coefficient. The value of this coefficient has been found to decrease with flow. Consequently, the production of flows for assimilative purposes is subject to the principle of diminishing marginal productivity. It is not flow augmentation that is desired per se but rather DO augmentation.

7.3 Physical System as a Uni-Directional Transportation Network

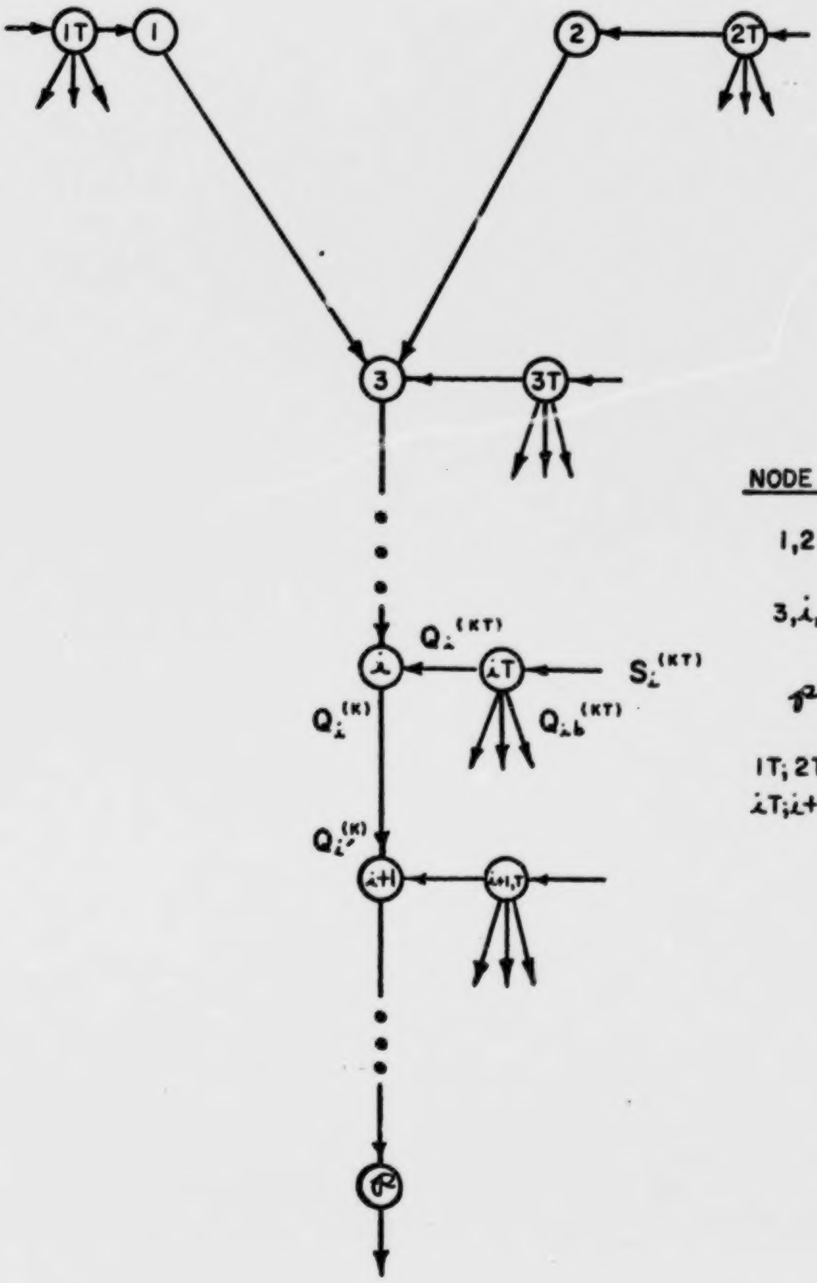
The region under study is partitioned into a set of reaches arrayed in a dendritic configuration. In a network format, a reach consists of two nodes connected by a transport branch, as shown in Figure 7-2. Reaches one and two are headwater reaches. Nodes have been classified into four types, as listed in Figure 7-2. The modeling procedure traces the movement of one or more commodities through the same or slightly modified network. In this study, water, biochemical oxygen demand (BOD) and dissolved oxygen (DO) are routed through the network resulting in a three commodity network problem. The following convention is used for identifying the three commodities.

Commodity	k
water	1
BOD	2
DO	3

In the simplest case, supply and demand related to these commodities as well as their transport properties do not depend on the activity levels of other commodities. In the more general case, which is considered in this section, interdependencies exist between commodity flows.

An important input to the analysis is the supply of water entering the region in each time period. Starting with historical hydrologic data, a simulation program has been developed to generate synthetic traces. The supply of BOD is determined as the product of a BOD coefficient times the appropriate activity level, e.g., population, crop acreage. The estimated demand for water is based on water use coefficients and activity levels. The commodity flow along a given branch is determined by equations of continuity with regard to each commodity. Water is considered to be a conservative commodity.

FIGURE 7-2
NETWORK REPRESENTATION OF RIVER SYSTEM



NODE No.	CLASSIFICATION
1,2	SOURCE
3, i, $i+1$	TRANSSHIPMENT
R	SINK
1T; 2T; 3T; iT; $i+1$, T	TREATMENT

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The flow of BOD and DO are considered to be nonconservative so that it is necessary to describe the associated attenuation and amplification factors.

It is assumed that the activity levels have been projected to the end of the planning horizon. In addition to projected changes in the demand pattern, future modifications in the supply system are also tabulated. Lastly, a criterion was needed to measure the desirability of transporting a commodity along a given branch. The criterion chosen in this study was to minimize the wastewater treatment costs required to satisfy a specified water quality goal for a given quantity of augmented flow. The notation used in the analysis is defined in Table 7-1.

7.4 Objective Function

The analysis of wastewater treatment indicates that the cost functions are convex in the range of concern. The objective is to find the combination of flows of commodity 2 (BOD) which minimize the value of the objective function, Z, or

$$\text{minimize } Z = \sum_{i \varepsilon V_i} f(Q_i^{(2T)}) \quad . \ . \ . \ . \ . \ . \ . \ \text{[Eq. 7.1]}$$

and $f(Q_i^{(2T)})$ is such that for any pair of values of $Q_i^{(2T)}$, say $Q_{i'}^{(2T)}$ and $Q_{i''}^{(2T)}$:

$$f[\zeta \, Q_{i''}^{(2T)} + (1 - \zeta) \, Q_{i'}^{(2T)}] \leq \zeta f(Q_{i''}^{(2T)}) +$$

$$(1 - \zeta) \, f(Q_{i'}^{(2T)}) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ \text{[Eq. 7.2]}$$

where $0 \leq \zeta \leq 1.0$

It is possible to separate this convex function of a single variable as accurately as desired using a piecewise linear function. Clough and Bayer's model (1) uses a logarithmic objective function and critizes the use of a piecewise linear objective function as being less accurate. However existing

TABLE 7-1
LIST OF NOTATION

A_i	Set of all contributing reaches above reach i
F_i	Set of all contributing reaches immediately above reach i
G_i	Set of all recipient reaches downstream from reach i
V_i	Set of all treatment plants in the region (q elements)
$S_i^{(kT)}$	Flow of commodity k into treatment plant in reach i
$S_i^{(k)}$	New supply of commodity k entering reach i
$Q_i^{(kT)}$	Flow of commodity k from treatment plant to reach i
$Q_i^{(k)}$	Flow of commodity k at beginning of reach i
$Q_{i'}^{(k)}$	Flow of commodity k at end of reach i
$\varepsilon_i^{(k)}$	Attenuation or amplification of commodity k in transit through reach i
$\rho_i^{(k'k)}$	Change in flow of one commodity k' through reach i per unit of flow of another commodity k through reach i
$C_{ib}^{(kT)}$	Cost of transporting a unit of commodity k along treatment branch b in ith reach
$Q_{ib}^{(kT)}$	Flow of commodity k along treatment branch b in ith reach
$\bar{Q}_{ib}^{(kT)}$	Upper bound on $Q_{ib}^{(kT)}$
$\underline{Q}_{ib}^{(kT)}$	Lower bound on $Q_{ib}^{(kT)}$
$\underline{Y}_i^{(k)}$	Minimum allowable flow of commodity k in reach i
Z	Value of the objective function
n	Number of treatment branches
p	Number of reaches
q	Number of treatment plants

computer codes permit comparable levels of accuracy to be attained efficiently with piecewise approximations.

The piecewise linear approximations are obtained by partitioning $Q_i^{(2T)}$ into n segments where b = 1,2,...,n. Then, breakpoints, $\bar{Q}_{i1}^{(2T)}, \bar{Q}_{i2}^{(2T)}, \ldots, \bar{Q}_{in}^{(2T)}$, can be defined such that

$$Q_i^{(kT)} + \sum_{b=1}^{n} \bar{Q}_{ib}^{(2T)} = S_i^{(2T)} \quad \ldots\ldots \quad [\text{Eq. } 7.3]$$

Thus the objective is to

$$\text{minimize } Z = \sum_{i=1}^{p} \sum_{b=1}^{n} (C_{ib}^{(2T)})\, Q_{ib}^{(2T)} \quad .\,. \quad [\text{Eq. } 7.4]$$

wherein $0 \leq Q_{ib}^{(2T)} \leq \bar{Q}_{ib}^{(2T)}$ for all treatment segments.

7.5 Physical-Technical Constraints

Given a network format, continuity equations can be written using Kirchhoff node laws. Water is assumed to be a conservative commodity. The description of the nonconservative BOD and DO movements follows the development by Loucks, Revelle, and Lynn (2) of the Camp (3) and Dobbins (4) formulations of the Streeter-Phelps equations. This formulation has been modified to express the commodities in mass units rather than as BOD and DO concentrations. The purpose of this modification is to simplify interpretation of the dual problem.

7.5.1 Water Continuity Equations

The flow of water from the treatment plant to the river in the ith reach, $Q_i^{(1T)}$, is simply the quantity of water entering the treatment plant, $S_i^{(1T)}$, or

$$Q_i^{(1T)} = S_i^{(1T)} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 7.5]$$

The flow of water in the ith interior reach, $Q_i^{(1)}$, is comprised of j upstream inflows, $\sum_{j \varepsilon F i} Q_j^{(1)}$, the inflow from the waste treatment plant, $Q_i^{(1T)}$, and the tributary inflow, $S_i^{(1)}$, or

$$Q_i^{(1)} = \sum_{j \epsilon F_i} Q_j^{(1)} + Q_i^{(1T)} + S_i^{(1)} \qquad ..[\text{Eq. } 7.6]$$

For headwater reaches, $\sum_{j \epsilon F_i} Q_j^{(1)} = 0$, so that

$$Q_i^{(1)} = Q_i^{(1T)} + S_i^{(1)} \qquad ..[\text{Eq. } 7.7]$$

7.5.2 BOD Continuity Equations

The rate of change of BOD concentration with time, dB/dt, is proportional to the concentration of BOD present, B; and to the rate of BOD addition, R, due to runoff and scour; or

$$dB/dt = -(K_1 + K_3)B + R \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad [\text{Eq. } 7.8]$$

where

K_1 = rate constant for deoxygenation: day^{-1}

K_3 = rate constant for sedimentation and absorption: day^{-1}

B = BOD concentration: #/MG

R = rate of BOD addition due to runoff and scour: #/MG/day

Integration of equation [7.8] yields

$$B_t = [B_o - R/(K_1+K_3)] \exp [-(K_1+K_3)t] + R/(K_1+K_3) \quad . \; . \; . \quad [\text{Eq. } 7.9]$$

where B_o = BOD concentration at time 0 : #/MG

B_t = BOD concentration at time t : #/MG

Rearranging equation [7.9] yields

$$B_t = B_o \exp [-(K_1+K_3)t] + R/(K_1+K_3)\{1 - \exp [-(K_1+K_3)t]\}. \quad [\text{Eq. } 7.10]$$

Let $\lambda_t = \exp [-(K_1+K_3)t]$, and

$$\mu_t = (1-\lambda)[R/(K_1+K_3)]$$

Then

$$B_t = \lambda_t B_o + \mu_t \quad \ldots\ldots\ldots\ldots \text{[Eq. 7.11]}$$

Using the notation in Table 7-1,

$B_o = Q_i^{(2)}/Q_i^{(1)}$, and

$B_t = Q_i'^{(2)}/Q_i'^{(1)}$

Substituting for B_o and B_t in equation [7.11] and noting that $Q_i^{(1)} = Q_i'^{(1)}$ yields

$$Q_i'^{(2)} = (\lambda_t)\, Q_i^{(2)} + (\mu_t)\, Q_i^{(1)} \quad \ldots \text{[Eq. 7.12]}$$

where
$\lambda_t = \varepsilon_i^{(2)}$, and
$\mu_t = \rho_i^{(1,2)}$

so that

$$Q_i'^{(2)} = (\rho_i^{(1,2)})Q_i^{(1)} + (\varepsilon_i^{(2)})Q_i^{(2)} \quad \text{[Eq. 7.13]}$$

Given equation [7.13], it is straightforward to write the continuity equations for BOD at the treatment plant node and the head of the reach. For the treatment plant node, the BOD leaving the treatment plant, $Q_i^{(2T)}$, plus the BOD removed at the treatment plant, $\sum_{b=1}^{n} Q_{ib}^{(2T)}$, equals the BOD inflow to the treatment plant, $S_i^{(2T)}$, or

$$Q_i^{(2T)} + \sum_{b=1}^{n} Q_{ib}^{(2T)} = S_i^{(2T)} \quad \ldots\ldots \text{[Eq. 7.14]}$$

The BOD entering the reach, $Q_i^{(2)}$, consists of a water dependent component,

$\sum_{j\varepsilon F_i} \rho_j^{(1,2)} Q_j^{(1)}$; the decayed upstream BOD quantity, $\sum_{j\varepsilon F_i} \varepsilon_j^{(2)}Q_j^{(2)}$; the treatment plant effluent, $Q_i^{(2T)}$; and the BOD contained in the tributary inflow, $S_i^{(2)}$; or

$$Q_i^{(2)} = \sum_{j\varepsilon F_i} [(\rho_j^{(1,2)})\ Q_j^{(1)} + (\varepsilon_j^{(2)})Q_j^{(2)}] + Q_i^{(2T)} + S_i^{(2)}. \quad \text{[Eq. 7.15]}$$

7.5.3 DO Continuity Equations

Lastly, the DO continuity equations are obtained by the following manipulations. The rate of change in DO deficit, dD/dt, is proportional to the concentration of BOD present, B; the existing DO deficit, D; and the rate of change of oxygen production or reduction, M, due to plant photosynthesis and respiration; or

$$dD/dt = K_1B - K_2D - M \quad . . . . \quad \text{[Eq. 7.16]}$$

where
K_2 = reaeration rate constant: day $^{-1}$
D = DO deficit: #/MG
M = oxygen production (M>0) or reduction (M<0) due to plants and benthal deposits: #/MG/day

Integrating equation [7.16] yields

$$D_t = K_1/[K_2-(K_1+K_3)]\left\{[B_o - R/(K_1+K_3)][\exp -(K_1+K_3)t - \exp(-K_2)t]\right\}$$

$$+K_1/K_2\left\{[R/(K_1+K_3) - M/K_1][1-\exp(-K_2)t] + D_o\exp(-K_2)t\right\} . \quad \text{[Eq. 7.17]}$$

Let $\alpha_t = \exp(-K_2)t$
$\gamma_t = K_1\{[\exp(-K_1-K_3)t - \exp(-K_2)t]\}/[K_2-(K_1 + K_3)]$, and
$\beta_t = \gamma_t[R/(K_1 + K_3)] + K_1/K_2[R/(K_1 + K_3) - M/K_1](1-\alpha_t)$

Then, equation [7.17] becomes

$$D_t = (\alpha_t)\, D_o + (\gamma_t)\, B_o + \beta_t \qquad \text{. . . . . [Eq. 7.18]}$$

where D_o = DO deficit at time 0 : #/MG
D_t = DO deficit at time t : #/MG

The following equations,

$$\bar{Q}_i^{(3)} = \bar{Q}_i{}'^{(3)} \qquad \text{. . . . . [Eq. 7.19]}$$

$$Q_i^{(1)} = Q_i{}'^{(1)} \qquad \text{. . . . . [Eq. 7.20]}$$

$$D_o = \bar{Q}_i^{(3)}/Q_i^{(1)} - Q_i^{(3)}/Q_i^{(1)} \qquad \text{. . . . . [Eq. 7.21]}$$

$$D_t = \bar{Q}_i{}'^{(3)}/Q_i{}'^{(1)} - Q_i{}'^{(3)}/Q_i{}'^{(1)} \qquad \text{. . . . . [Eq. 7.22]}$$

where $\bar{Q}_i^{(3)}$ = saturation DO quantity at the beginning of the reach, and
$\bar{Q}_i{}'^{(3)}$ = saturation DO quantity at the end of the reach,

are used to express equation [7.18] in terms of oxygen resource availability. Thus

$$Q_i{}'^{(3)} = (1-\alpha_t)\, \bar{Q}_i^{(3)} + (\alpha_t)Q_i^{(3)} - \gamma_t\, Q_i^{(2)} - (\beta_t)Q_i^{(1)} \qquad \text{. . [Eq. 7.23]}$$

In the above relationship, let

$$\bar{Q}_i{}'^{(3)} = \bar{k}_t^{\ (3,1)}\, Q_i^{(1)}, \text{ and} \qquad \text{. . . . . [Eq. 7.24]}$$

$$\psi_t = \bar{k}_t^{\ (3,1)}\, (1 - \alpha_t) - \beta_t \qquad \text{. . . . . [Eq. 7.25]}$$

where $\bar{k}_t^{\ (3,1)}$ = saturation DO concentration : #/MG.
Then the quantity of DO at the end of the reach is found by substituting Equations [7.24] & [7.25] into Equation [7.23] to obtain

$$Q_i{}'^{(3)} = (\gamma_t)\, Q_i^{(1)} - (\gamma_t)\, Q_i^{(2)} + (\alpha_t)Q_i^{(3)} \qquad \text{. . . . . . . [Eq. 7.26]}$$

where $\psi_t = \rho_i^{(1,3)}$
$\gamma_t = \rho_i^{(2,3)}$
$\alpha_t = \varepsilon_i^{(3)}$
so that

$$Q_i{}'^{(3)} = (\rho_i^{(1,3)})\, Q_i^{(1)} - (\rho_i^{(2,3)})\, Q_i^{(2)} + (\varepsilon_i^{(3)})Q_i^{(3)} \qquad \text{. . [Eq. 7.27]}$$

The DO leaving the treatment plant node, $Q_i^{(3T)}$, is assumed to equal the specified DO availability in the wastewater, $S_i(3T)$, or

$$Q_i^{(3T)} = S_i^{(3T)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [\text{Eq. } 7.28]$$

The DO leaving the reach node, $Q_i^{(3)}$, is comprised of

a. the water dependent upstream DO term, $\sum_{j \varepsilon F_i} (\rho_j^{(1,3)}) Q_j^{(1)}$;

b. the BOD dependent upstream DO term, $\sum_{j \varepsilon F_i} (\rho_j^{(2,3)}) Q_j^{(2)}$;

c. the upstream DO reaeration term, $\sum_{j \varepsilon F_i} (\varepsilon_j^{(3)}) Q_j^{(3)}$;

d. the DO in the wastewater effluent, $Q_i^{(3T)}$; and

e. the DO in the tributary, $S_i^{(3)}$; or

$$Q_i^{(3)} = \sum_{j \varepsilon F_i} [(\rho_j^{(1,3)}) Q_j^{(1)} + (\rho_j^{(2,3)}) Q_j^{(2)} + (\varepsilon_j^{(3)}) Q_j^{(3)}] + Q_i^{(3T)} + S_i^{(3)} . [\text{Eq. } 7.29]$$

7.6 Water Quality Constraints

The DO standard for each reach requires that the quantity of DO leaving the reach, $Q_i{}^{,(3)}$, be not less than a specified minimum quantity, $\underline{Y}_i^{(3)}$. Thus, equation [7.27] is restated as the inequality

$$(\rho_i^{(1,3)}) Q_i^{(1)} - (\rho_i^{(2,3)}) Q_i^{(2)} + (\varepsilon_i^{(3)}) Q_i^{(3)} \geq \underline{Y}_i^{(3)} \quad \ldots\ldots [\text{Eq. } 7.30]$$

The constraints are formulated such that the quantity of DO at the end of the reach shall be greater than or equal to a specified minimum value. This assumes that ${}^{dD}/dt \leq 0$ in equation [7.16] so that the time of travel in the reach is less than or equal to the critical time. This is not a significant restriction on the applicability of the analysis, since the reach selected is arbitrary and so this condition can be satisfied by manipulating the number of reaches.

7.7 Summary of the Model

The complete primal problem is presented below in matrix notation:

$$\text{Minimize } Z = [C^{(2T)}]' \, [Q^{(2T)}]$$

subject to

$$\begin{bmatrix} A^{(1)} & 0 & 0 & 0 \\ P^{(1,2)} & A^{(2)} & 0 & T^{(2)} \\ P^{(1,3)} & P^{(2,3)} & A^{(3)} & 0 \\ D^{(1,3)} & D^{(2,3)} & D^{(3,3)} & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} Q^{(1)} \\ Q^{(2)} \\ Q^{(3)} \\ Q^{(2T)} \end{bmatrix} = \begin{bmatrix} S^{(1)} \\ S^{(2)} \\ S^{(3)} \\ \underline{Y}^{(3)} \\ \underline{Q}^{(2T)} \end{bmatrix} \quad \ldots [\text{Eq. } 7.31]$$

$$[Q^{(1)}],[Q^{(2)}],[Q^{(3)}],[Q^{(2T)}] \geq 0$$

where $[A^{(1)}]= (p+q) \times (p+q)$ matrix of water continuity coefficients;
$[A^{(2)}]= (p+q) \times (p+q)$ matrix of BOD continuity coefficients;
$[A^{(3)}]= (p+q) \times (p+q)$ matrix of DO continuity coefficients;
$[P^{(1,2)}]= (p+q) \times (p+q)$ matrix of water-BOD interdependency coefficients;
$[P^{(1,3)}]= (p+q) \times (p+q)$ matrix of water-DO interdependency coefficients;
$[P^{(2,3)}]= (p+q) \times (p+q)$ matrix of BOD-DO interdependency coefficients;
$[D^{(1,3)}]= (p) \times (p+q)$ matrix of water-DO quality coefficients;
$[D^{(2,3)}]= (p) \times (p+q)$ matrix of BOD-DO quality coefficients;
$[D^{(3,3)}]= (p) \times (p+q)$ matrix of DO quality coefficients;
$[T^{(2)}]= (pxq) \times (nxq)$ matrix of BOD treatment alternatives; and
$[I]= (nxq) \times (nxq)$ identity matrix of upper bounds on BOD removal.

Associated with the primal, or resource allocation, problem is the dual, or resource valuation, problem. The dual problem, presented below in matrix notation, provides important insights as will be shown later.

$$\text{Maximize } Z' = S^{(1)}\,\pi^{(1)} + S^{(2)}\,\pi^{(2)} + S^{(3)}\,\pi^{(3)} + \underline{Y}^{(3)}\pi^{(4)} + \bar{Q}^{(2T)}\pi^{(5)}$$

subject to

$$\begin{bmatrix} A^{(1)'} & P^{(1,2)'} & P^{(1,3)'} & D^{(1,3)'} & 0 \\ 0 & A^{(2)'} & P^{(2,3)'} & D^{(2,3)'} & 0 \\ 0 & 0 & A^{(3)'} & D^{(3,3)} & 0 \\ 0 & T^{(2)'} & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \pi^{(1)} \\ \pi^{(2)} \\ \pi^{(3)} \\ \pi^{(4)} \\ \pi^{(5)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ C^{(2T)} \end{bmatrix} \quad \ldots\ldots \text{[Eq. 7.32]}$$

$[\pi^{(1)}],[\pi^{(2)}],[\pi^{(3)}],[\pi^{(4)}],[\pi^{(5)}]$ unrestricted in sign

where the vector of dual variables are associated as follows:
$[\pi^{(1)}]$ with $[S^{(1)}]$; $[\pi^{(2)}]$ with $[S^{(2)}]$; $[\pi^{(3)}]$ with $[S^{(3)}]$;
$[\pi^{(4)}]$ with $[\underline{Y}^{(3)}]$; and $[\pi^{(5)}]$ with $[\bar{Q}^{(2T)}]$.

The analyst may solve either the primal or dual problem since they are equivalent. Available computer codes such as IBM's Mathematical Programming System/360 contain efficient algorithms for solving either problem. The MPS/360 code employs the bounded variable -- product form of the inverse revised simplex method. The bounded variables routine permits a significant reduction in the size of the problem since the bounds would otherwise have to be included as explicit constraints.

7.8 Post-Optimal Analysis to Determine Regional Waste Management Strategy

The various solutions which may result when attempting to find the optimal combination of wastewater treatment plants for a given water quality standard are described. Then, the effects of considering waste treatment and flow augmentation simultaneously are discussed.

7.8.1 States of the Treatment System

7.8.1.1 Infeasible Solution

Maximum wastewater treatment is insufficient to satisfy one or more of the water quality standards. In this case low flow augmentation is the only available alternative.

7.8.1.2 Present Facilities Adequate

The value of the objective function in this case would equal zero and accordingly the value of $[\pi^{(2)}] = [0]$. The existing wastewater treatment system is adequate and the analysis terminates.

7.8.1.3 Competitive Headwater Treatment Facilities

If $\pi_i^{(2)} > 0$ for one or more of the headwater reaches and $\pi_i^{(2)} = 0$ for all interior reaches then there is no further need to analyze the multiple reach problem since the headwater reaches are, by definition, independent of each other. The analysis may then proceed to independently comparing low flow augmentation with waste treatment at each individual reach.

7.8.1.4 Competitive Interior Treatment Facilities

If $\pi_i^{(2)} > 0$ for one or more interior reaches then it is necessary to further analyze the regional interdependencies which exist to define the solutions of interest. If $\pi_i^{(2)} > 0$ for an interior reach then it necessarily follows that $\pi_j^{(2)} > 0$, for $j \varepsilon A_i$, i.e., a change in BOD discharge of any contributing reach, j, will have some positive impact on the cost of waste treatment at reach i. However, it is not yet known whether the shadow price represents the value of an incremental unit of BOD removal to only one, or more than one, downstream reach. Consequently, it is necessary to examine the dual price vector $[\pi^{(4)}]$ with regard to the water quality standards. If $\pi_j^{(4)} = 0$ for any contributing reaches then these units are treating more than their required amount, and thus reduce the waste treatment costs of downstream units. Assume that $\pi_i^{(4)} > 0$ for only a single reach, i, so that it is known that upstream shadow prices $(\pi_j^{(2)})$ represent the value to the ith reach of a unit of BOD removal at any of the upstream reaches for the specified water quality conditions.

The interpretation of these shadow prices provides important information. $\pi_i^{(2)}$ represents the marginal cost of BOD removal at reach i. Recall that since this waste discharger is located on an interior reach an upstream plant could be requested to increase their waste treatment so that the BOD load entering the ith reach is reduced by one unit. However, because of instream self-purification it is necessary to remove more than one unit upstream in order to effect an equivalent removal of one unit of BOD measured at the inlet to reach i. Accordingly the value of upstream waste treatment diminishes in the upstream direction so that

$$\pi_j^{(2)} < \pi_i^{(2)} \text{ if } \begin{cases} \pi_j^{(4)} = 0 \\ \pi_i^{(4)} > 0 \end{cases} \text{ for } j \varepsilon A_i \quad \ldots\ldots\ldots\ldots \text{[Eq. 7.33]}$$

It is desired to know the relationship

$$\Theta_{ij} = \pi_j^{(2)}/\pi_i^{(2)} \text{ for } j \varepsilon A_i \quad \ldots\ldots\ldots\ldots \text{[Eq. 7.34]}$$

for all upstream reaches. Knowing that $\pi_i^{(4)} > 0$ implies that the associated constraint in the primal problem is binding, i.e., is an equality. Consequently it is possible to calculate Θ_{ij} for all $j \varepsilon A_i$. Knowing this relationship it is then possible to proceed upstream as far as desired by substituting for $Q_i(1)$, $Q_i^{(2)}$, and $Q_i^{(3)}$ and analyzing the commodity 2 terms. From before

$$Q_i^{(2)} = \sum_{j \varepsilon F_i} [(\rho_j^{(1,2)}) Q_j^{(1)} + (\varepsilon_j^{(1,2)}) Q_j^{(2)}] + Q_i^{(2T)} + S_i^{(2)} \text{ . [Eq. 7.15]}$$

$$Q_i^{(3)} = \sum_{j \varepsilon F_i} [(\rho_j^{(1,3)} Q_j^{(1)} - (\rho_j^{(2,3)}) Q_j^{(2)} + (\varepsilon_j^{(3)}) Q_j^{(3)}] + Q_i^{(3T)}$$

$$+ S_i^{(3)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{[Eq. 7.29]}$$

$$(\rho_i^{(1,3)}) Q_i^{(1)} - (\rho_i^{(2,3)}) Q_i^{(2)} + (\varepsilon_i^{(3)}) Q_i^{(3)} = \underline{Y}_i^{(3)} \quad \ldots\ldots \text{[Eq. 7.30]}$$

wherein the last equation is now written as an equality. For a given $j \varepsilon F_i$, substituting equations[7.15] & [7.29] into equation [7.30] & combining all coefficients of $Q_i^{(2T)}$ & $Q_j^{(2)}$ yields

$$-(\rho_i^{(2,3)}) Q_i^{(2T)} - [(\rho_i^{(2,3)} \varepsilon_j^{(2)} + (\rho_j^{(2,3)}) \varepsilon_i^{(3)}] Q_j^{(2)} + \ldots = \underline{Y}_i^{(3)} \text{. [Eq.7.35]}$$

Then, the rate of substitution of $Q_j^{(2)}$ for $Q_i^{(2T)}$ is

$$\Theta_{ij} = [(\rho_i^{(2,3)}) \varepsilon_j^{(2)} + (\rho_j^{(2,3)}) \varepsilon_i^{(3)}]/\rho_i^{(2,3)} \quad \ldots\ldots\ldots \text{[Eq. 7.36]}$$

Equation [7.36] gives the trade-offs between reach i and any contributing reach immediately above reach i. This recursive relationship can be extended further upstream to obtain the rate of substitution between BOD removal at reach i and any upstream reach, $j \varepsilon A_i$. This general relationship is shown in Equation [7.37].

$$(\rho_i^{(2,3)}) \Theta_{ij} = [\rho_i^{(2,3)} (\varepsilon_j^{(2)} \varepsilon_{j-1}^{(2)} \ldots) + \rho_j^{(2,3)} (\varepsilon_{j-1}^{(2)}, \varepsilon_{j-2}^{(2)} \ldots)(\varepsilon_i^{(3)})$$

$$+ \ldots + \rho_2^{(2,3)} (\varepsilon_1^{(2)})(\varepsilon_3^{(3)}, \varepsilon_4^{(3)} \ldots)] \text{ for } j \varepsilon A_i \text{.[Eq. 7.37]}$$

Thus precisely what waste discharger i would pay to have upstream waste treatment at some point j can be found if Θ_{ij} and the marginal cost of waste treatment in reach i are known. Furthermore, the Θ_{ij} are independent of the value of the standard at reach i so that a vector of upstream shadow prices, $[\pi_j^{(2)}]$, can be generated for any binding waste treatment standard at reach i.

7.8.1.5 Competitive Headwater and Interior Treatment Facilities

If $\pi_i^{(2)} > 0$ for at least one interior reach and if $\pi_i^{(4)} > 0$ for at least one headwater reach then the interpretation presented in the last section needs to be modified slightly. Recall that the shadow price with respect to BOD removal, $\pi_i^{(2)} = \partial Z^*/\partial S_i^{(2)}$. But $\pi_i^{(2)}$ can consist of two components: the marginal value of treating at reach i, and the marginal value of treatment at reach i relative to a recipient downstream reach. Thus

$$\pi_i^{(2)} = \partial Z_i^*/ S_i^{(2)} + \sum_{j \varepsilon G_i} (\partial Z_j^*/\partial S_j^{(2)}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 7.38]$$

where G_i = set of all recipient reaches downstream from reach i, so that the total value of the shadow price with respect to $S_i^{(2)}$ can be partitioned into its individual components. The value of each component can be determined using parametric programming in such a way that the water quality constraints in each affected reach are systematically relaxed so that $\partial Z_i^*/\partial S_i^{(4)} = 0$ and the marginal value in all reaches is determined accordingly. This procedure determines $\partial Z^*_i/\partial S_i^{(4)}$ over a prespecified range of $\underline{S}_i^{(4)} \leq S_i^{(4)} \leq \bar{S}_i(4)$.

7.8.2 Flow Augmentation Analysis

Simultaneous consideration of waste treatment and flow augmentation with water of varying quality encompasses a wide range of alternatives. Thus it would be impractical to specify a priori, procedures for analyzing all of these cases. Therefore, approaches for analyzing specific regional situations are presented to illustrate the interpretation of the model. Similar methods can be used when investigating related questions.

7.8.2.1 Assumed Sources of Augmented Flow

The problems of investigating the most effective single source or combination of sources of augmented flow are very similar. First it is desired to determine the value of an additional unit of flow in any of the headwater & interior reaches. The information needed for this analysis is obtained from the optimal solution to the primal problem shown in equation [7.31] or the equivalent dual problem shown in equation [7.32]. The required information is the vector of dual variables associated with $[\underset{\sim}{S}^{(1)}]$. These values are the shadow prices with respect to flow, or $\partial Z^*/\partial S_i^{(1)}$, where $i = 1,2, \ldots, p$. Since the potential sources of flow are at the beginning of the headwater reaches one would deduce that the shadow prices in the upper reaches are greater than or equal to the shadow prices in the recipient reaches downstream, $[G_i]$ or

$$\partial Z^*/\partial S_i^{(1)} \geq \partial Z^*/\partial S_j^{(1)} \text{ For } j \varepsilon G_i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 7.39]$$

Using parametric programming, the relationship for determining flow augmentation benefits can be obtained for a single source or a combination of sources. A graphical illustration relating flow augmentation benefits to the quantity of augmented flow is shown in Figure 7-3. Specific analysis of these cases and illustrations of the applied procedures are shown in more detail in Section 9.

7.8.2.2 Effect of Variation in Quality of Augmented Flow

Once the sources of augmented flow have been investigated for the region, then analyses can begin on the effect of water quality, and the varying activity coefficients which are dependent on flow. The importance of these initial quality conditions would tend to decrease as the water moves downstream. For example, release of a water with zero DO might have a detrimental effect on the first reach but could subsequently contain enough DO to be of positive benefit to users further downstream. An analysis of the effect of augmented flow, with zero DO, and with high BOD is presented in Section 9.

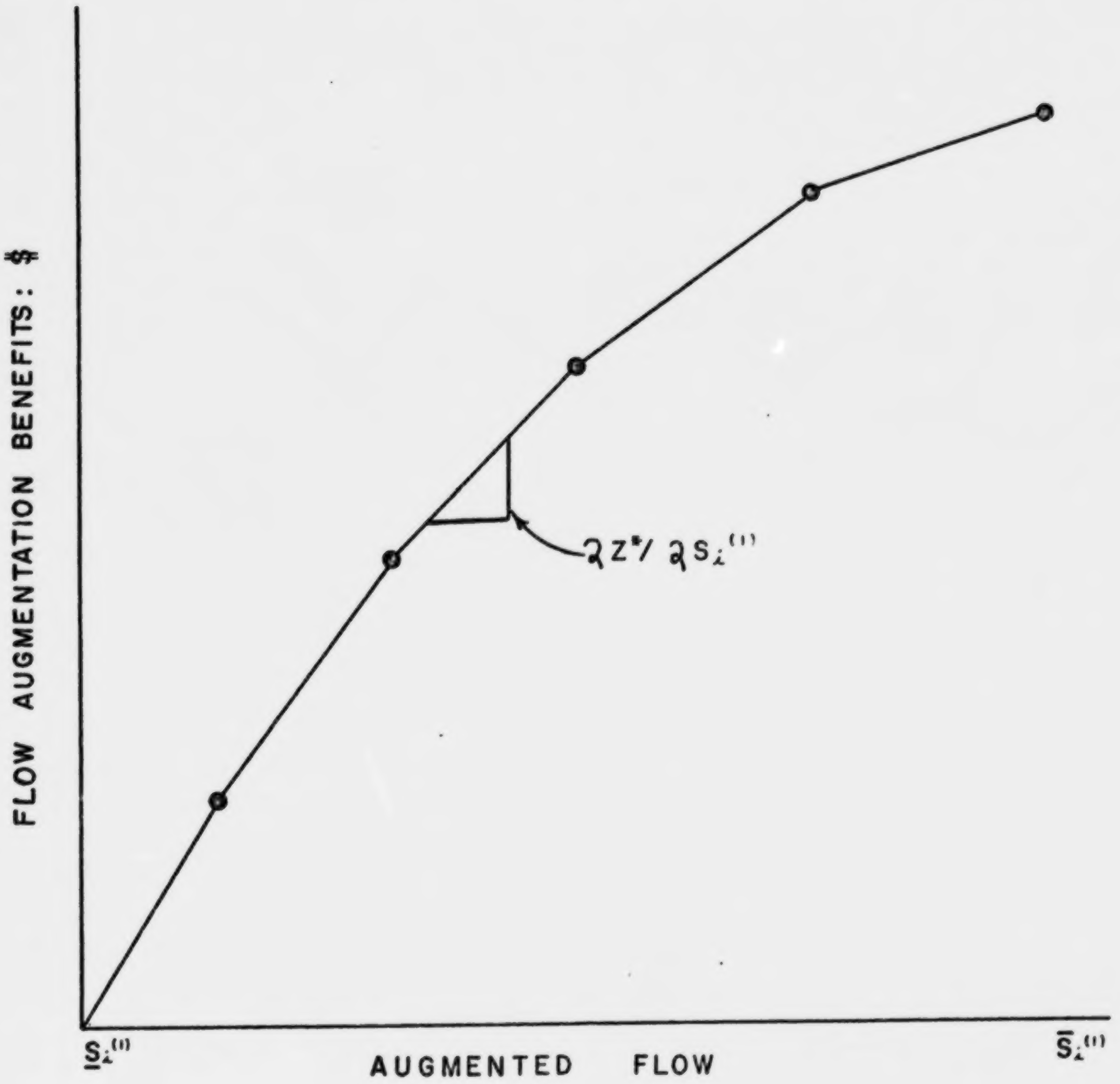
The effect of varying activity coefficients when flow is augmented is an interesting question. If the coefficients in the activity matrix shown in equation [7.31] were independent of flow, then it would be simple to determine the value of low flow augmentation using parametric programming. Unfortunately this is not the case. The dependence of the reaeration coefficient on the river's depth and velocity is well established (5). Also, the rate of deoxygenation is dependent on river discharge. With these factors taken into consideration, the curve shown in Figure 7-3 could be derived by determining the optimal solution for selected quantities of augmented flow. However, it is relatively laborious to manually regenerate the matrix for each selected flow value and the corresponding activity coefficients and then determine the optimal solution. Thus, a computer program was written to expedite the matrix regeneration.

An efficient way to approximate the flow augmentation benefits is by examining limiting conditions. First assume that the coefficients in the model are independent of flow. Using parameteric programming, trace an estimate of the benefit function for the augmented flow with the coefficients of the base flows, $\underline{S}_i^{(1)}$. From these results, select an upper bound on the quantity of low flow that might be desired, $\bar{S}_i^{(1)}$. Then rerun the model with the new matrix for this upper bound. The parameterization in this case takes place from $\bar{S}_i^{(1)}$ to $\underline{S}_i^{(1)}$; i.e., from the maximum flow conditions to the base flow. A specific analysis is shown in Section 9 to illustrate the procedure in more detail.

7.9 Conclusions

The purpose of this section was to present the theoretical developments associated with the optimization model formulation. It is felt that this approach can be applied, in a general sense, to analyzing multiple commodity flows of water, and selected water quality conditions in river systems.

FIGURE 7-3
FLOW AUGMENTATION BENEFITS



The model structure permits direct interpretation of the dual in terms of resource valuation of water, BOD & DO and describes how these valuations change as one moves along the region. A wide variety of analyses can be made without significant modifications in the model. Discussion of the analyses germane to this study are contained in Section 9.

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SECTION 7

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SECTION 8
APPLICATION OF SIMULATION MODEL

8.1 Description of Study Area

The study area selected for application of the methodology developed in this work was the Farmington River Basin in north-central Connecticut and southwestern Massachusetts. This river basin was chosen as being representative of the type of river basin prevalent in the populous eastern and northeastern sections of the United States, where stream pollution problems are and will be most acute.

8.1.1 Overview

The Farmington River is not a large river nor has it been devastated by pollution. In fact, organic waste pollution has affected only one tributary, the Pequabuck River, which drains the industrial area of Bristol. The size of the river, in terms of length, and the river basin, in square miles of drainage area, are of advantage in the application of the methodology because they are large enough to contain most of the features of very large basins yet are small enough so that the exercise is not bogged down by masses of data. As for pollution, the potential is there, for it is anticipated that the eastern third of the basin will develop into very desirable bedroom communities for residents who work in the Hartford business and commercial center and the New Britain-Bristol and Windsor-Windsor Locks industrial areas.

The Farmington River Basin is important to central Connecticut. It supplies water for most of Greater Hartford in a unique system which, by gravity, brings filtered water to much of Hartford. The Basin is an excellent place for the urban population to recreate and its facilities are well used the year around. Sixteen state parks and forests, more than 44 square miles in extent, provide bathing, boating, camping, fishing, picnicking and skiing only a few minutes drive from the state's populated centers (1). Industry in the Basin can be classified as light, but small manufacturing plants are to be found in each of the Connecticut villages. Industry plays the major role in the internal economy of the Basin. Agriculture is important too in that significant acreage in the Basin produces fine leaf tobacco used in wrapping cigars. The southeastern area, between Granby and Plainville, has some of Connecticut's richest and most productive land. Large dairy herds also contribute to the value of the land. And later, the main water courses of the Basin will become mundanely important as the carriers of its waterborne wastes.

8.1.2 Streams

Figure 8-1 is a map of the Farmington River Basin showing the towns and major tributaries. A concise description of the Basin and its streams is contained in the Corps of Engineers Interim Report (2). For the convenience of the reader, this description is reproduced below.

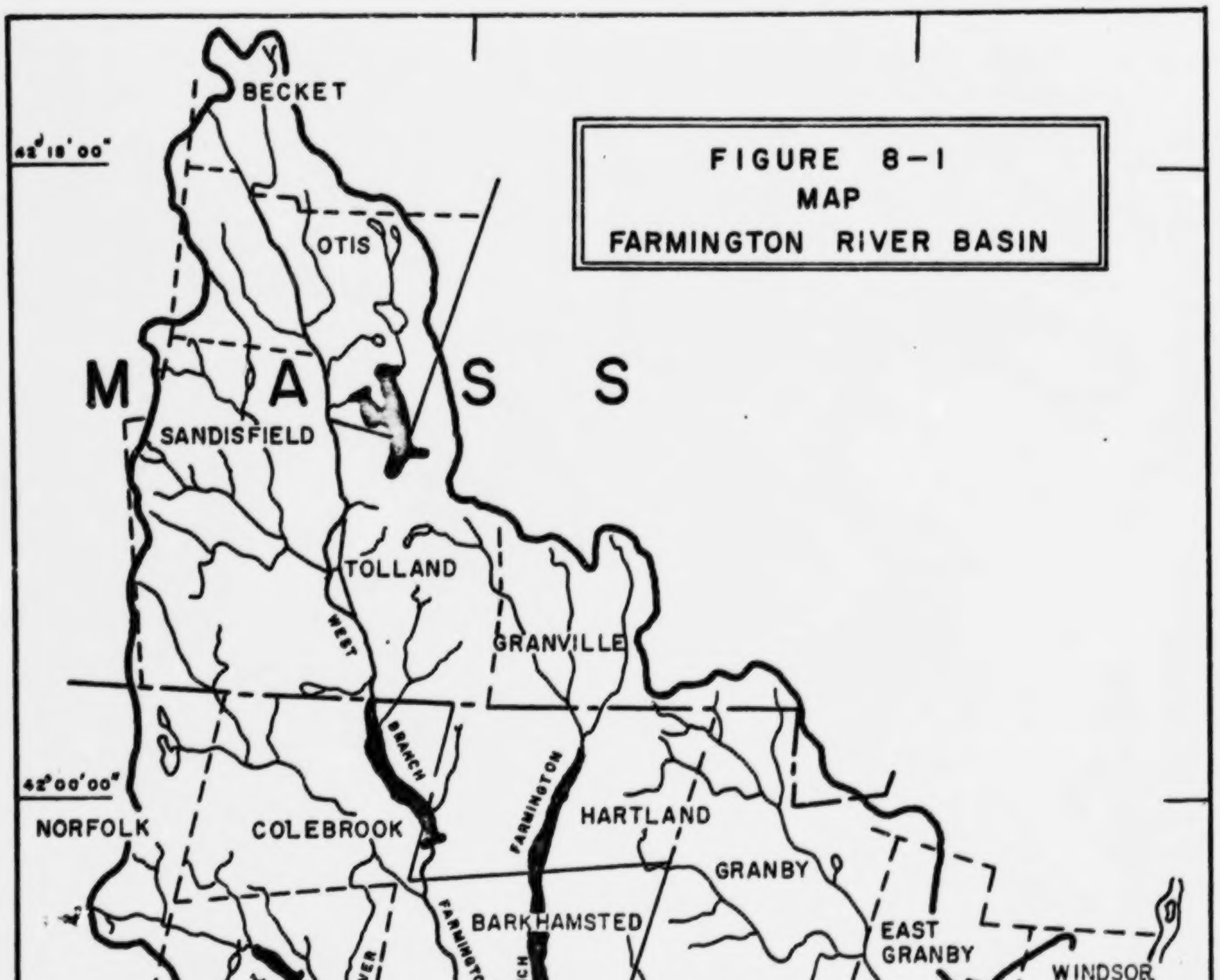


FIGURE 8-1
MAP
FARMINGTON RIVER BASIN

"LOCATION AND EXTENT

The Farmington River Basin, the fourth largest subbasin of the Connecticut River system, is located in southwestern Massachusetts and north-central Connecticut within the confines of Berkshire and Hampden Counties in Massachusetts, and Litchfield and Hartford Counties in Connecticut. The Farmington watershed has a maximum length of 46 miles, a maximum width of 29 miles, and a total drainage area of 602 square miles.

"STREAMS

a. Main stream. The Farmington River is founded by the confluence of the East Branch Farmington and West Branch Farmington Rivers in the northeast corner of the town of New Hartford, Connecticut, at an elevation of about 350 feet above mean sea level. The river flows generally southeast for 15 miles to the mouth of the Pequabuck River in the town of Farmington, Connecticut, thence north for 18 miles to the East Granby-Simsbury, Connecticut, town line, thence southeasterly, joining the Connecticut River in the town of Windsor, Connecticut, 57 miles above Long Island Sound. The Farmington River has a total length of approximately 56 miles and a total fall of about 350 feet.

b. Tributaries. The principal tributaries of the Farmington River are the West Branch Farmington, the East Branch Farmington, the Nepaug, and the Pequabuck Rivers, and Salmon Brook.

(1) The West Branch Farmington River. The West Branch Farmington River rises at Shaw Pond on the Otis-Becket, Mass., town line and flows in a general southeasterly direction for about 33 miles to its confluence with the East Branch Farmington River in the town of New Hartford, Connecticut. It has a drainage area of 236 square miles, and a total fall of about 290 feet.

(2) The East Branch Farmington River. The East Branch Farmington River is formed by the confluence of Hubbard River and Valley Brook in the town of Hartford, Connecticut, a short distance from the Massachusetts-Connecticut border. It flows south for about 12 miles to its confluence with the West Branch Farmington River. It has a drainage area of 65.8 square miles, and a total fall of about 200 feet.

(3) The Nepaug River. The Nepaug River rises at Marsh Pond in New Hartford, Connecticut, flows generally southeast for about 3 miles to the village of Nepaug and thence easterly to its confluence with the Farmington River in the town of Canton, Connecticut. It has a total length of about 10 miles, a drainage area of 31.8 square miles, and a total fall of approximately 448 feet.

(4) The Pequabuck River. The Pequabuck River has its source in Harwinton, Connecticut, flows southeast for about 5 miles, east for about 7 miles, and thence northeast for about 5 miles to its confluence with the Farmington River at Farmington, Connecticut. It has a drainage area of 58.4 square miles and a total fall of about 780 feet.

(5) Salmon Brook. Salmon Brook is about 2.1 miles in length with a total fall of about 10 feet. It is formed by the confluence of 2 branches, each of which has an approximate length of 11.5 miles. The West Branch Salmon Brook rises in Hartland, Connecticut and flows in an east-southeast direction; and the East Branch Salmon Brook has its source in Granville, Massachusetts, and flows in a southerly direction. The 2 branches merge near Granby Station in the town of Granby, Connecticut, to form Salmon Brook, which in turn flows southeast to its confluence with the Farmington River near the village of Tariffville, Connecticut. The West Branch Salmon has a total fall of about 965 feet, and the East Branch Salmon has a total fall of about 505 feet. The combined drainage area of the Salmon Brook and its 2 branches is 67.3 square miles.

(6) Minor tributaries. There are also a large number of smaller streams in the Farmington River system, many of which are sources of high runoff during periods of intensive rain or rapid snowmelt."

Table 8-1 contains information about the locations and drainage areas of the principal tributaries of the Farmington River.

The tributaries, in general, begin as drainage ways from the swamps and ponds which are formed in the upland valleys. The velocity of flow in these upper reaches is often low. Where the tributary leaves these upland areas, the slopes increase and the stream becomes a rushing "mountain" brook. Downstream, where the main stream starts, the slopes are flatter, the channel is wider and deeper and the flow velocity again becomes low.

Consider the main stream (also called the main stem in this work) is the Farmington River extended up the West Branch, Farmington River for its full length. This results in a "main stem" that has a total length of 79 miles.

The upstream-most four miles of the main stem has flat slope and flow is from swamp to pond. For the next 32 miles (to mile 43), the stream slope is steep and the water rushes over the rocky bed. From mile 43 to mile 39, the water passes through a small impoundment over two small dams and through a steep, rocky gorge. At mile 39 the river enters the main valley; slopes are flat, and the channel becomes deeper. Excepting for passage through another gorge, from mile 14 to mile 12, where the slope is again steep, the river flow to the outlet into the Connecticut River is characterized as valley flow.

TABLE 8-1

Principal Tributary Streams
Farmington River

Stream	Drainage Area (sq. miles)	Miles from Mouth of Farmington
Farmington River, at mouth	602.0	0
Salmon Brook	67.2	14.3
East Branch Salmon Brook	33.5	16.4
West Branch Salmon Brook	19.9	16.4
Bissell Brook	6.4	17.7
Hop Brook	13.0	19.5
Pequabuck River	58.4	31.3
Copper Mine Brook	18.1	39.0
Negro Hill Brook	3.9	42.0
Roaring Brook	7.6	35.5
Nepaug River	31.8	42.5
Cherry Brook	13.1	43.1
East Branch Farmington River	65.8	46.3
Hubbard River	20.5	58.3
Valley Brook	7.1	58.3
West Branch Farmington River	236.0	46.3
Still River	86.6	55.0
Sandy Brook	33.8	56.3
Mad River	33.3	60.9
Slocum Brook	9.2	60.7
Clam River	32.0	65.2
Buck River	8.9	67.3

Source: Interim Report on Review of Survey, Farmington River Basin, New England Division, Corps of Engineers, Waltham, Mass. December, 1958.

8.1.3 Streamflow Regulation

Stream flow in the Farmington Basin is regulated by no less than ten reservoirs or impoundments, which are located as shown on Figure 8-2. For a description of the reservoir use, operating rules and other features of each reservoir and control structure, the reader is referred to Appendix A3.2.

Other reservoirs exist in the Farmington Basin, but their effects on the Basin have been neglected. The Whigville Reservoir, owned by the city of New Britain, is a part of that city's water supply system. The tributary area is relatively small, 3.95 square miles, and the reservoir contains only 5 million cubic feet when full. The reservoir was built in 1908. The regulatory effects of Whigville Dam have been neglected because they are embedded in the data of gage 1890, which is located downstream. Similar reasoning accounts for neglecting the several small reservoirs in the city of Bristol water supply system.

8.1.4 Streamflow Data

Records of observed stream flow in the Farmington Basin are obtained and processed by the U. S. Geological Survey. Data are available for thirteen locations in the Basin, as shown in Figure 8-3. A brief description of the location and years of record are listed in Table A3-3 of Appendix A3.

At the time the historical streamflow data on magnetic tape were obtained, in the early months of the project, data were available only through the 1963 water year which ended in September, 1963.

8.1.5 Population and Wastewater Discharges

Population figures, current and projected, for the Farmington Basin are contained in Tables A3-20 and A3-21 of Appendix A3. Waste loads are dependent upon population and, in this work, are a primary reason for interest in numbers of people and their locations. Waste load figures, current and projected, are contained in Table A3-22, Appendix A3. To convey a feeling to the reader of the distribution of population and waste loads in the Basin, Figures 8-4 and 8-5 are included. The principal population centers, with projected 1970 and year 2000 populations indicated, are shown on Figure 8-4 while wastewater discharge rates for like periods are shown on Figure 8-5.

The population and wastewater information was obtained from the report, Water Resources Planning Study of the Farmington Valley by The Travelers Research Center, Inc., Hartford, Connecticut, February, 1965.

8.1.6 Water Supply

The hydrology of the Farmington River Basin, and as a consequence, the water quality in its rivers, is affected to a considerable extent by the diversion of water from the Basin for water supply.

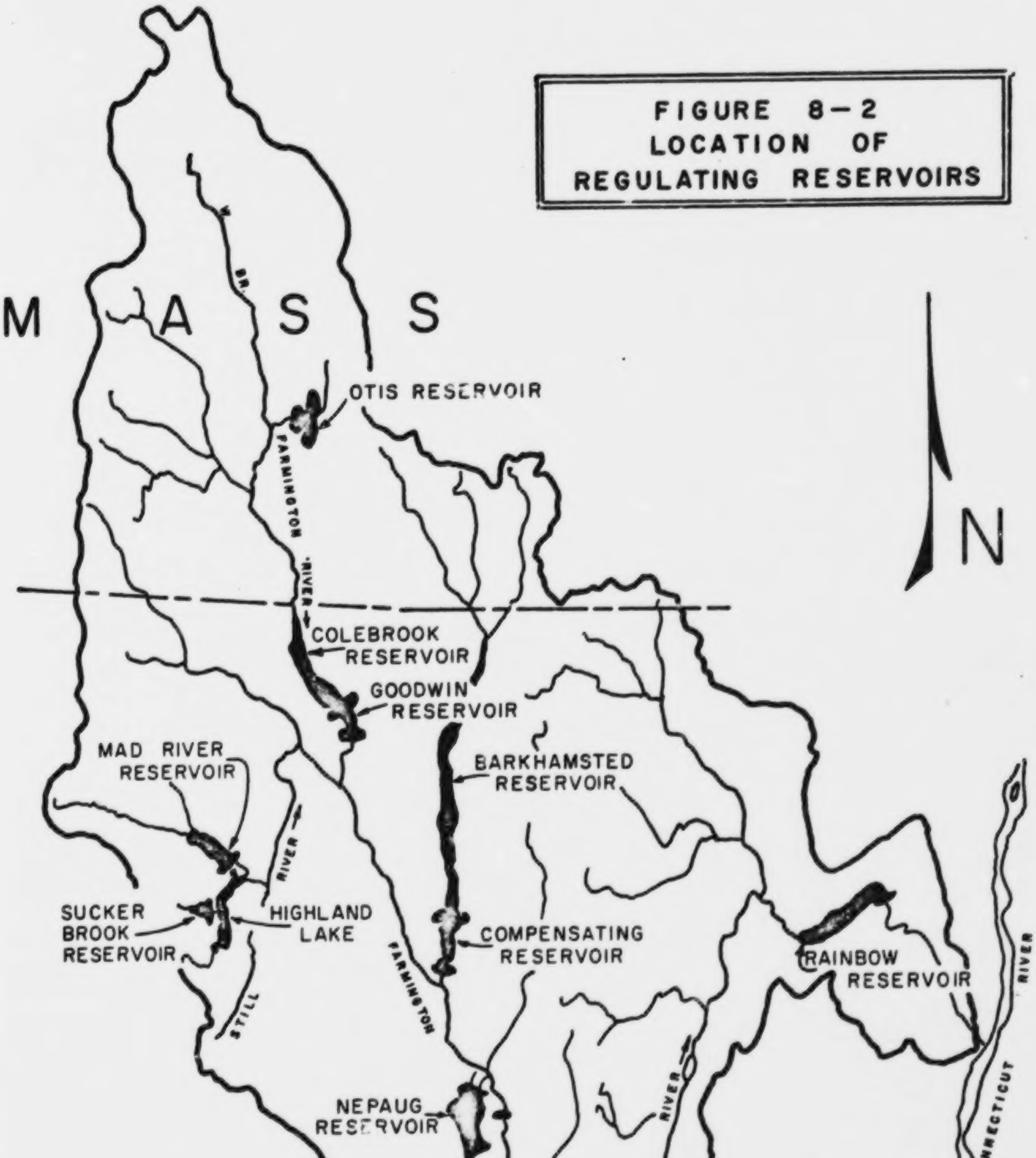


FIGURE 8-2
LOCATION OF
REGULATING RESERVOIRS

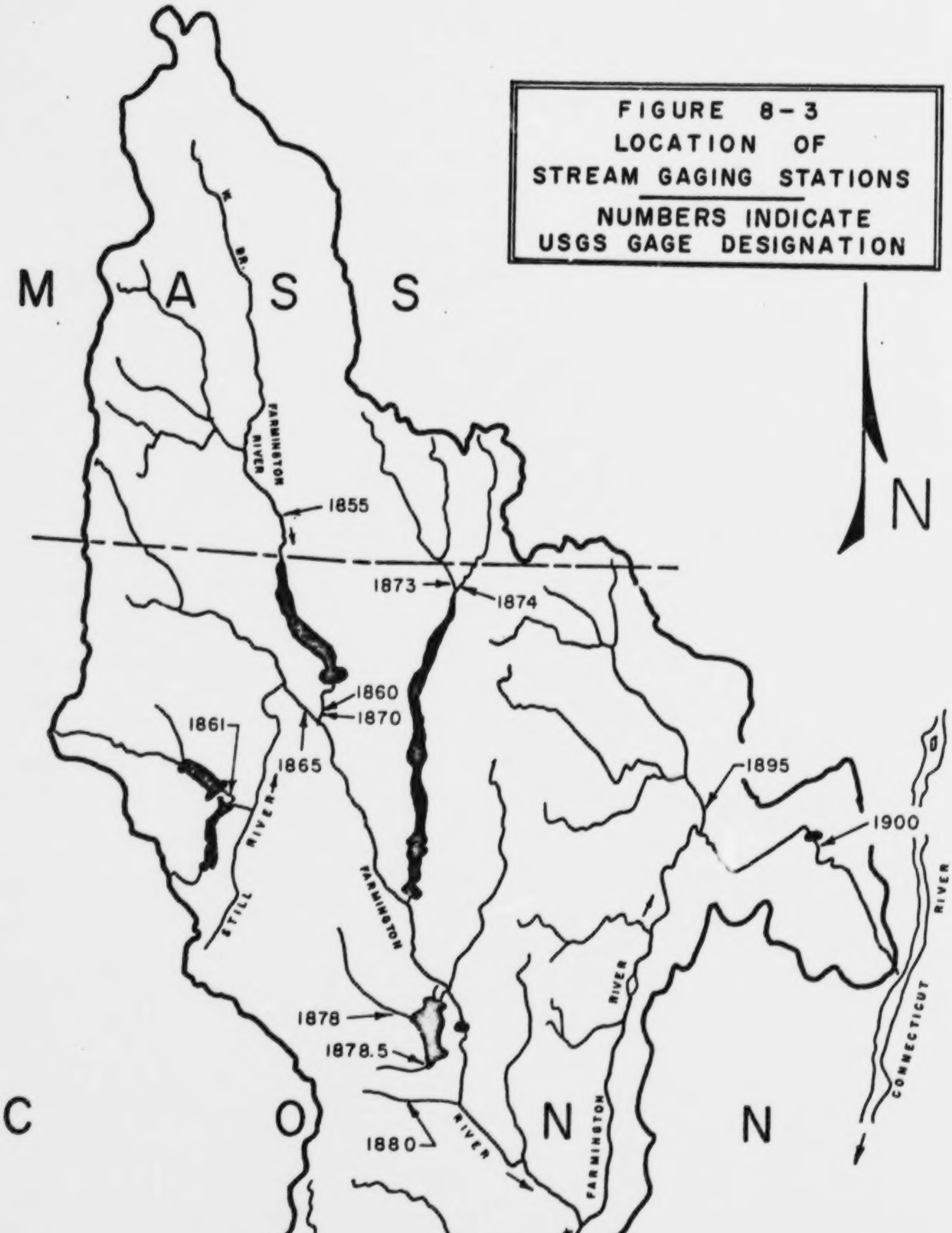
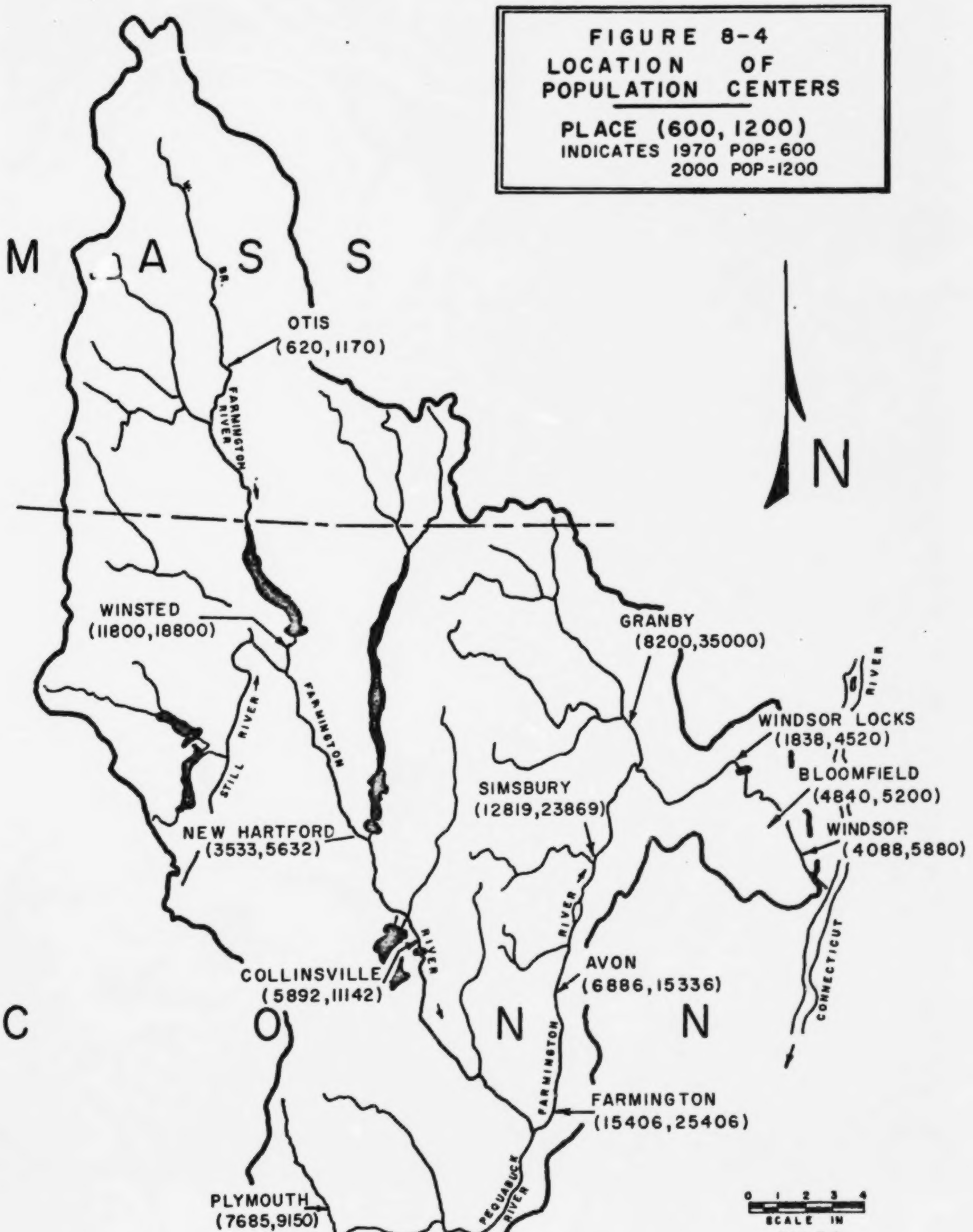


FIGURE 8-3
LOCATION OF
STREAM GAGING STATIONS

NUMBERS INDICATE
USGS GAGE DESIGNATION

FIGURE 8-4
LOCATION OF POPULATION CENTERS

PLACE (600, 1200)
INDICATES 1970 POP=600
2000 POP=1200



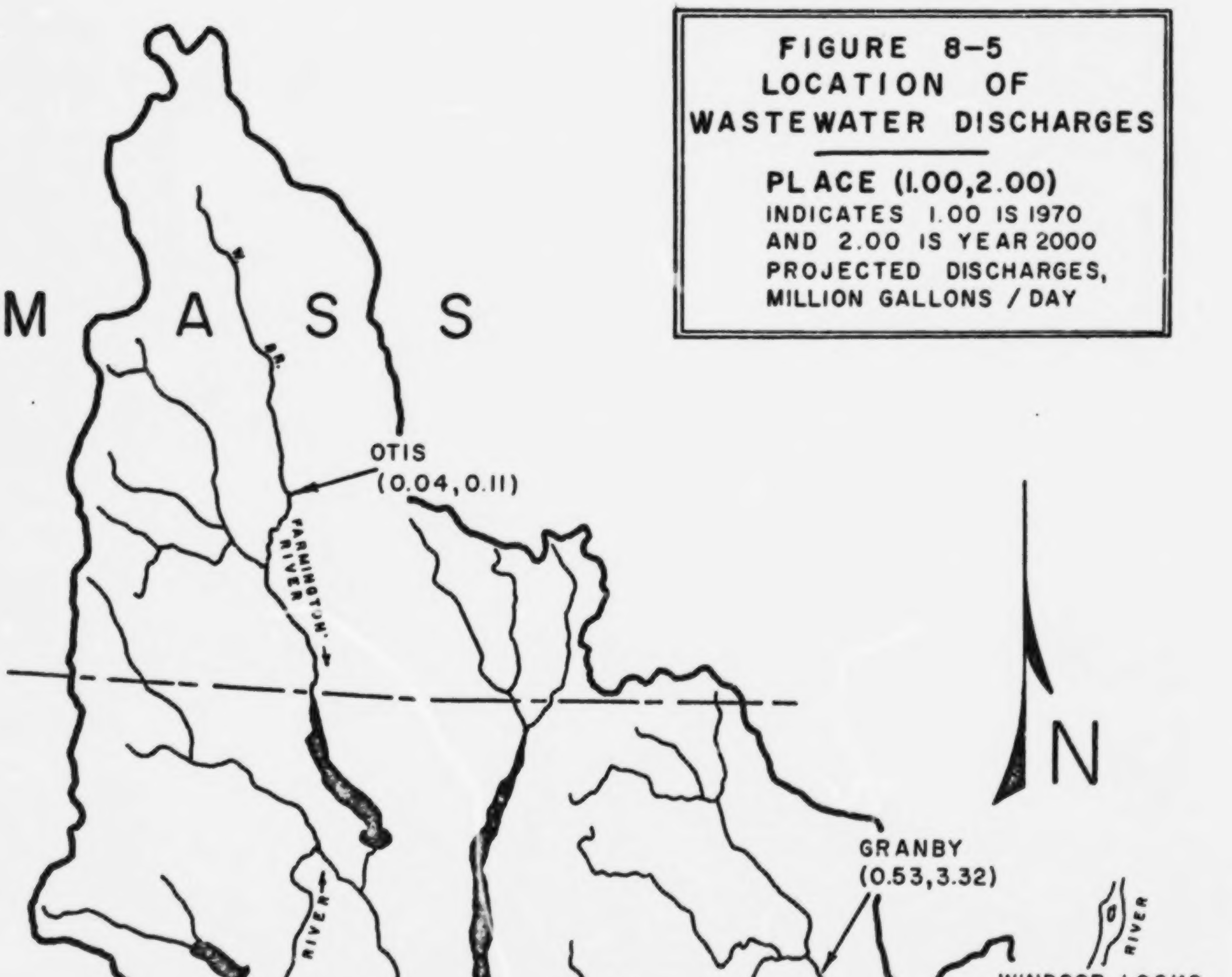


FIGURE 8-5
LOCATION OF
WASTEWATER DISCHARGES

PLACE (1.00,2.00)
INDICATES 1.00 IS 1970
AND 2.00 IS YEAR 2000
PROJECTED DISCHARGES,
MILLION GALLONS / DAY

There is available for diversion approximately 100 million gallons per day (mgd) firm supply (3). At present, Metropolitan District Commission (MDC), the water utility for a major portion of the greater Hartford area, is diverting about 55 mgd from the Barkhamsted and Nepaug Reservoirs. MDC considers that the firm supply from these two reservoirs is 82 mgd. The demand is expected to reach that rate about 1978.

The history of the Hartford water supply is interesting and is briefly related here as background information. Prior to 1916, Hartford obtained water from the eastern slopes of Talcott Ridge which, for a considerable distance, forms the eastern boundary of the Farmington Basin. Refer to the location map, Figure 8-6. In 1913, construction was begun on Nepaug Reservoir and by 1916 water was diverted from the Farmington. To compensate the downstream riparian owners for the loss of water during low flow periods resulting from this diversion, Compensating Reservoir was built on East Branch Farmington River. In 1929, MDC was formed and became the regional water utility. When the demand for diverted water increased, Barkhamsted Reservoir was built a short distance upstream of Compensating Reservoir. Later, in 1961, to replace the compensating water lost for that use by its storage in Barkhamsted Reservoir, Goodwin Dam (Hogback Reservoir) was built on West Branch Farmington River. Compensating water was then supplied from Hogback Reservoir with the stored water in Compensating Reservoir held in reserve.

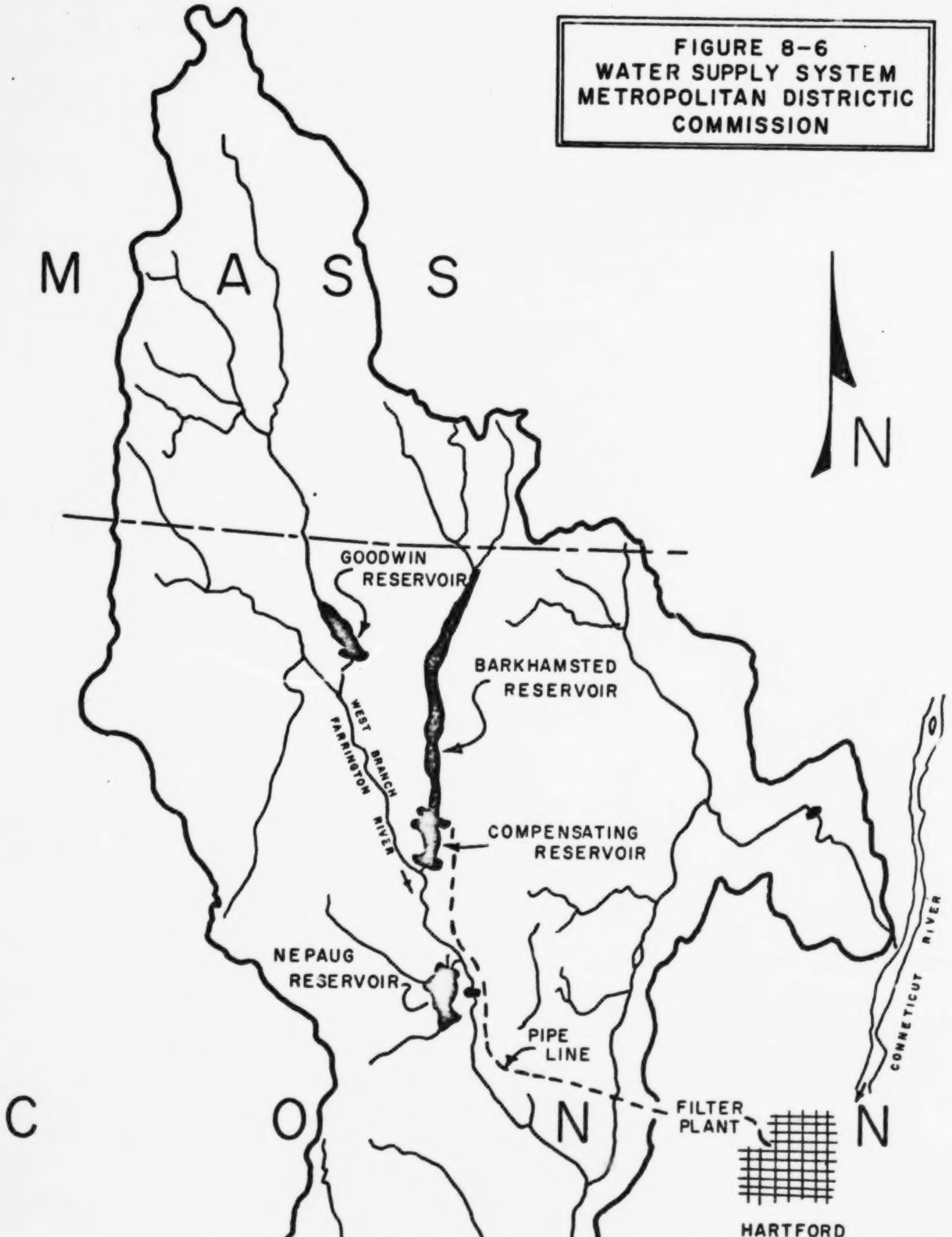
The latest change is the construction, by the Corps of Engineers with MDC participation, of the multi-purpose Colebrook Dam and Reservoir located in the pool of Hogback Reservoir. Additional water will be available for water supply and for compensating riparian owners from Colebrook Reservoir. The plan for development of this additional water supply calls for construction of a tunnel from Hogback Reservoir to Barkhamsted Reservoir to divert the needed additional supply from the West Branch to the MDC system (4). The empty level at Hogback Reservoir is at elevation 540 (overflow is at elevation 640) and the overflow level of Barkhamsted Reservoir is at elevation 530. This system is expected to develop the full 100 mgd water supply capacity of the Farmington River, estimated to provide adequate water supply to MDC until the year 2000 (1).

The operating plan for the completed system is to hold the Barkhamsted pool at elevation 520, ten feet below the spillway level and divert from Hogback Reservoir only enough water to maintain that level. The balance of the system will be operated as it is now (4).

8.1.7 Hydroelectric Development

In the past, the Farmington Basin contained several hydroelectric generating systems but only one of any consequence remains. The system at Rainbow, near Windsor Locks, Connecticut, remains in operation for peaking purposes. The system is owned and operated by the Farmington River Power Company. The fall is about 50 feet at Rainbow Dam and the

FIGURE 8-6
WATER SUPPLY SYSTEM
METROPOLITAN DISTRICTIC
COMMISSION



operating pool contains about 100 million cubic feet of water.

Generation is usually scheduled to meet peak loads without exceeding the limits of pond capacity. Normal gate and flashboard leakage amounts to 25-50 cfs (5). The flow in the Farmington River immediately downstream from Rainbow Dam is this leakage plus the water released through the turbines, the latter being intermittent. Spills, water passing over the overflow, are infrequent.

8.2 Preparation of Input

The simulation model is in two parts, the stream gage flow generator, FLASH, and the stream flow and water quality simulator, WASP. These two parts have been set up as separate entities because the combined program size, about 426,000 bytes, exceeds the normal core storage capacity of many computers. With only minor programming changes, the two parts can be combined.

8.2.1 Input to FLASH

The input to FLASH is prepared by using the three preliminary programs, CHKDATA, NORMAL and TFLOW described in Section 6 and Appendix A4. CHKDATA receives raw stream gage data on cards or tape, edits, fills missing data and computes weekly gage data and outputs this information on tape for input to FLASH. NORMAL scans the tape prepared by CHKDATA and outputs information which guides the operator in selecting the proper transformation. TFLOW also uses the tape prepared by CHKDATA to output information which guides the operator in selection of basis gages.

The input to FLASH consists of the edited data tape from CHKDATA and values, selected by the operator, for a series of control variables which instruct the computer how to proceed and what to output. No deviation from this general method was needed to apply the Farmington River Basin data.

8.2.2 Input to WASP

8.2.2.1 Hydrologic Data

Simulation of hydrology requires synthetic gage data from FLASH, reach and drainage area information and flow regulation information. Reach and drainage area information were prepared from maps and information gathered in reconnaissance of the Basin. The basic data are contained in Appendix A3. The programming in subroutines TGEN and TRAN prepares, internally, the data necessary for simulation of unregulated flows.

Regulation of flows requires that operating rules and physical characteristics of the regulating device, for all possible conditions of operation, be programmed. Operating rules have been programmed for the basic data and information described in Appendix A3.

The reservoir operating rules have been programmed in relation to the depth of water in the reservoir pool and the reservoir capacity at overflow. The physical characteristics of each reservoir have been cast in mathematical terms which are listed in Table 8-2. The equations are "least squares" best fit equations developed from curves or tabulated data.

The correction for evaporation in reservoirs was made using the equation:

$$E = 0.6125 \sin(7.5L+252°44') + 0.8175 \quad . . . . \text{[Eq. 8.1]}$$

which was developed by "least squares" fit of the data contained in Table A3-5 in Appendix A3. E is the evaporation rate in inches per "week" and L is the week of the year (L=1,...48).

The corrections for withdrawals for irrigation were indexed by reach and week and applied at the proper time and place by program checks made in reach and week "do loops." The corrections for the Farmington River are contained in Table A3-23 in Appendix A3.

Water supply diversions from Barkhamsted and Nepaug Reservoirs were programmed as periodic equations in six harmonics, prepared using the program FITCRV (see Appendix A2). The equations are: for Barkhamsted,

$$\begin{aligned} D_B = {} & 21.10 + 10.4923 \cos(L+1.3312) + 4.5649 \cos(2L+0.2589) \\ & + 3.5464 \cos(3L-2.3756) + 2.2172 \cos(4L+1.6153) \\ & + 0.9459 \cos(5L+3.0593) + 1.2392 \cos(6L-0.8807) \quad . . . \text{[Eq. 8.2]} \end{aligned}$$

and for Nepaug:

$$\begin{aligned} D_N = {} & 25.25 + 7.9133 \cos(L-2.7662) + 2.9285 \cos(2L-3.0423) \\ & + 1.9733 \cos(3L+0.6910) + 2.1480 \cos(4L-1.6692) \\ & + 0.7230 \cos(5L+2.1691) + 0.6434 \cos(6L-0.8756) \quad . . . \text{[Eq. 8.3]} \end{aligned}$$

D is the diversion rate in million cubic feet per "week" and L is the week of the year. These equations were developed from data contained in Tables A3-10 and A3-16, Appendix A3.

8.2.2.2 Water Quality Data

Simulation of water quality requires the regulated flow in each reach, furnished by the hydrology simulation described above, temperature data, waste load data including rate of flow, BOD and DO concentration, values for parameters K_1, r and s for river reaches, values for parameters K_2 and K_3 for reservoir reaches and initial values for the BOD and DO in the reservoirs. In addition, values for constants c,f,k and m are needed

TABLE 8-2

Capacity-Depth and Area-Depth Equations
Farmington Basin Reservoirs

Reservoir		Equation**
Otis	Capacity:	$C = -0.55 + 6.0d + 0.86d^2$
	Area:	$A = 1.65 + 1.95d$
Colebrook	Capacity:	$C = 537 - 12.01d + 0.1614d^2$
	Area	$A = 0.4 + 0.1692d + 0.0005d^2$
Barkhamsted*	Capacity	$C = 2009 + 71.5d + 0.567d^2$
	Area:	$A = 70 + 1.167d$
Sucker Brook	Capacity	$C = -0.535 + 0.104d + 0.0205d^2$
	Area:	$A = 0.0408d + -.161$
	Outlet:	$Q = 16.25d^{0.4874}$
Rainbow	Capacity:	$C = 81.8 - 5.33d + 0.15d^2$
	Area:	$A = 1.69 + 0.006d + 0.0031d^2$
Goodwin	Capacity:	$C = -14 + 0.073d + 0.04d^2$
	Area:	$A = -2.63 + 0.14d - 0.0004d^2$
Nepaug*	Capacity:	$C = 494 + 26.5d + 0.21d^2$
	Area:	$A = 25 + 0.5d$
Compensating	Capacity:	$C = 90.8 + 7.2d + 0.21d^2$
	Area:	$A = 7.10 + 0.41d$
Highland Lake	Capacity:	$C = 233.5 + 13.46 + 0.32d^2$
	Area:	$A = 13.72 + 0.624d$
Mad River	Capacity:	$C = 25.3 - 1.944d + 0.0276d^2$
	Area:	$A = 0.1122 - 0.0015d + 0.0003d^2$
	Outlet:	$Q = 42.2d^{0.48}$

*For top 24 feet in reservoir.

**C in million cubic feet, A in million square feet, d in feet, Q in cfs.

to compute velocity and depth by equations [6.7] and [6.8].

River water temperature data were available for only one location in the Farmington Basin and for slightly more than one year. The data are contained in Table A3-6 in Appendix A3. These data are expressed by the equation:

$$T = 12 \sin (7.5L+240°) + 11 \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. } 8.4]$$

obtained by a least squares fit of the data. In equation [8.4], T is the temperature in degrees centigrade in the Lth week of the year. The mean recorded temperature is 11°C. There were not enough data to be able to compute the standard deviation. In the Farmington simulation, temperature was considered deterministic and had the value given by equation [8.4].

Data on waste loads discharged by treatment plants in the Farmington River Basin are meager. Determinations are not made by the plant operators, but the Connecticut Water Resources Commission does make infrequent (average of two per year) determinations on one-day composite samples.

In the simulation runs made, the waste discharge rates have been computed on the basis of: (1) 100 gallons per capita per day, (2) a raw sewage BOD of 220 mg/l and 90 per cent BOD removal by treatment, leaving a BOD concentration of 22 mg/l in the discharged waste, (3) a DO concentration of 2.0 mg/l in the discharged waste, (4) K_1 at 20°C equal to 0.276 and (5) values of r and s equal to 0.005. In the reservoir reaches, values of K_2 at 20°C have been assumed 0.10 for Otis, Colebrook, Sucker Brook, Goodwin, Nepaug and Highland Lake and 0.15 for Barkhamsted, Rainbow, Compensating and Mad River Reservoirs; the lower value is used for deeper and more wind-protected impoundments. The value for K_3 of 0.005 was used in each reservoir. Values of initial BOD and DO concentrations in the reservoir were each placed at 0.2 mg/l, excepting for Rainbow Reservoir which is downstream of waste discharges. Values of 0.5 and 1.5 mg/l, respectively, were assumed for Rainbow. The effect of these initial values is removed after one or two weeks of simulation.

The evaluation of the constants c,f,k and m for the velocity and depth equations was made by analyzing the plots of data from Table A3-4, Appendix A3.

Recall that a requirement is that the frequency distribution of the flows must be the same for various points along the river to be able to use the relationships in equations [6.6], [6.7] and [6.8]. A plot of the cumulative frequency of the data for the four stations for which cross section-velocity data were available is shown as Figure 8-7. This plot shows that the frequency distributions of the flows at the four stations are nearly the same, so that it will be acceptable to use these relationships. Incidentally, the cumulative frequency data used to plot Figure 8-7 are obtainable from the output of NORMAL.

Data for stations 1-1860, 1-1875, 1-1890 and 1-1895, for flow versus width, flow versus depth and flow versus velocity are plotted as shown on Figures 8-8 through 8-19. Straight lines were drawn by eye through the plotted points and the coefficients were determined. The values found result in the following equations:

$$w = aQ^b = 36.68Q^{0.11} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 8.5]$$

$$d = cQ^f = 0.241Q^{0.33} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 8.6]$$

$$v = kQ^m = 0.113Q^{0.56} \quad \ldots\ldots\ldots\ldots \quad [\text{Eq. } 8.7]$$

The values of c,f,k and m determined above give reasonable values of velocity and depth in the Farmington River. During August, 1968 cross sections were measured* and, where possible, current meter velocity readings were taken. Unfortunately, the flow in the river was so low at that time that depths were too small to accommodate the meter, or, if the depth was great enough, the flow was pooled by downstream obstructions. In one cross section, at the Old Farmington Bridge near Farmington, Connecticut, the flow was measured at 117 cfs, average depth was 0.96 feet, width at surface was 78 feet and the average velocity was 1.58 feet per second. Application of equations [8.5], [8.6] and [8.7], using Q=117 cfs, gives d=1.16 feet, w=62 feet and v=1.63 feet per second.

8.3 Simulation Results

8.3.1 Preliminary Programs

The application of the programs CHKDATA, NORMAL and TFLOW to the Farmington River historical stream gage data is described below.

The program CHKDATA was used to check, edit and compute average weekly or average monthly historical gage data. The "raw" data used were obtained on magnetic tape from the U. S. Geological Survey, Washington, D. C. These data contained no missing values. However, data values were removed to test the program's ability to: (1) detect and fill missing data values if less than 30 consecutive days of data were missing, (2) detect and call EXIT if more than 30 consecutive days of data were missing, and (3) detect and call EXIT if, for all years of data being checked, all data for one day of the year were missing. The program functioned properly in these tests. In addition, CHKDATA was asked to output successively, daily data, average weekly data and average monthly data. It performed as required.

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*by G. R. Grantham

FIGURE 8–7
HISTORICAL DATA
CUMULATIVE FREQUENCY

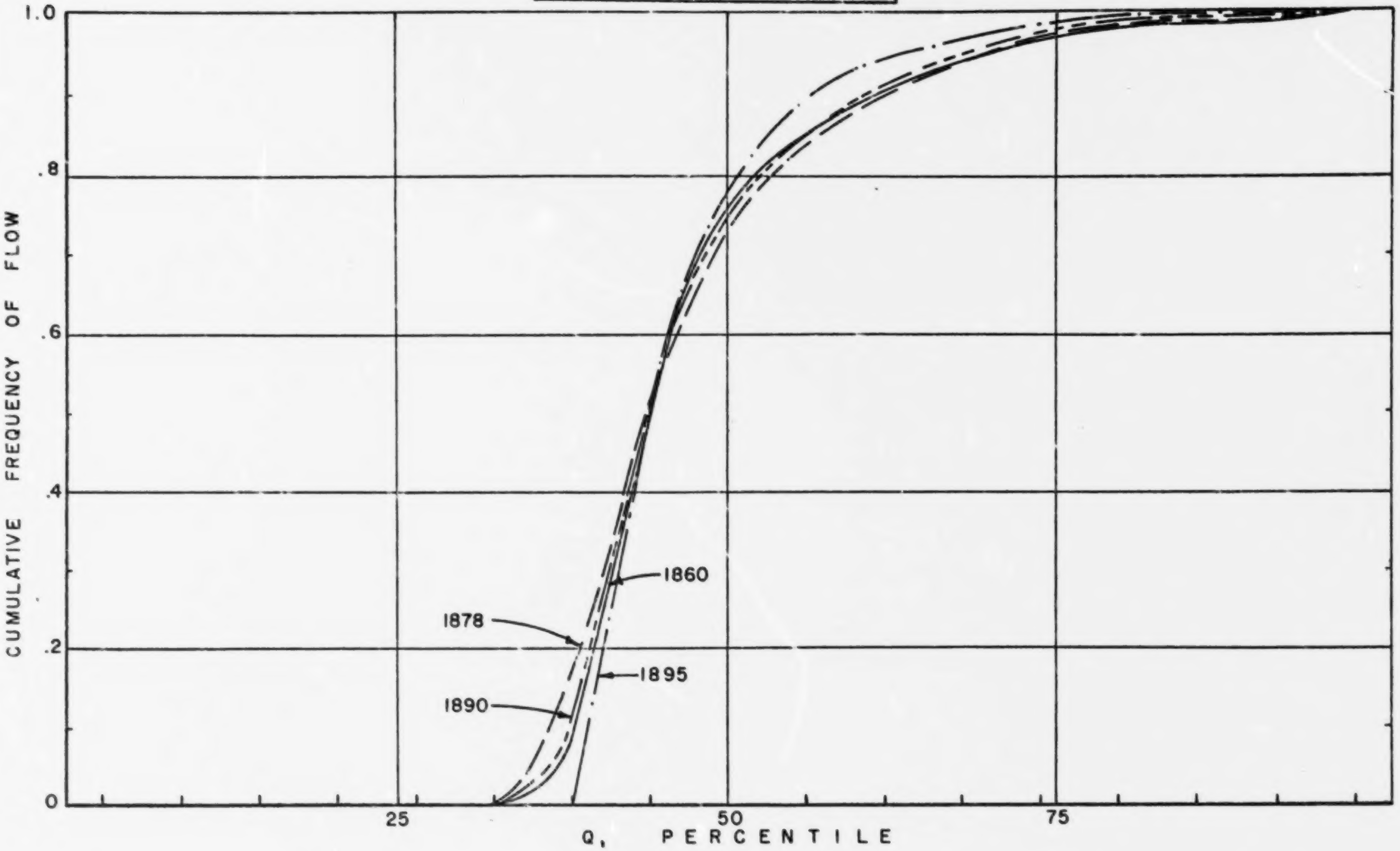


FIGURE-8-8
CONSTANT EVALUATION
HYDRAULIC FORMULAS

GAGE 1860

WIDTH- Q

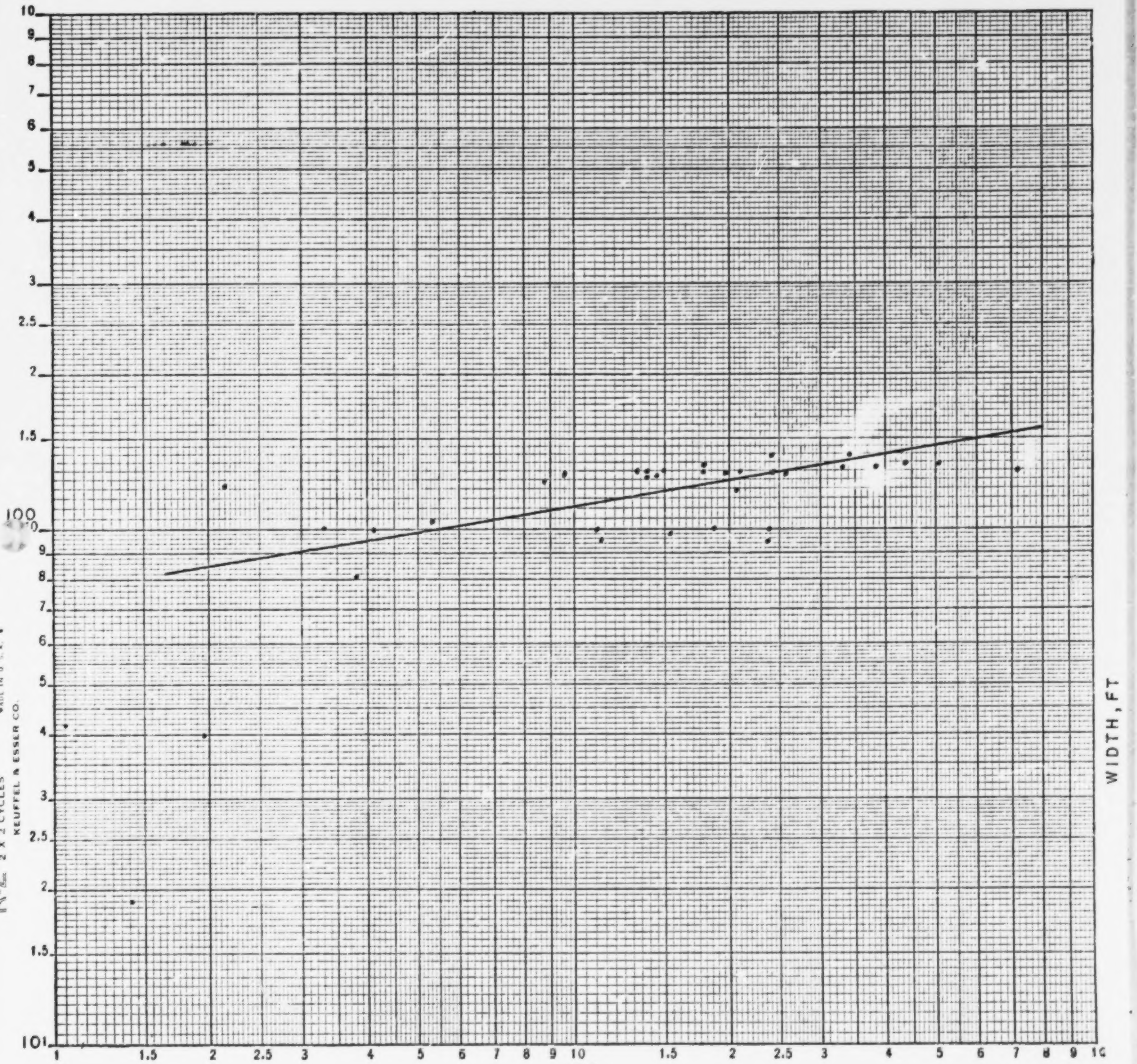


FIGURE 8-9
CONSTANT EVALUATION
HYDRAULIC FORMULAS

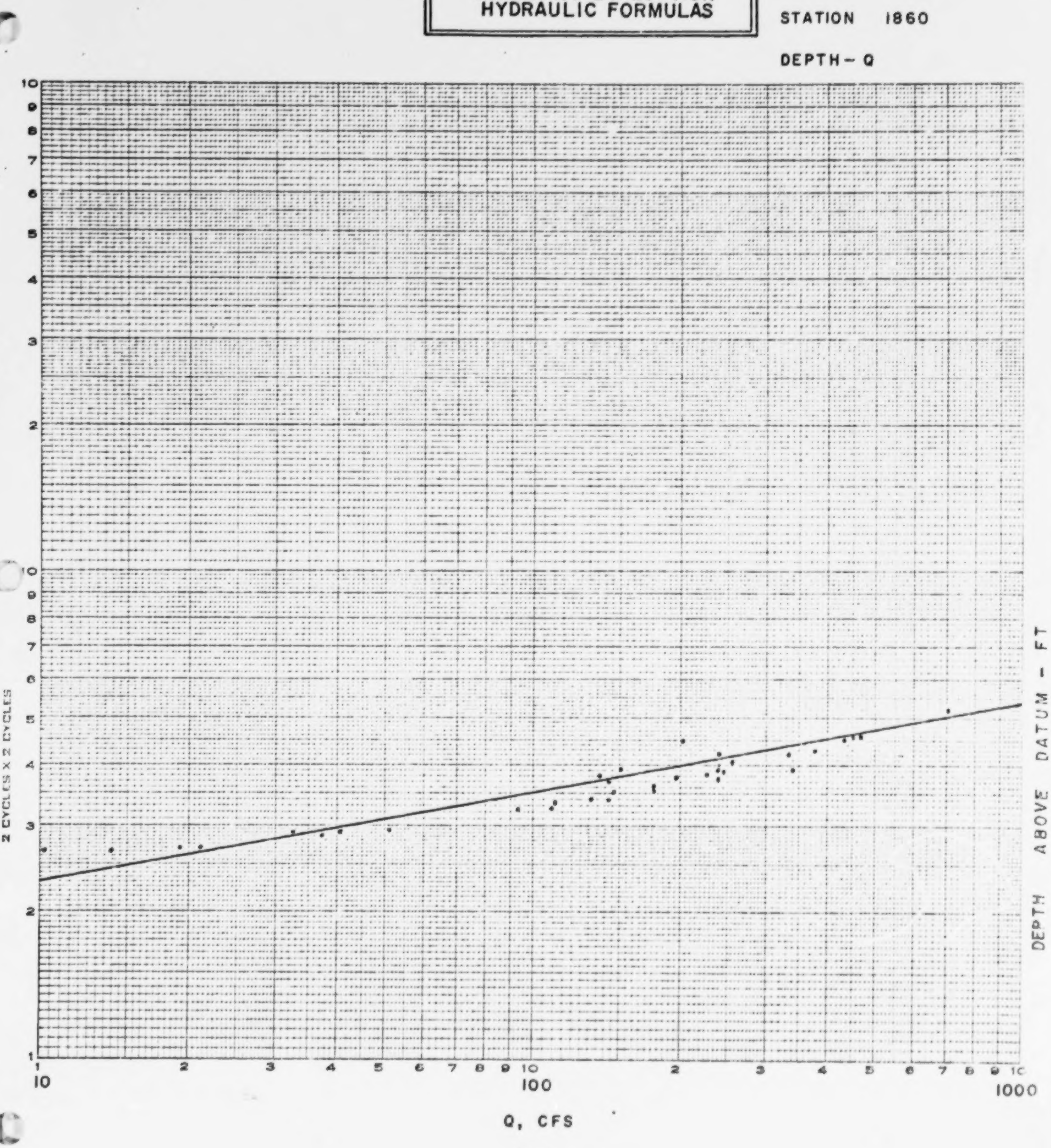


FIGURE 8-10
CONSTANT EVALUATION
HYDRAULIC FORMULAS

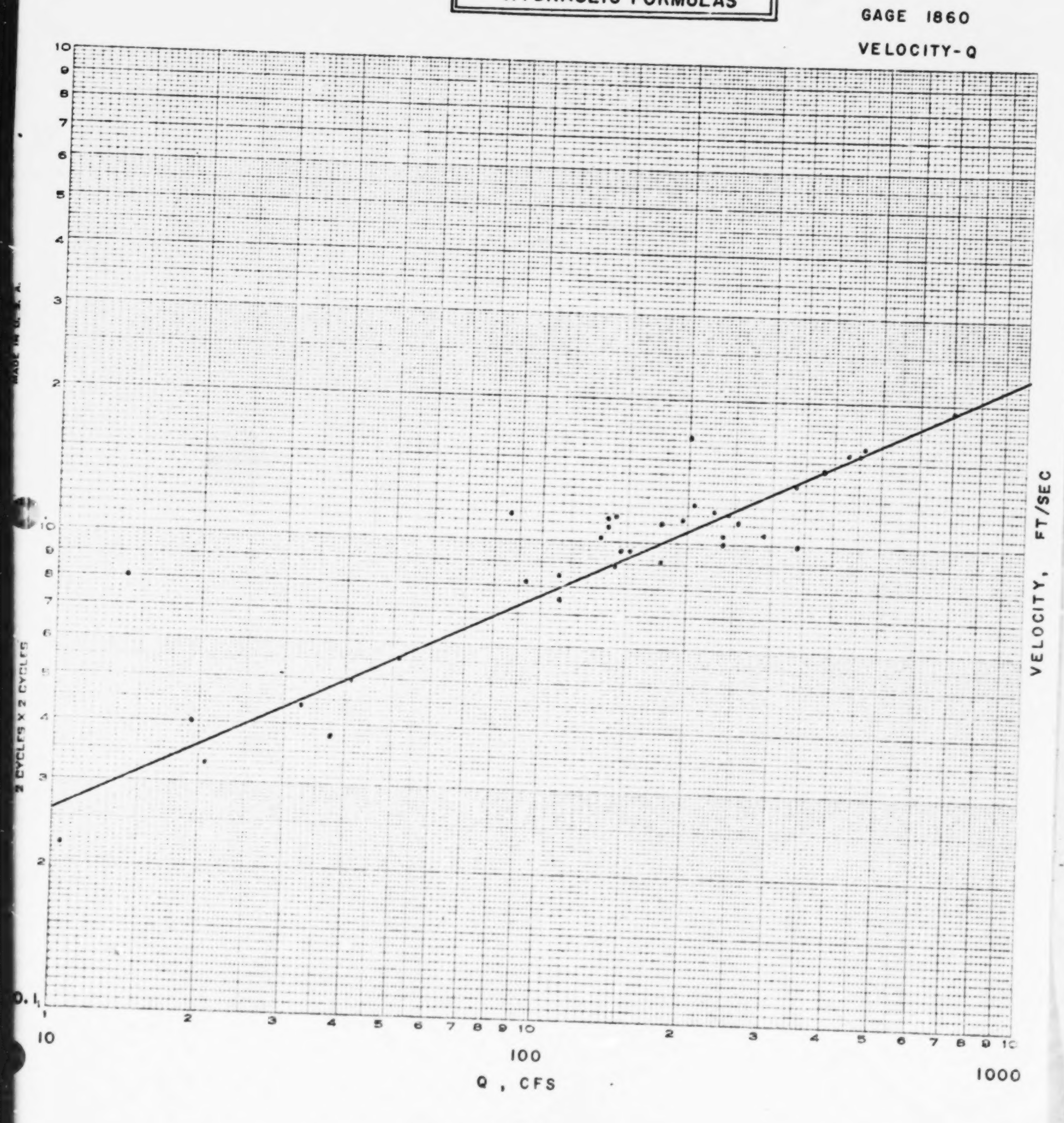


FIGURE 8-11
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1878

WIDTH – Q

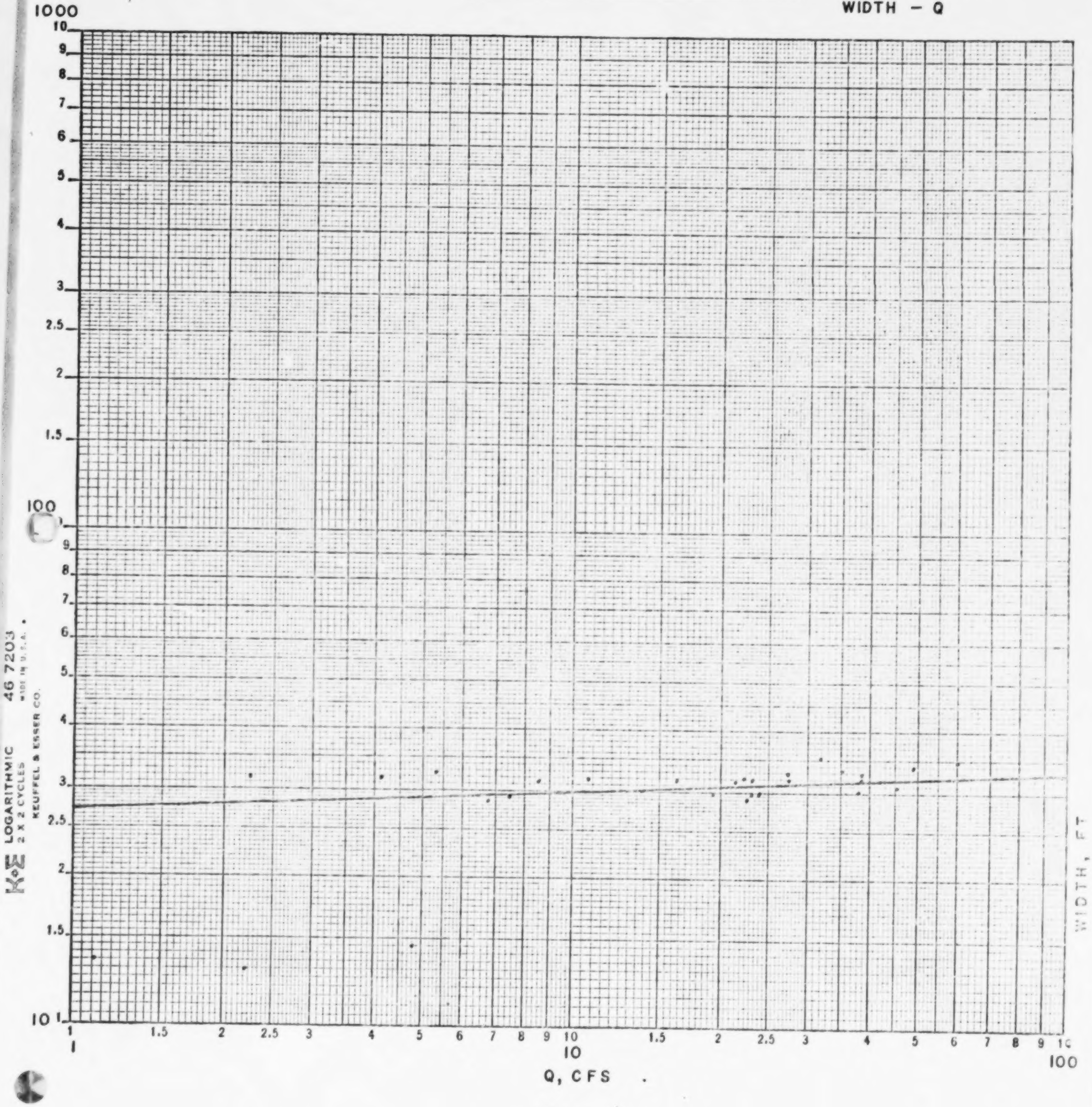


FIGURE 8-12
CONSTANT EVALUATION HYDRAULIC FORMULAS

STATION 1878
DEPTH — Q

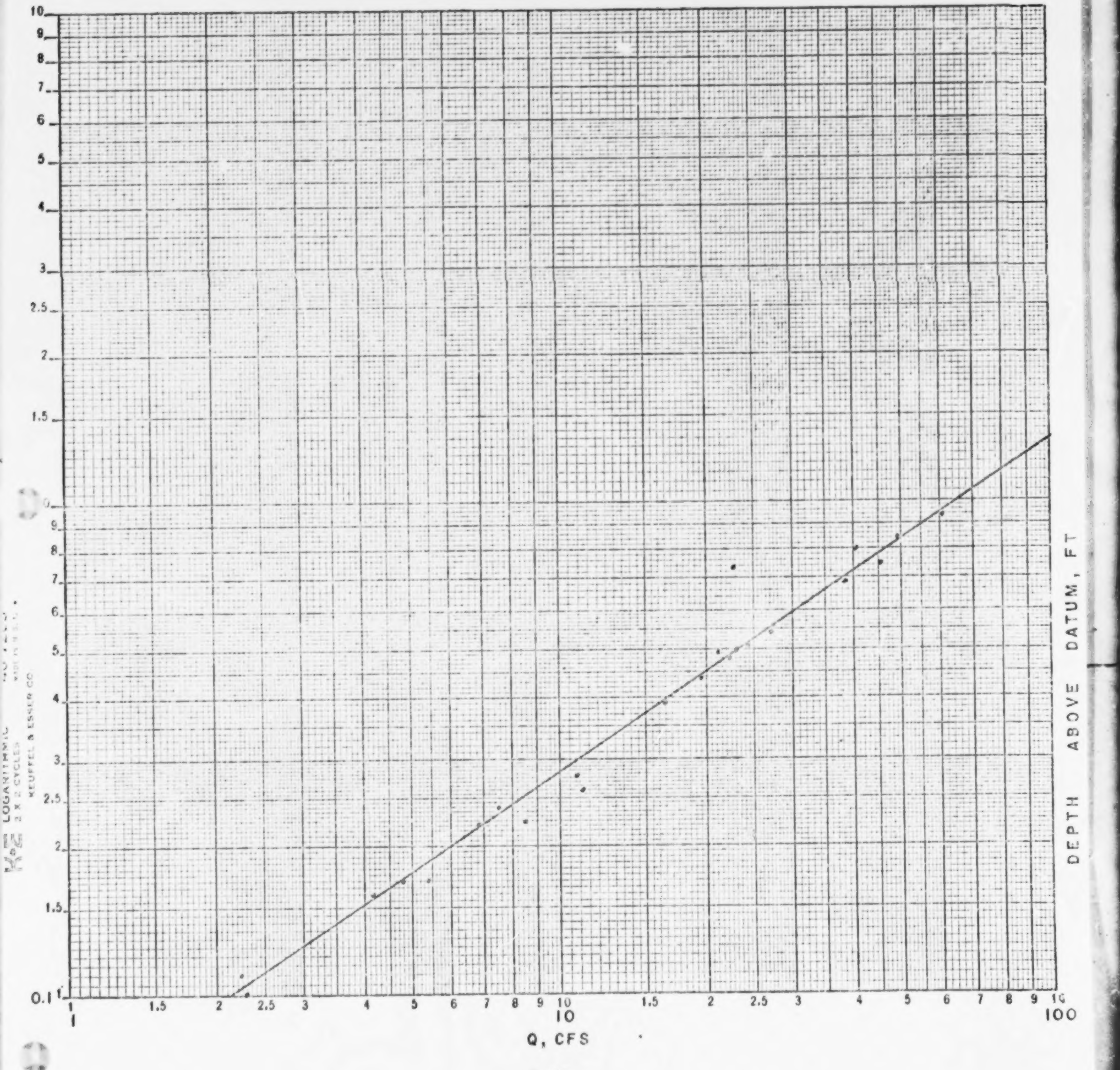


FIGURE 8-13
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1878

VELOCITY – Q

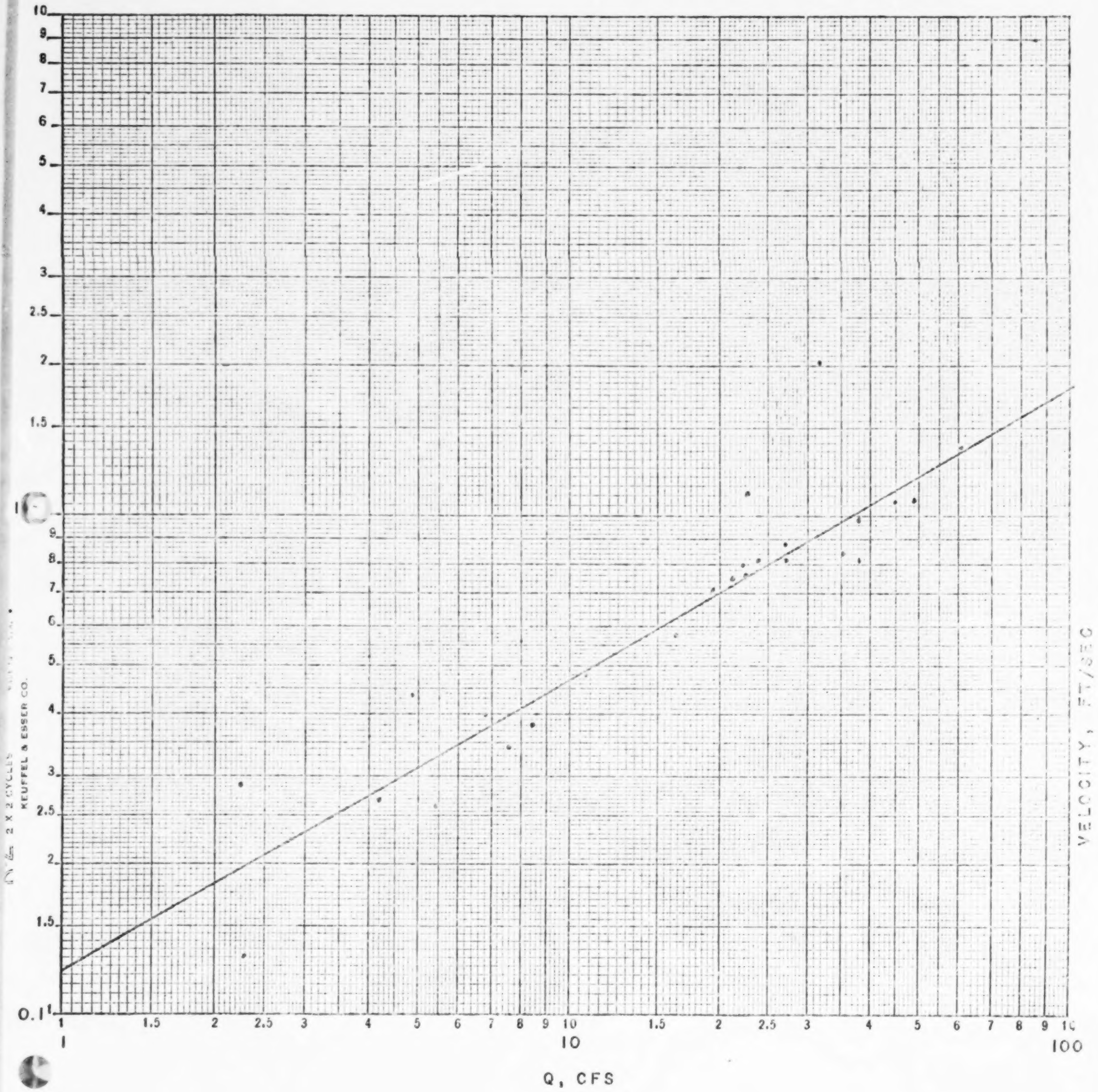


FIGURE 8-14
CONSTANT EVALUATION HYDRAULIC FORMULAS

STATION 1890

WIDTH - Q

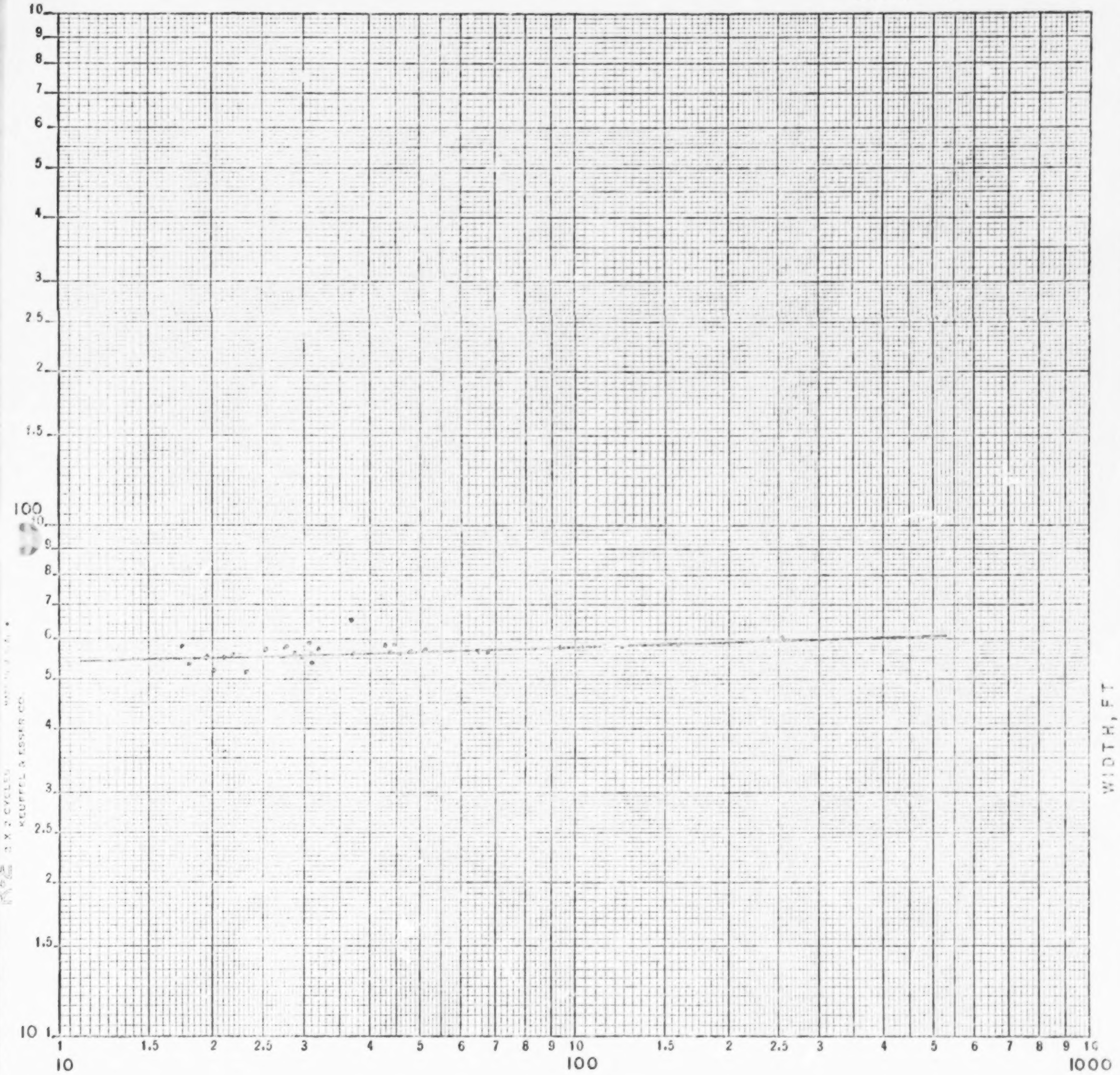


FIGURE 8-15
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1890

DEPTH—Q

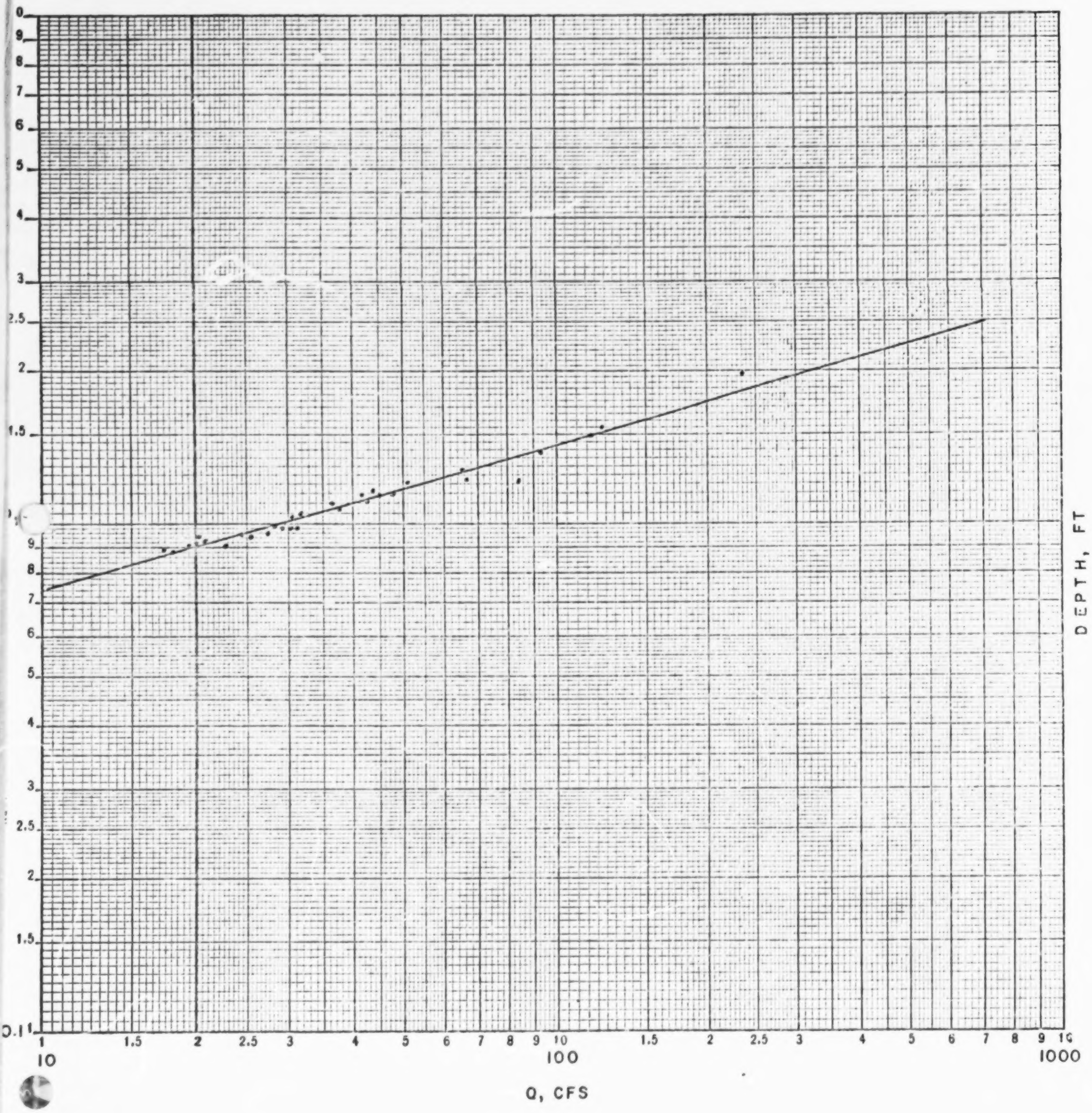


FIGURE 8-16
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1890
VELOCITY – Q

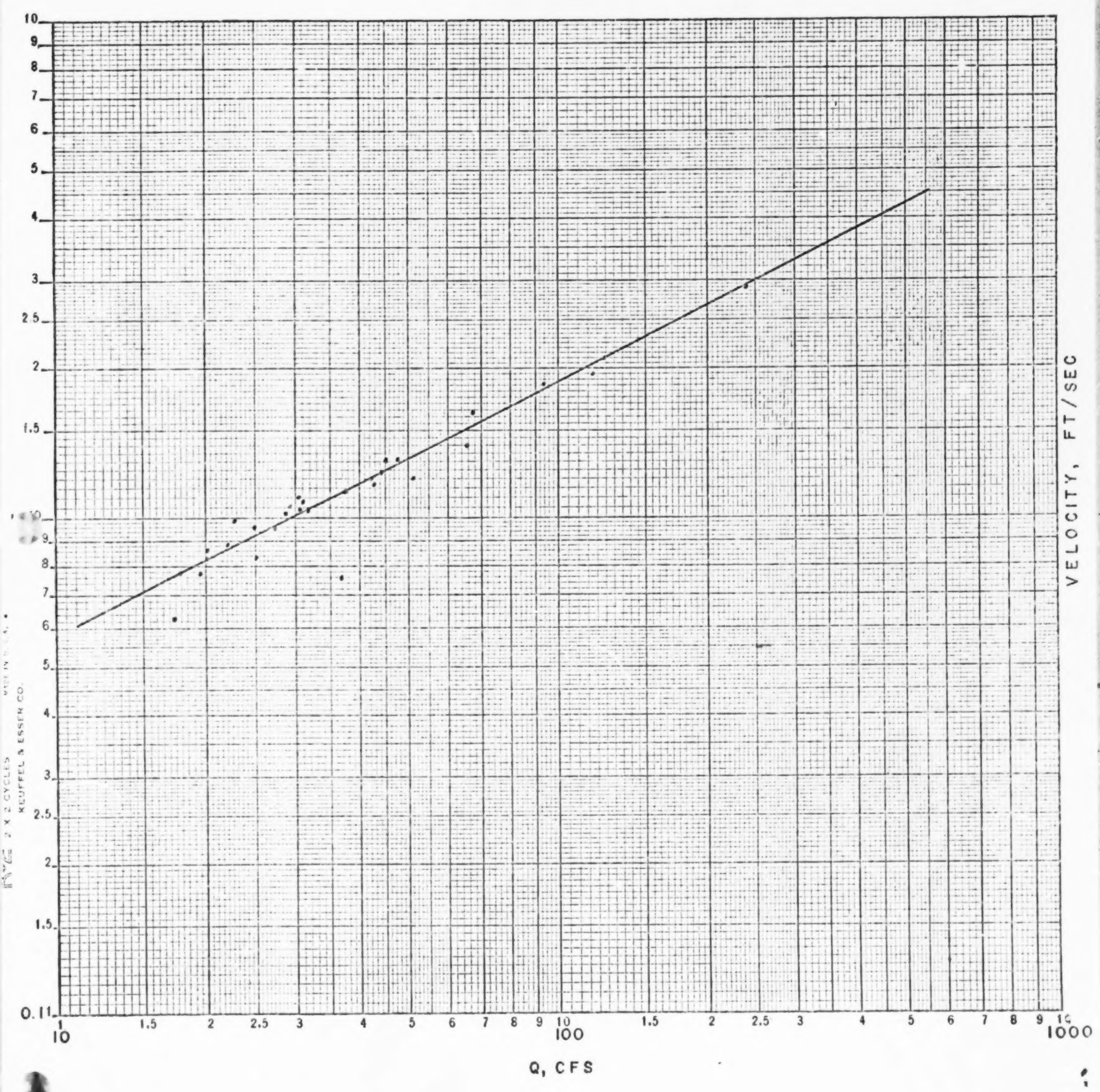


FIGURE 8-17
CONSTANT EVALUATION HYDRAULIC FORMULAS

STATION 1895

WIDTH - Q

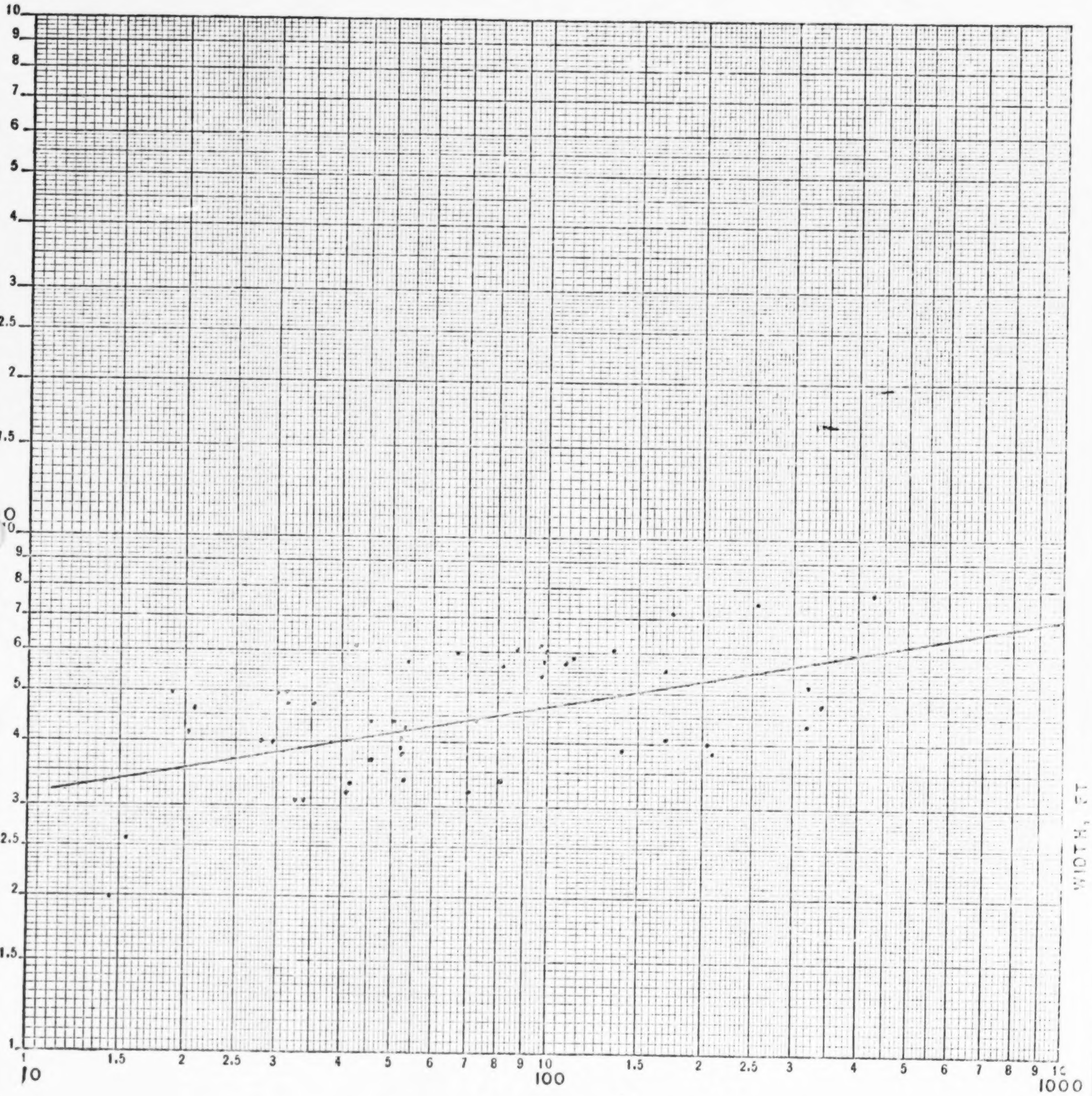


FIGURE 8-18
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1895

DEPTH – Q

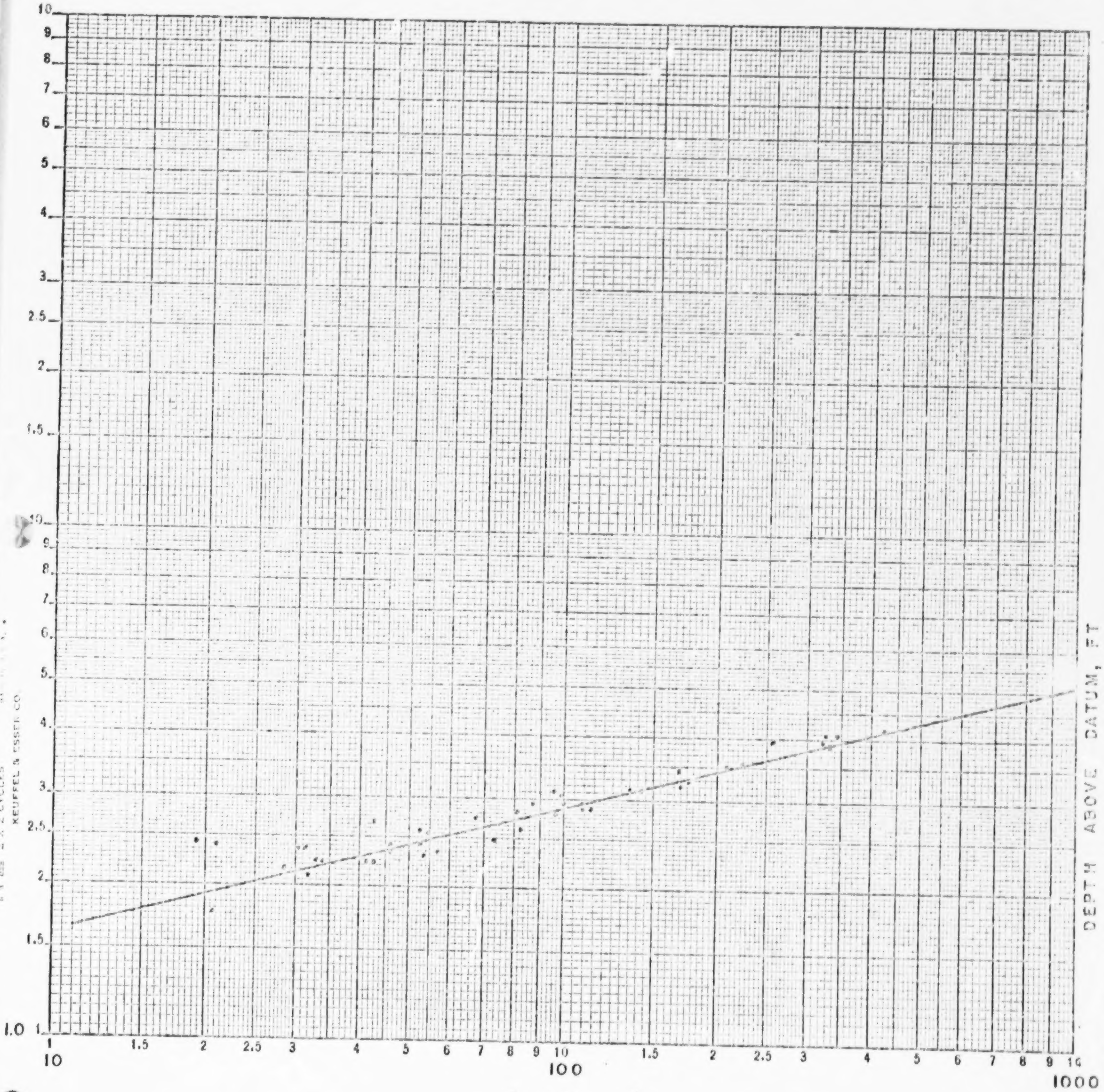
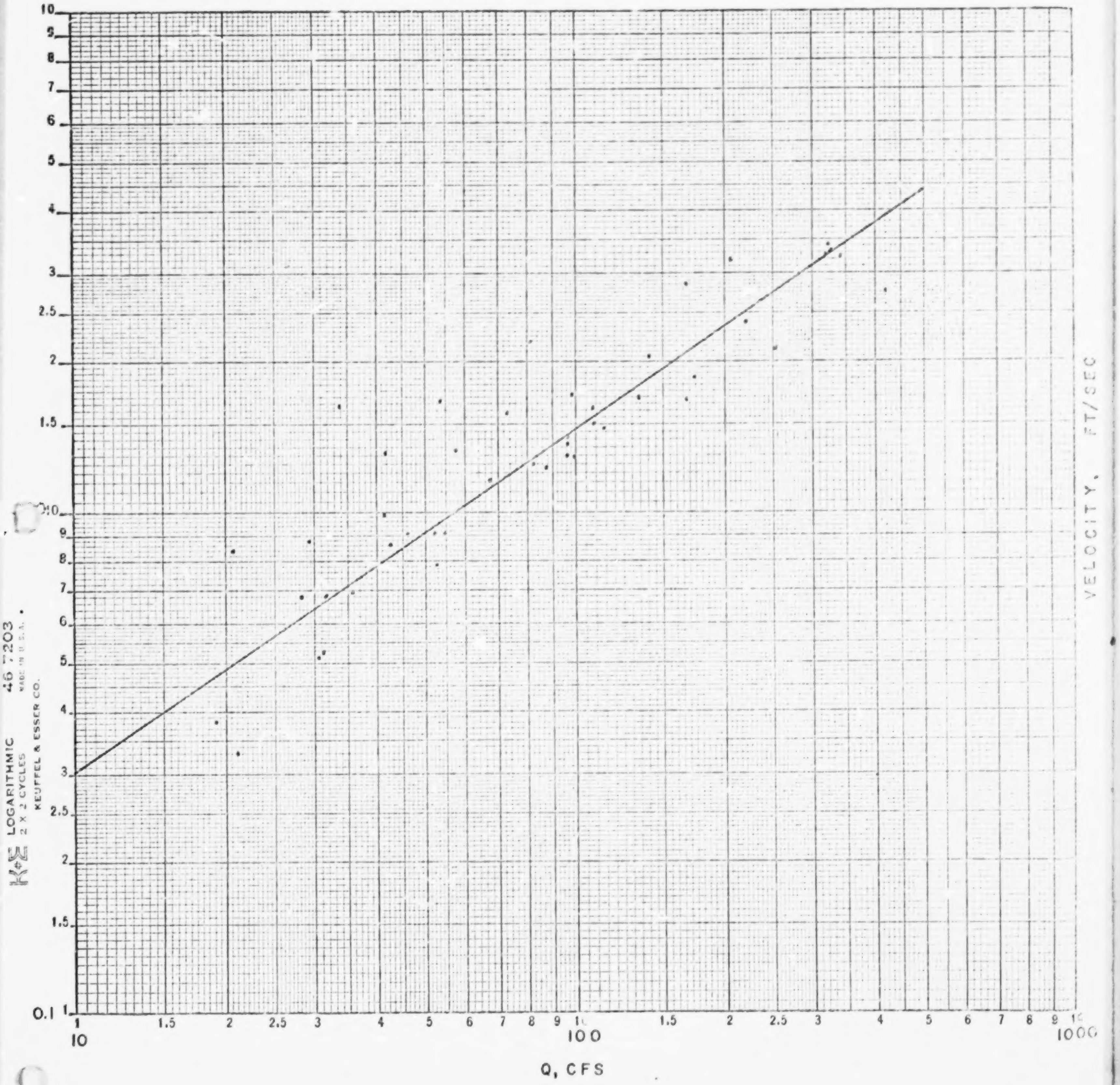


FIGURE 8-19
CONSTANT EVALUATION
HYDRAULIC FORMULAS

STATION 1895
VELOCITY - Q



The program CHKDATA read, checked and edited all available Farmington River Basin USGS historical gage data (see Table A3-3), computed average weekly flows and both printed and punched cards as output in 8.77 minutes execution time on an IBM 360/50 computer. In doing so, 4,510 lines were printed and 3,648 cards were punched. The total charge, at $200 per hour of computer time, with adjustments made for punching cards, was $48.70.

The program NORMAL was given average weekly gage data, prepared by CHKDATA, for all the USGS gaging stations in the Farmington River Basin and was instructed to determine if the data were normally distributed and, if not, which of six transformations normalized the data within the test limitations imposed. The program performed as directed and produced the results given in Table 8-3.

The data in Table 8-3 indicate that for the Farmington Basin the transformation most likely to normalize streamflow data is the log transformation. It was found, however, that the log transformation does not always normalize these data and that a check should be made to assure that it does so. Wherever the log transformation failed to normalize these data, it was the best of the transforms tried, based upon the criterion of minimum value of the maximum cell difference (see Section 6.3.2.2).

Five additional scale-changing transformations were tried on the five stations which were not normalized by the transformations shown in Table 8-3. The results are shown in Table 8-4.

Note that none of the twelve transformations normalizes the data from stations 1-1855 and 1-1800. Of those tried, the transformation, $q=Q^{0.10}$, produced the distribution nearest to normal for both stations. In both instances, the test failed in only one of the 17 cells and by 0.0025 for stations 1-1855 and by 0.0049 for station 1-1880, based upon the total cumulative frequency equalling 1.000. The implication is that, for these stations, the normalizing transform is not of the type $q=Q^b$.

The program NORMAL, running on all Farmington River Basin historical stream gage data, and using the seven initial transformations, as indicated in Table 8-3, required 4.94 minutes execution time on the IBM 360/50 computer at a cost of $16.46. The number of lines printed was 3,198.

Recall that NORMAL contains programming to determine the normalizing transform(s) for the average flows for each week of the year, the distribution being computed over the number of years of data. This feature of the program was tested on data from two stations and was found to give the desired result. Forty-eight times as much data are produced when distributions of weekly data are obtained as when the distributions are computed using all the data for each station. The execution time will be, however, only about three times longer.

TABLE 8-3

Results of Tests for
Normalizing Transformation

Gaging Station	Years of Data**	Transformation Number* 1	2	3	4	5	6	7
1-1855	49	-	-	-	-	-	-	-***
1-1860	7	+	-	-	-	+	+	+
1-1861	6	+	-	-	-	+	+	+
1-1865	14	+	-	-	-	+	-	-
1-1870	25	-	-	-	-	-	-	-
1-1873	16	+	-	-	-	+	-	-
1-1874	22	+	-	-	-	-	-	-
1-1878	33	-	-	-	-	-	-	-
1-1878.5	41	-	-	-	-	-	-	-
1-1880	31	-	-	-	-	-	-	-
1-1890	21	-	-	-	-	+	-	-
1-1895	16	-	-	-	-	+	-	-
1-1900	23	-	-	-	-	+	-	-

*Transformations are: (1) $q = Q^{0.25}$, (2) $q = Q^{0.5}$, (3) $q = Q^{0.75}$. (4) $q = Q$, (5) $q = \log Q$, (6) $q = \log(Q+0.25\mu)$ (7) $q = \log(Q+0.5\mu)$,

**One year of data lost in converting from "water year" to calendar year (see Table A3-3).

***+ indicates transform normalizes; - indicates transform does not normalize.

TABLE 8-4

Results of Tests
Additional Transforms

Gaging Station	Transformation Number*				
	8	9	10	11	12
1-1855	-	-	-	-	-
1-1870	-	-	-	-	+
1-1874	+	+	+	+	-
1-1878	-	-	+	-	-
1-1880	-	-	-	-	-

*Transformations are: (8) $q = Q^{0.10}$, (9) $q = Q^{0.05}$, (10) $q = Q^{0.15}$, (11) $q = Q^{0.20}$, (12) $q = \log(Q-0.14)$.

The program TFLOW, designed to guide the operator in selection of basis gages for simulation, was given edited historical average weekly gage data for the eight gaging stations from which suitable data are available for simulation and was instructed to compute correlation coefficients for various combinations of basis and "estimate" gages. The program performed as directed and produced the data given in Table 8-5.

Table 8-5 contains only a few of the possible basis-estimate gage combinations (there are 5714 possible combinations if there is used at least one basis gage and one estimate gage). To find the optimum combination, defined as the combination having the highest overall correlation coefficient, would be a lengthy process. It would be an interesting exercise, beyond the scope of this work, to develop an algorithm that would find the optimum combination, or the combination having the least number of stations, while still having a correlation coefficient of .9500.

If the correlation coefficient value of 0.9500 is considered indicative of acceptable correlation, then the test results in Table 8-5 indicate that:

(1) gages 1874 and 1900 likely could be eliminated as basis gages if gages 1855, 1865, 1880, 1890 and 1895 were retained as basis gages.
(2) gage 1865 likely could be eliminated as a basis gage if gages 1874, 1878.5, 1890, 1895 and 1900 were retained as basis gages.

TABLE 8-5

Correlation Coefficients For Various Basis-Estimate Gage Combinations

Run No.	Basis Gage Numbers*	Estimate Gage Numbers	Correlation Coefficients
1	1855, 1865, 1880, 1890, 1895	1874	.9697
		1878.5	.6742
		1900	.9532
2	1855, 1865, 1874, 1878.5, 1900	1800	.9143
		1890	.9198
		1895	.9326
3	1855, 1874, 1878.5, 1895, 1900	1865	.9483
		1800	.9042
		1890	.9150
4	1855, 1865, 1874, 1880, 1890	1878.5	.6584
		1895	.9464
		1900	.9470
5	1855, 1865, 1874, 1878.5, 1880	1890	.8973
		1895	.9347
		1900	.9384
6	1874, 1878.5, 1890, 1895, 1900	1855	.9469
		1865	.9595
		1880	.9389
7	1878.5, 1880, 1890, 1895, 1900	1855	.9213
		1865	.9345
		1874	.9365
8	1865, 1880, 1890, 1895, 1900	1855	.9459
		1874	.9578
		1878.5	.7062

*USGS gage designations

(3) gage 1874 likely could be eliminated as a basis gage if gages 1865, 1880, 1890, 1895 and 1900 were retained as basis gages.
(4) gage 1878.5 cannot be adequately represented by combinations of basis gages and therefore should not be eliminated. By the same token, however, gages located near gage 1878.5 should not be eliminated because the effect of gage 1878.5 extrapolated too far would result in poor representation of the expected flow values.

A subsequent run of TFLOW using the six gages, 1855, 1865, 1878.5, 1880, 1890, and 1895, as basis gages and 1874 and 1900 as estimate gages resulted in correlation coefficients of 0.9697 and 0.9533, respectively. It is interesting to compare these correlation coefficients with those in Table 8-5, run number 1. Note that including gage 1878.5 as a basis gage had negligible effect on the correlation coefficients of these two estimate gages.

The above analysis of the results of TFLOW is not intended to be exhaustive or conclusive; rather, it is included here to indicate how TFLOW may be used in the selection of basis gages so that a minimum of them can be employed.

The eight runs of TFLOW that produced the results shown in Table 8-5, required a total of 9.68 minutes of execution time for a University of Florida billing cost of $46.58. The output for each run produced 849 lines when the source deck was used and 239 lines when the object deck was used.

8.3.2 Hydrologic Simulation

The hydrologic simulation is made in two parts. The first part generates simulated gage data through the use of the program FLASH. The second part converts the gage data generated in the first part to simulated stream flow at the reach points in the river and makes the necessary corrections to account for regulation. This second part is carried out in the program, WASP, through subroutines TRAN and REG.

The program coding as set forth in Appendix A4 separates the two parts of the hydrologic simulation for the reason that together the total program length presents a problem to all except the very large computers or requires extra data storing facilities. The procedure used in the Farmington River application has been to place the output from FLASH on magnetic tape which is read as an input to WASP. It is not difficult to combine the two programs.

The programs FLASH and WASP have satisfactorily simulated the streamflow conditions in the Farmington Basin. All of the regulation conditions have been checked and the programmed operating rules control the releases and diversions in the desired manner. The programs output the simulated gage data, unregulated stream flows, regulated stream flows, reservoir inventories, releases, diversions and evaporation losses, all on a weekly basis.

Figures 8-20, 8-21, and 8-22 are included to illustrate the hydrologic and water quality simulation results. Understandably, these figures only show a very small portion of the total output.

The results of 100 weeks of simulated operation of Colebrook Reservoir are illustrated in Figure 8-19. These data were generated using a trial set of release rules wherein the minimum release was 150 cfs and incoming

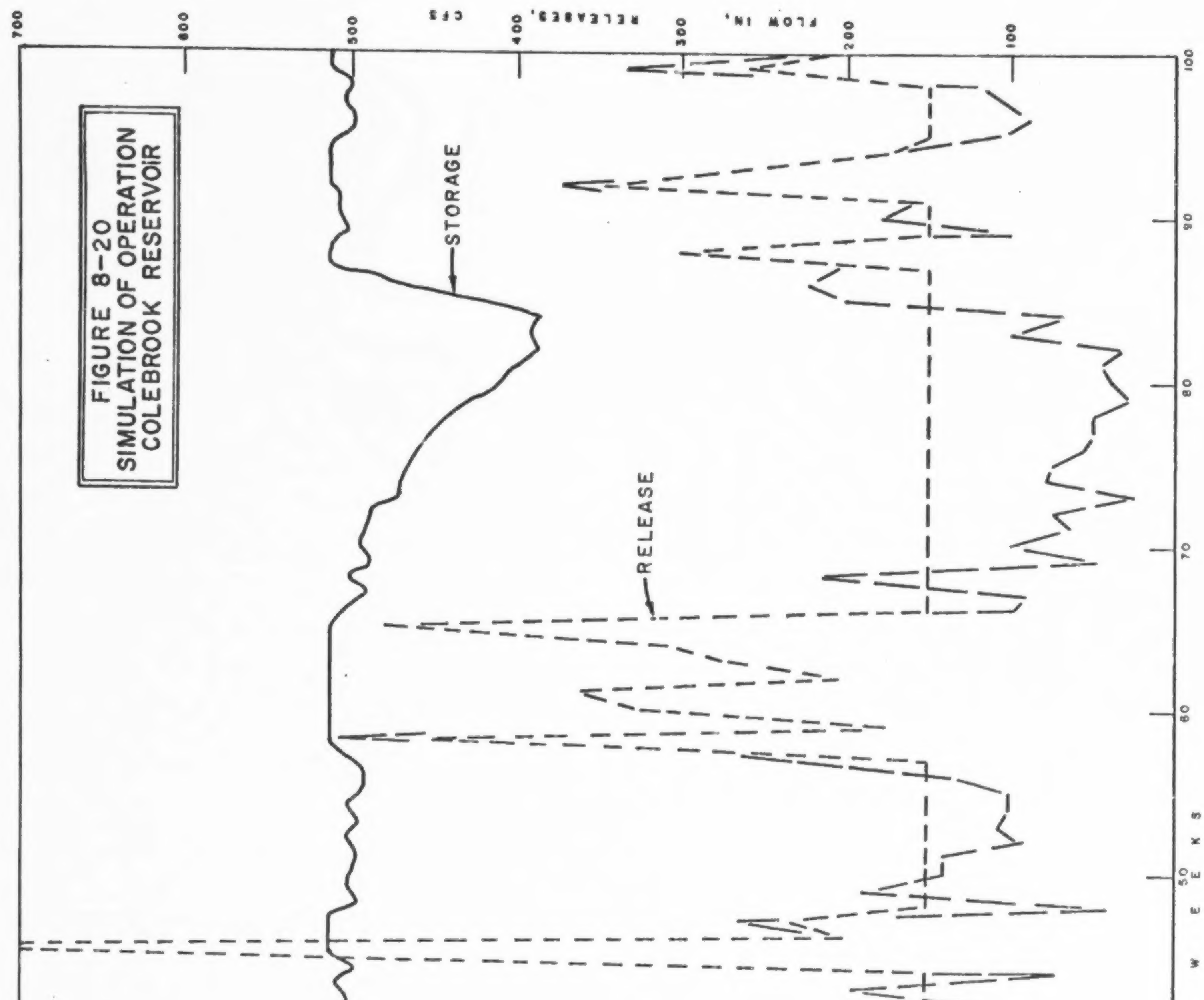


FIGURE 8-20
SIMULATION OF OPERATION
COLEBROOK RESERVOIR
STORAGE
RELEASE
RESERVOIR FULL
FLOW IN
FLOW IN, RELEASES, CFS
700
600
500
400
300
200
100
STORAGE, MCF
2000
1000
0
WEEKS
10
20
30
40
50
60
70
80
90
100

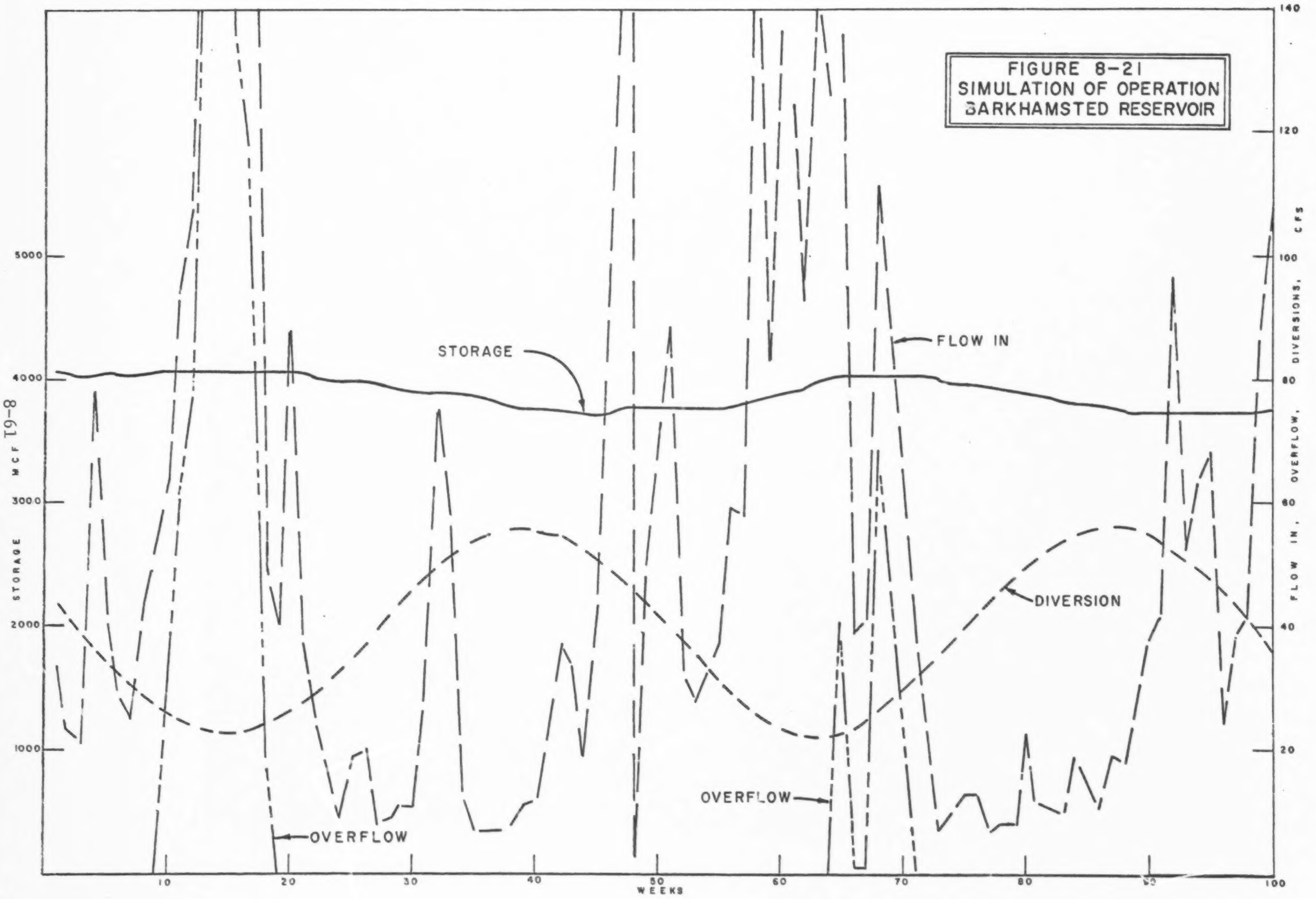


FIGURE 8-21
SIMULATION OF OPERATION
BARKHAMSTED RESERVOIR

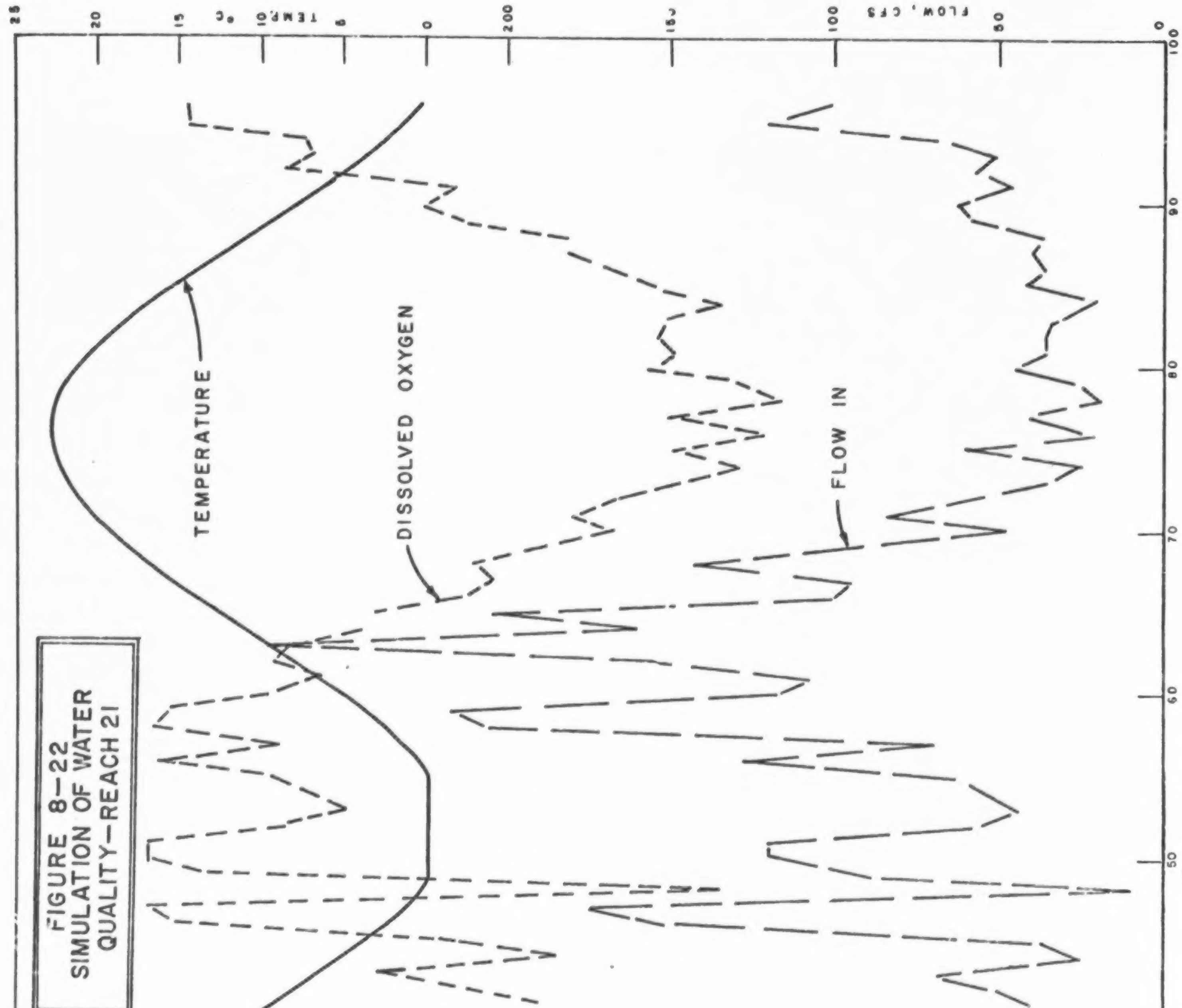


FIGURE 8-22
SIMULATION OF WATER
QUALITY—REACH 21

flow greater than 150 cfs was stored unless the water supply and fisheries pools were full. Reservoir inventory, flow in and release rates are shown.

The results of 100 weeks of simulated operation of Barkhamsted Reservoir are shown on Figure 8-21. The operating program called for diversion according to the demand curve (periodic curve) and release of water in excess of capacity at spillway level. Note that overflow (release to downstream) occurs when the reservoir is full and the inflow exceeds the diversion, according to plan.

The results shown on Figure 8-22 are generated by hydrologic and water quality simulation in reach 21 for a two-year period (96 weeks). The dissolved oxygen (DO) concentration, flow in the reach and temperature are plotted. It is evident that the DO concentration is closely related to flow but also it can be seen that the temperature has an effect. Note that the DO concentration recovers in the fall while the flow is still low and that the DO concentration decreases in the spring while the flow is still relatively high.

The computer can be programmed to produce the simulation data results in graphical form. Although the program for these plots is not difficult, its preparation is not within the scope of this work. Plotting programs designed to output pictorial data would need to be prepared with a specific objective in mind, such as the operation of a particular reservoir or the quality of water in a given reach. The programs contained in this model could easily be extended to include this type of output.

In the application to the Farmington Basin an interesting problem was encountered. The span of historical data used contained record of the storm of mid-August, 1955, which was a very extreme event. On August 19, 1955, a hurricane centered over the Basin and the rainfall and resulting runoff exceeded all records. Pertinent data are shown in Table 8-6. The flow at gage 1900 was affected by considerable flood-plain storage.

The effect of such a storm is compounded when it is realized that the normal August flows are usually at their minimum, often about 5 percent of mean flows. The problem encountered was in the computation of the statistical parameters for the third week in August, wherein the standard deviation turned out to be a relatively enormous value. This resulted in some wildly varying flows being generated for that week. The variance factor (third factor, right hand side of the generating equation [6.66]) can, for that week, override the effect of the regression factor and result in a set of generated flows where an upstream flow is somewhat larger than the flow at a downstream point. While this is possible in long rivers and at a given instant of time, it is not possible for rivers having a total time of flow less than the averaging interval of the simulation.

TABLE 8-6

Records - Storm of August 19, 1955
Farmington River Basin

Gaging Station	Tributary Area, Sq.Mi.	Recorded Flow, cfs	Flow, cfs/sq.mi.	Rainfall in./24 hr.	Annual Mean Flow, cfs
1855	90.5	16,100	178	6.6	192.0
1861	18.4	10,200	555	20.6	23.9
1865	89.6	24,800	277	10.3	190.0
1870	127.0	52,000	409	15.2	--
1873	20.4	5,000	245	9.1	36.2
1874	7.2	2,040	284	10.5	13.8
1878	23.6	4,000	170	6.3	38.3
1878.5	0.6	34	58	2.2	1.6
1880	4.1	673	164	6.1	78.0
1890	45.6	6,500	143	5.3	82.7
1895	33.6	18,400	548	20.4	--
1900	585.0	69,200	118	4.3	1029.0

To eliminate the problems caused by this condition in the data, the programming was modified slightly to check for upstream flows in excess of downstream flows and when found, to set the downstream flows equal to the next upstream flow.

The occurrence, during one of the simulation test runs, of an extreme event as a result of the high variance for the third week in August allowed a system check of the operating rules of the flood control reservoirs. In this run, Colebrook, Mad River and Sucker Brook Reservoirs were filled to overflowing and the program routed the water in accordance with the programmed rules.

Although the overall thrust in this work is to simulate water quality in a stream, the programs first simulate the natural and regulated stream flows and can be used, without modification, if only hydrologic simulation is needed. A saving in computer time will result, however, if subroutines QUAL and RQUAL and their supporting subroutines are removed during the hydrologic simulation.

The program FLASH read, from punched cards, 21 years of historical weekly average gage data from six basis gages and generated thirty years of simulated data for these six gages in 10.3 minutes execution time at a University of Florida billing of $34.33. The output was both placed on magnetic tape and printed on paper. The printed output amounted to 2736 lines.

8.3.3 Water Quality Simulation

The water quality simulation is carried out by the program WASP and particularly by subroutines QUAL and RQUAL. Simulation runs have been made using Farmington River Basin data and the results appear to be accurate. Desk calculator computations have been made for situations selected at random from the simulation output to check the computer-made computations. These have shown that the programming is correct.

In the application of the Farmington River data, the year 2000 waste loadings, assuming 90% BOD removal in treatment plants, have been applied and it has been found that violations of the minimum stream standard of 4.0 mg/l dissolved oxygen have been few. Violations that did occur invariably coincided with low flow and high temperature conditions. Several test runs wherein special water quality and/or regulation situations have been imposed have been made and the results of these are described in Section 8.5 below.

The effect of placing a single extra heavy BOD loading on one reach during one week, to simulate a possible industrial waste spill, was investigated. A loading of 5.1 mgd having a BOD of 650 mg/l was introduced at the upper end of reach 19 (at Granby, Connecticut on Salmon Brook). The rapid time of flow carried the waste downstream before it exerted much of its demand, until it was detained in Rainbow Reservoir. Here, even with the substantial dilution afforded by the main stream flow, the BOD was exerted, driving the dissolved oxygen concentration to zero. The DO also was zero in reach 1 below Rainbow Reservoir. Actually, the printed output showed negative DO concentrations in the Reservoir and downstream. Because the system equations (Streeter-Phelps) do not describe the negative DO situation, nor does negative DO have accurate physical meaning, the program was modified to check for a DO concentration less than zero and, if it is, EXIT is called and the execution is stopped.

The program WASP, in object form, reading from magnetic tape the synthetic gage data produced by FLASH, simulated the stream hydrology and water quality for 30 years in an execution time of 25.38 minutes. This is at a rate of 0.846 minutes per year. When the source deck is used, 3.4 minutes additional time per run is required. Off-line printing by IBM 1401 computer is used for output of the generated data. The cost of a 30 year run, using object deck and off-line printing is $84.60 for execution and $38.50 for printing, for a total billing of $133.10.

The output, amounting to 69,300 lines, contains weekly natural flows, regulated flows, reservoir inventories, reservoir releases and diversions, K_1 values, K_2 values, minimum DO concentrations, BOD concentrations and DO deficit concentrations for each reach. For 48 weeks, 30 years and 43 reaches in the Farmington Basin, the output contains 61,920 sets of data.

TABLE 8-6

Records - Storm of August 19, 1955
Farmington River Basin

Gaging Station	Tributary Area, Sq.Mi.	Recorded Flow, cfs	Flow, cfs/sq.mi.	Rainfall in./24 hr.	Annual Mean Flow, cfs
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8.4 Sensitivity Tests

Sensitivity tests were run for eleven variables or parameters using the basic Farmington River data and year 2000 waste loads. In the base or comparison run, the normal values of all variables and parameters were used and the water quality conditions were simulated for a two-year period. Then, one by one, each of the eleven variables and parameters were given values different from their normal values and the water quality conditions were simulated, also for two years. The same tape of gage data was used for each test so that the stream flow at each point in space and time was the same for each run, thus eliminating the variability caused by flow.

The runs were analyzed by determining the mean difference in the DO concentration for each reach and week between the base run and the run in which a variable was changed. This was done for each of 18 reaches affected by the loading, for 15 weeks of the year and for two years, a total sample of 540 differences. The 15 weeks selected were the summer weeks when the effect of the change is most pronounced.

The variable change and resulting effect on the average stream DO for each run are summarized as follows:

(1) Run S-1. The temperature of the water was increased 10 percent, from 21.3°C to 23.4°C mean temperature. The effect was to decrease the stream DO concentration an average of 0.3591 mg/l with a variability indicated by a standard deviation of 0.0564. Using the dimensionless form, equation [6.132], the value of sensitivity function $S_K^{F(K)}$ was given by -0.3591/8.2173/2.1/21.3 = -0.4430. The mean DO concentration in the base run was 8.2173 mg/l - standard deviation, 0.3456.

(2) Run S-2. The value of the reoxygenation velocity constant, K_2, was decreased 50 percent, from a mean of 1.8770 to a mean K_2 of 0.9385. The effect was to decrease the mean DO concentration by 0.2494 mg/l with a standard deviation of 0.1206. The value of the sensitivity function was given by 0.2494/8.2173/0.9385/1.8770 = 0.0606.

(3) Run S-3. The value of the regulated flow was decreased 10 percent. The effect was to decrease the mean DO concentration 0.0406 mg/l with a standard deviation of 0.0057. The value of the sensitivity function was 0.0406/8.2173/0.1 = 0.0490.

(4) Run S-4. The value of the reoxygenation "error" term was changed from +0.005 to -0.05. The effect was to increase the average DO concentration by 0.0131 mg/l, standard deviation of 0.0029. The value of the sensitivity function was 0.0131/8.2173/-0.055/0.005 = 0.0001.

(5) Run S-5. The value of the deoxygenation "error" term was changed from +0.005 to +0.05, a ten-fold increase. The mean DO concentration was de-

creased 0.0014 mg/l with a standard deviation of 0.0003. The value of the sensitivity function was -0.0014/8.2173/10 = -0.000017.

(6) Run S-6. The value of the deoxygenation velocity constant, K_1, at 20°C was increased 10 percent from 0.276 to 0.307. The effect was to decrease the mean DO concentration by 0.0105 mg/l - standard deviation of 0.0033. The sensitivity function = -0.0105/8.2173/0.1 = -0.0128.

(7) Run S-7. The value of the evaporation loss from the reservoirs was increased 10 percent. The effect on the DO concentration in the stream averaged 0.0017 mg/l decrease - standard deviation, 0.0010. The sensitivity function = -0.0017/8.2173/0.1 = -0.0021.

(8) Run S-8. The value for the waste loading to the stream, expressed as mg/l BOD, was increased 10 percent from 22 to 24.2. The effect was to decrease the average DO concentration in the stream 0.0148 mg/l - standard deviation, 0.0048. The sensitivity function = -0.0148/8.2173/0.1 = 0.0180.

(9) Run S-9. This is a companion to Run S-8 in that the value of the waste loading to the stream was decreased 10 percent from 22 to 19.8 mg/l. The effect was to increase the average DO concentration in the stream 0.0160 mg/l - standard deviation 0.0078. The sensitivity function = -0.0160/8.2173/0.1 = -0.0195.

(10) Run S-10. The value of the DO concentration of the waste load discharges to the stream was decreased 10 percent from 2.0 mg/l to 1.8 mg/l. The effect was to decrease the average stream DO concentration 0.0137 mg/l - standard deviation, 0.0037. The sensitivity function equals 0.0137/8.2173/0.1 = 0.0167.

(11) Run S-11. The values of the constants m and f in the Leopold and Maddock equations [6.7] and [6.8] were changed from m = 0.56 to m = 0.51 and from f = 0.33 to f = 0.38. The change made was about 10 percent decrease in m. To preserve the requirement that the sum of exponents in these equations (also equation [6.6]) equal 1.00, the value of f was changed accordingly. The effect of this dual change was to decrease the velocity and increase the depth. The average stream DO concentration was decreased 0.4939 mg/l with a standard deviation of 0.1476. The sensitivity function, computed with respect to the change in m, was 0.4939/8.2173/0.05/0.56 = 0.6732.

The overall water quality conditions in the Farmington Basin are affected by the relatively fast time of flow throughout the length of the river. The mean time of flow from Winsted, mile 60.9 (on the Still River), to Rainbow Reservoir, mile 10, is 1.34 days, an average velocity of 2.32 feet per second. Winsted is the load point most distant from the mouth of the Farmington. This rapid flow (and relatively short distance) does not provide time for the time-dependent decay forces of the deoxygenation reaction to exert themselves to drive the DO down to its least value. Only part of the total effect of the waste loading is felt within the limits of the river being investigated, the rest is being imposed downstream. For example, at a temperature of 21.3°C, the mean temperature during the sensitivity runs, a waste load having a K_{120} value of 0.276 discharged at Winsted would have only 39 percent of its BOD satisfied at the mean time of flow of 1.34 days, when it arrives at Rainbow. In addition, the relatively high values of K_2, obtained using the Langbein and Durum equation ([6.38]), consistently replenish the DO used so that the function value, the DO concentration, remains high and causes low-appearing sensitivity function values.

Considering the values of the sensitivity functions of the various runs it may be concluded that DO concentrations in the stream are (1) relatively sensitive to the values of the constants in the velocity and depth equations and the water temperature, (2) somewhat less sensitive to the reoxygenation velocity constant, K_2, and the regulated flow, (3) sensitive to a low degree to the magnitude of waste loads, DO concentration in the waste and the deoxygenation velocity constant K_1, and (4) insensitive to the deoxygenation and reoxygenation error terms, r and s, and the evaporation loss in the reservoirs.

The analysis of Run S-3, where the regulated flow was decreased 10 percent, showed that the time of flow from Winsted to Rainbow was increased 5.8 percent. The effect of this change in flow on the DO concentration proved to be of low significance. This is contrary to other evidence, as shown in Figure 8-22 for instance, and common knowledge. The small amount of the change in DO concentration in this test is probably due to two offsetting effects. One, tending to increase the DO concentration, is the loss in diluting water by the decrease in flow, and the other, tending to decrease the DO concentration, is the increase in time afforded the natural deoxygenation process. Under normal circumstances, the latter would predominate but, with the short time of flow, it has not had time to exert its full effect.

If water quality simulation is the only objective, the correction for evaporation loss may be omitted. However, if the results of hydrologic simulation are also of interest, evaporation should not be neglected. The maximum evaporation rate in the Farmington Basin is 5.72 inches per month, which, when Barkhamsted Reservoir is full, amounts to a weekly loss to evaporation of 11.76 million cubic feet (about 18 cfs).

8.5 Results of Special Studies

A series of special studies were made to simulate the system when different reservoir management practices are employed in augmenting flows for quality control and when the released water quality has been impaired by detention in a stratified reservoir. The studies were applied to the Farmington River system and the special conditions were imposed only upon the management and releases of Colebrook Reservoir.

The Colebrook Reservoir operating rules used in these special studies were those expected to be in force in the year 2000 when diversions to Barkhamsted Reservoir will be made for MDC water supply (4). These operating rules are: (1) minimum release is 50 cfs; (2) release at the inflow rate when the inflow is greater than 50 cfs but less than 150 cfs; (3) release 150 cfs when the inflow exceeds 150 cfs and store the balance; (4) operate flood control and fisheries pools as at present; (5) divert to Barkhamsted Reservoir at a rate required to maintain the level in Barkhamsted at elevation 520; (6) make no diversions if the stored volume in Colebrook decreases to 315 million cubic feet (mcf) and, (7) release only that amount of inflow that causes the volume to exceed 315 mcf.

A simulation run was made to determine the effect of releasing, from Colebrook Reservoir, water which has a BOD of 10 mg/l. The normal year 2000 waste loads were also imposed on the system, all of which are downstream from Colebrook and Goodwin Reservoirs. The result was that most of the released BOD was satisfied in Goodwin Reservoir where the detention time during the summer months is usually in excess of 30 days. The DO in the Goodwin release was somewhat lower than when no loading is applied but recovery was rather rapid downstream. This is illustrated in Figure 8-23 on which is plotted the result of the 27th week of the second year, the week in which the poorest water quality was found.

The effect of Goodwin Reservoir under these conditions was determined by a second simulation run, identical to the first excepting that the BOD load of 10 mg/l was shifted from the Colebrook release to the Goodwin release. This result also is shown in Figure 8-23. Note that in this latter case, the BOD is being exerted in the stream below Goodwin Dam decreasing the stream DO a significant, but not critical, amount. Much of the remaining BOD is carried into Rainbow Reservoir where the detention for this particular week was 5.32 days. The DO decreases still further in and below Rainbow Reservoir.

The BOD at each main stem reach point below Colebrook Dam is shown in Table 8-7. These data substantiate the conditions described above. Note that the BOD was reduced from 10.00 to 0.62 mg/l in Goodwin

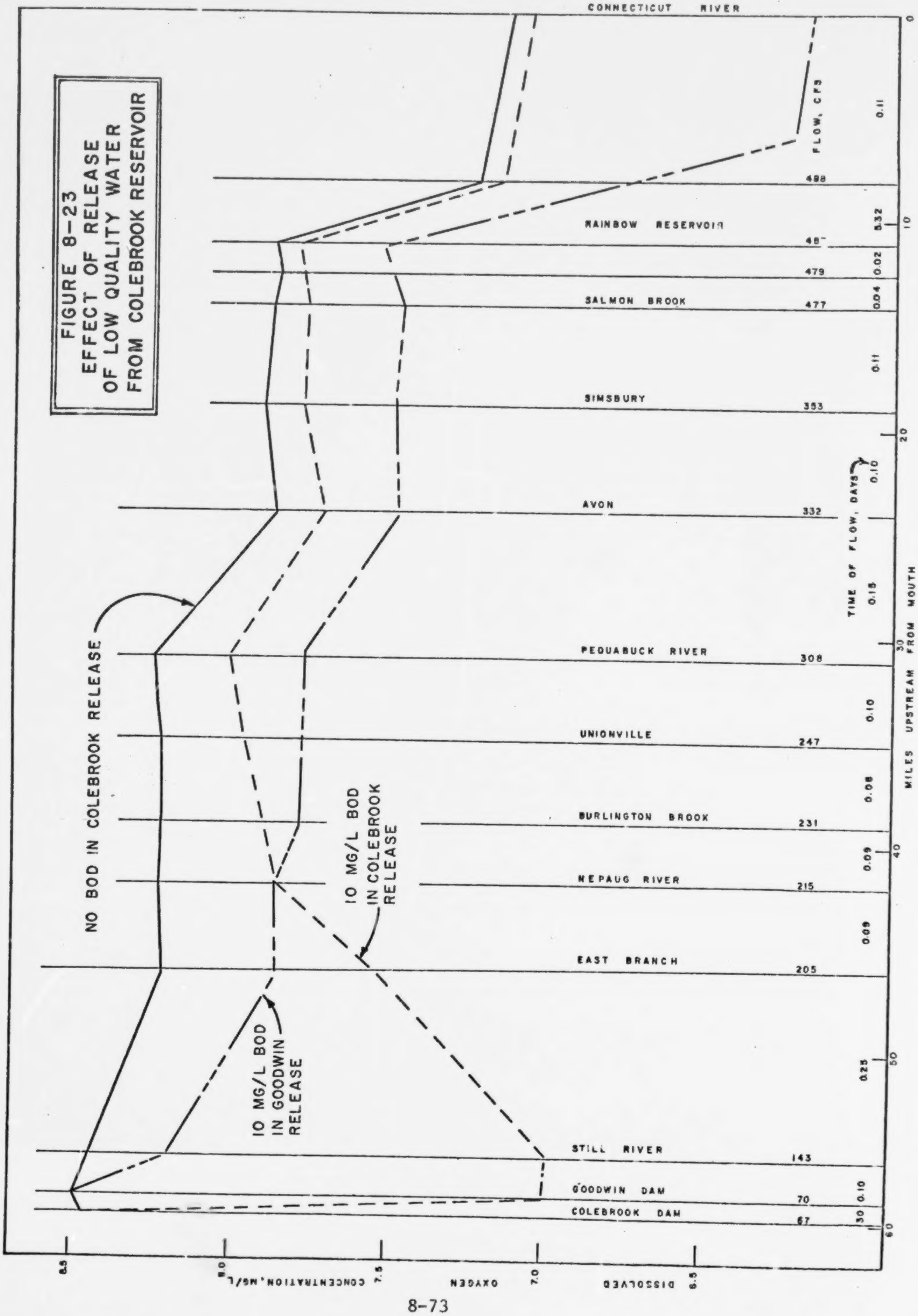


FIGURE 8-23
EFFECT OF RELEASE
OF LOW QUALITY WATER
FROM COLEBROOK RESERVOIR

Reservoir when the Colebrook release contained 10 mg/l BOD. Note also that the downstream BOD values are nearly the same as the run where no BOD was released from either dam. When the Goodwin release was loaded to 10 mg/l BOD, a portion of the BOD was exerted in the river reaches, considerable BOD was exerted in Rainbow Reservoir and an increased BOD loading was discharged in the Connecticut River.

A second special simulation run was made setting the DO concentration equal to zero in the water released from Colebrook Reservoir. The result for week 27, year one of this simulation run is plotted on Figure 8-24. The 30 days detention and the assumption of complete mixing in Goodwin Reservoir resulted in recovery of the DO concentration to 7.53 mg/l in the Goodwin release.

Reaeration in the stream below Goodwin Dam resulted in complete recovery from the imposed condition in the stretch of river between Goodwin Dam and its confluence with East Branch, a distance of about ten miles.

The condition of zero DO in the Goodwin Reservoir release was also simulated. The resulting DO concentration in the stream was also plotted on Figure 5-24. Note that recovery was rapid from the release point to reach 10 (Nepaug River). Recall that the rate of reaeration is a function of the undersaturation. The effects of decreasing reaeration rates and waste loadings cause the rate of recovery to decrease. Full recovery had occurred at the upper end of Rainbow Reservoir.

The results obtained from these test runs are as expected.

Another test run was made to determine where, when and how much water would be needed for low flow augmentation. Again, the Farmington River Basin year 2000 waste loads and conditions were imposed. The operation of Colebrook Reservoir was as described above. The run was for 30 years giving 30 one-year samples of the state of the river at the year 2000.

The simulation output recorded 24 water quality violations in the 30 year run. A quality violation is where the average DO in the reach for the week is less than 4.0 mg/l. The location, week, minimum reach DO and average weekly regulated flow for each violation are listed in Table 8-8. Note that there was only one main stem violation, reach 7 during the 33rd week of year 12. Where the DO was 2.124 mg/l and the average flow was 1 cfs. This was a particularly dry week following a period when the water for maintaining the minimum release from Colebrook Reservoir could not be met.

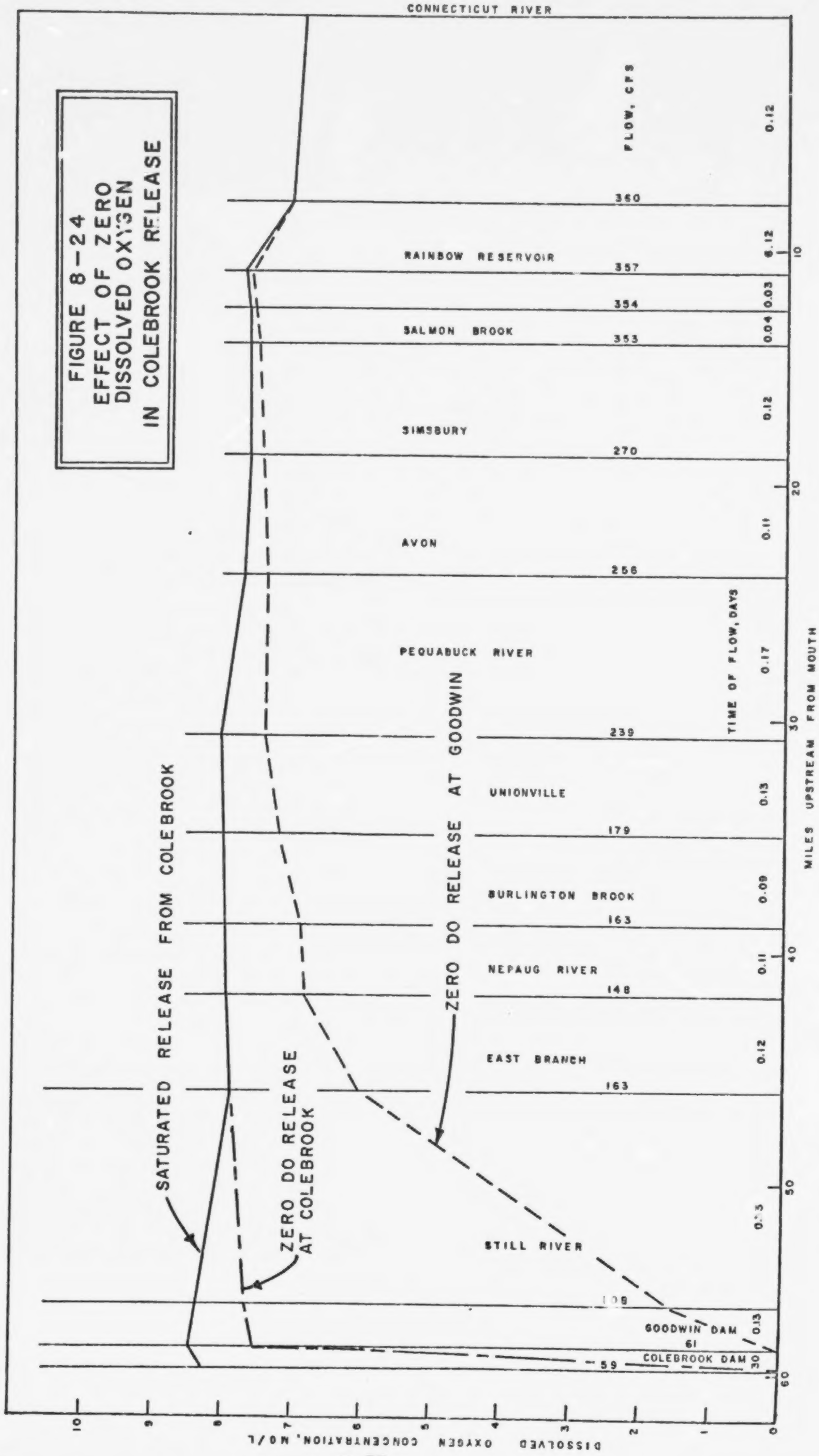


FIGURE 8-24
EFFECT OF ZERO
DISSOLVED OXYGEN
IN COLEBROOK RELEASE

TABLE 8-7
SIMULATED BOD DATA
SPECIAL STUDY NO. 1

Reach No. (Downstream end)	Location	BOD in Stream mg/l*	BOD in Stream mg/l**	BOD in Stream mg/l***
14	Colebrook Dam	10.00	0.00	0.00
13	Goodwin Dam	0.62	10.00	0.00
12	Still River	0.66	9.74	0.06
11	East Branch	1.22	5.78	0.92
10	Nepaug River	1.25	4.91	1.00
9	Burlington Brk.	1.30	5.10	1.04
8	Unionville	1.28	4.85	1.05
7	Pequabuck River	2.43	5.21	2.29
6	Avon	2.56	5.17	2.36
5	Simsburg	2.67	5.19	2.50
4	Salmon Brook	2.78	4.93	2.64
3	Spoonville Br.	2.76	4.89	2.62
2	Rainbow Rsvr.	0.99	1.76	0.94
1	Connecticut R.	1.29	2.04	1.24

* Load in Colebrook release.
** Load in Goodwin release.
*** No load in Colebrook or Goodwin release.

TABLE 8-8
WATER QUALITY VIOLATIONS

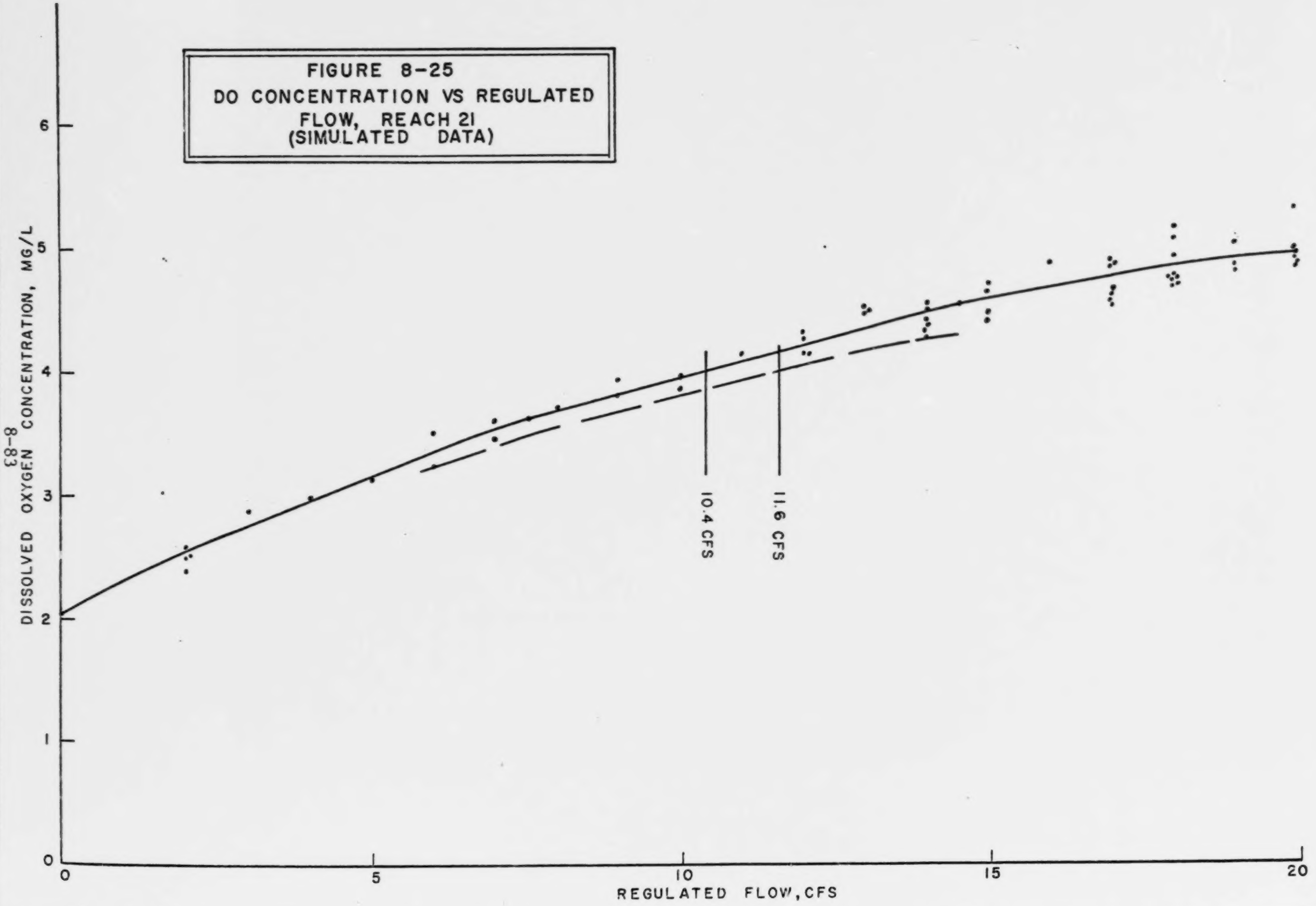
Year	Violations in Year	Location, Reach	Week of Year	DO Conc. mg/l	Reg. Flow cfs.
1	0				
2	0				
3	0				
4	1	21	33	3.4640	7
5	1	21	33	3.2479	6
6	1	21	36	3.9352	9
7	0				
8	0				
9	1	21	33	3.1265	5
10	1	21	33	3.6988	8
11	2	21	33	2.4915	2
		21	34	3.9439	10
12	3	21	33	2.3865	2
		34	33	3.0198	1
		7	33	2.1240	1
13	0				
14	0				
15	3	19	33	3.2907	1
		21	33	2.0942	0
		21	34	2.5281	2
16	3	21	29	3.8156	10
		21	32	3.9580	10
		21	36	2.5844	2
17	2	21	32	3.8470	10
		21	33	2.5844	4
18	1	21	36	2.5256	2
19	0				
20	0				
21	0				
22	1	21	33	3.8145	9
23	1	21	33	3.7176	8
24	1	21	36	2.8902	3
25	1	21	36	3.5252	6
26	0				
27	1	21	36	3.6283	7
28	0				
29	0				
30	0				

Excepting one violation each in reaches 7, 19 and 34, all violations were recorded in reach 21 into which the Bristol, Connecticut sewage treatment plant discharges. Reaches 19, 21 and 34 are tributary reaches and flow augmentation releases from Colebrook Reservoir would not relieve the conditions causing the violation. It would be necessary to obtain water for flow augmentation upstream on the appropriate tributary.

The rate of release of augmenting flow needed to forestall a water quality violation can be determined only from information fed back from some point downstream where a condition which indicates a violation will occur can be sensed. The condition must be sensed and the water must be released in the same time frame. The internal sequence for computing flow regulation and water quality must start upstream and proceed downstream. This presents a problem and would require that a subroutine be set up which, when a violation is detected in a reach, would return the sequence to the upstream reach where the release is to be made and recompute regulated flow and water quality in the reaches downstream therefrom. The amount of flow to add to the original release must be determined.

The additional or augmenting flow can be determined by analysis of the simulation run wherein no return for recomputation is made. Plot the DO concentration versus regulated flow rate for only those flows which result in the DO being near the minimum quality standard. Data points both above and below the standard should be used. The rate of flow needed to maintain the quality standard can be determined from the plot, either from a line drawn through the median of the plotted points or, better, through the lower of the plotted points. This has been done for reach 21 and the result is shown in Figure 8-25. Note that, on the average, 10.4 cfs will result in a DO concentration of 4.0 mg/1. The lower curve on Figure 8-25 shows that for the 30 samples used, 11.6 cfs will assure at least 4.0 mg/1 DO. This can be done for all reaches that experience a violation in the simulation run. The recomputation routine can then be set up to add, to the original flow, the required augmenting flow, which was determined by the graphical method described above, to obtain a new value for the regulated flow. Then, using the new regulated flow, the water quality values for reaches downstream from the reservoir can be recomputed. The total added flow may be expressed in terms of volume to give the reservoir space needed for the flow augmentation use.

FIGURE 8-25
DO CONCENTRATION VS REGULATED FLOW, REACH 21
(SIMULATED DATA)



If few violations occur in the simulation run, it is an easy matter to compute the volume needed in a reservoir for flow augmentation using a desk calculator or by hand. For instance, in Table 8-8, note that in year 11, the additional flow needed to assure 4.0 mg/l DO in reach 21 is 11.6-2 = 9.6 cfs = 6.4 million cubic feet (mcf) for week 34 for a total of 7.5 mcf. In year 15, it would require (11.6 + 9.6) 0.657 = 13.9 mcf. This is the maximum requirement for flow augmentation reservoir space as determined by the 30 samples from the simulation run. This volume must be provided upstream of reach 21 on the upper end of the Pequabuck River.

A similar plot of the reach 7 data yielded two data points (see Figure 8-26) and an indicated flow of 19.8 cfs is the minimum that will result in no violation. The single main stem violation would then require low flow augmentation volume of 19.8-1 = 18.8 cfs = 12.4 mcf upstream of reach 7. Similarly, it was found that 0.85 mcf would be required to forestall the violation in reach 19 and 0.72 mcf would be required in reach 34.

These computations and plots were made by hand in a few minutes time from data contained in the simulation run. If the number of violations is great and many reaches experience violations, it may be of advantage to program these computations for the computer.

8.6 Transfer Functions

Because the development of the transfer function program was beyond the scope of the original project, low priorities for time and computer funds were assigned to the application of this phase. The program AIJ was not afforded a final run to produce transfer function coefficients for the Farmington Basin.

8.7 Summary and Conclusions - Simulation Model

8.7.1 Summary

The objective to develop a simulation model that can be used to simulate the hydrology and water quality in a stream system having a dendritic pattern has been accomplished. The capability of the model to simulate for various conditions imposed on the system has been demonstrated. Tests have been made to assure the correct operation of the model under all feasible conditions and safequards have been introduced to prevent simulation under impossible real-life situations. Desk calculator computations have been made to

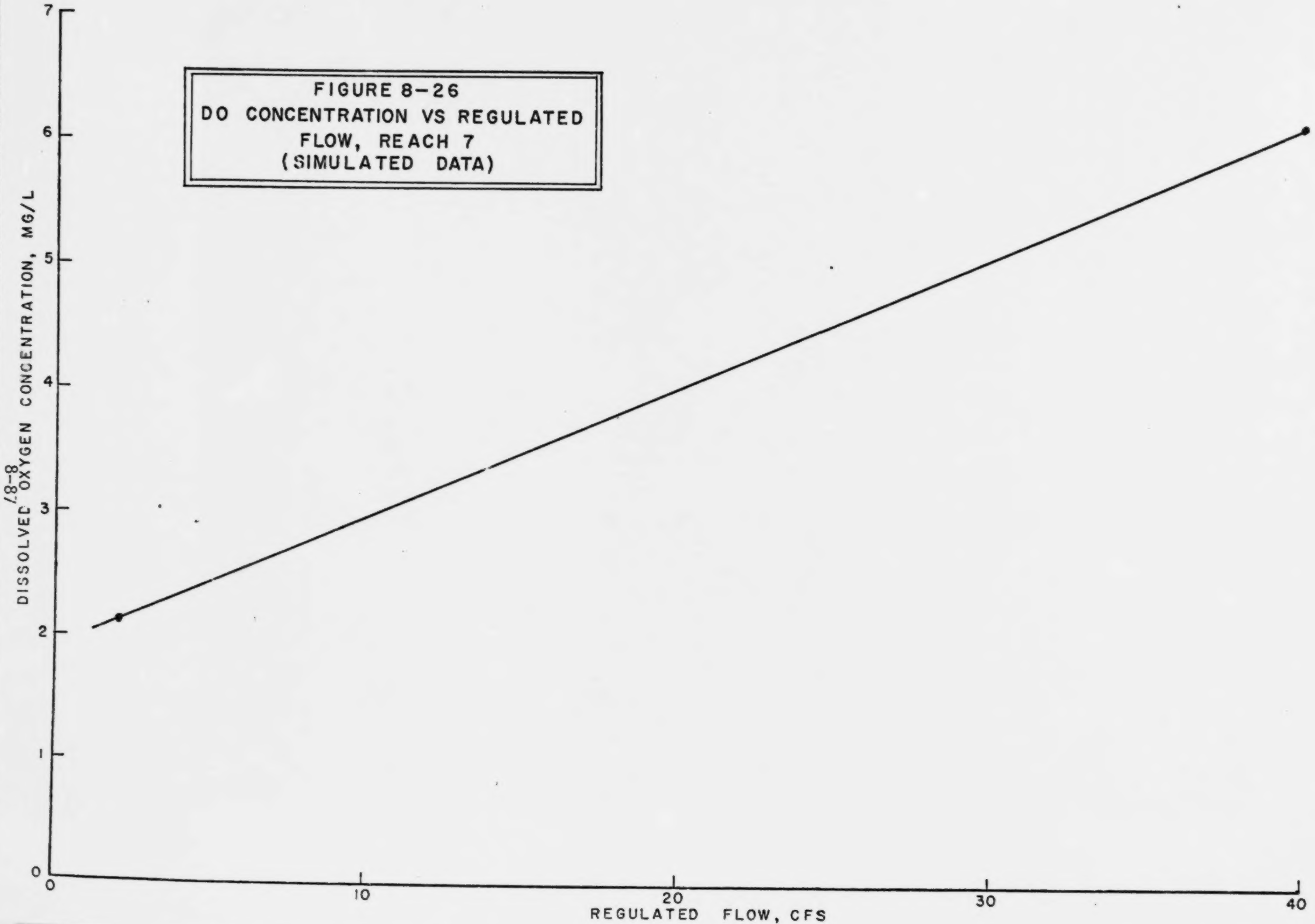


FIGURE 8-26
DO CONCENTRATION VS REGULATED
FLOW, REACH 7
(SIMULATED DATA)

assure that the mathematical models and program logic have functioned correctly.

The model is flexible and can be readily adapted to any watershed if proper attention is paid to obtaining accurate base data. The model has been applied successfully to the Farmington River Basin in Connecticut and Massachusetts.

8.7.2 Conclusions

Simulation techniques can be applied to a river system to determine the effects of the various decisions available to those whose task is to manage, or police the management of, the system. The system has been programmed to the digital computer which, if basic data inputs are accurate, will quickly and inexpensively provide a reasonable representation of the system response to the applied condition. A few of the advantages of this method of analysis over other methods which accrue to the user are: (1) The user is afforded an answer not only of an expected value but also a valuable indication of its variability by obtaining several simulation runs wherein the random variables in the system are allowed to vary as observed in nature, (2) The expected answer is the mean of several samples of system response to the set of imposed conditions, (3) The model can simulate a given set of conditions to determine the time when a resulting event is likely to occur, and the associated probability, (4) The model can simulate the future state of the system under as many different conditions as desired and at various levels of each condition variable by merely making minor changes in the input data, (5) The simulation will be completed in a matter of minutes and at a fraction of the cost of other methods, (6) Although the simulation model produces much detailed information, the output of information can be programmed to suppress all unwanted data to result in a concise data package that is readily analyzed, all without affecting the simulation, and (7) The rapidity and relative inexpense at which runs can be made and the ease with which variable values may be changed affords the opportunity to determine the sensitivity of the system to small changes in each variable. In short, a hydrology and water quality simulation model which accurately represents the watershed is a tool that can be of considerable use in making decisions relative to the planning and/or operating of a river system.

The application and testing phase of the project has shown that the simulation of hydrology in the Farmington River system produces an accurate representation of actual conditions. The lack of water quality data on the Farmington prevents drawing conclusions as to the accuracy of the water quality simulation. It can be said that the water quality values produced in simulation are reasonable and in line with those that could be expected under the imposed conditions.

This inability to check the water quality simulation prevents drawing conclusions about values of K_1 and K_2 which were used. It is suspected that the values of K_2 which result from the use of the Langbein and Durum formula (equation 3.38) are too high. Midway in the testing period the coefficient in that equation was arbitrarily changed from 3.3 to 1.3 and more reasonable results were obtained. This latter value may even prove to be somewhat high. A conclusion in this regard is to, if at all possible, find a way to determine the values of these constants from stream surveys of the watershed.

The sensitivity tests show that the dissolved oxygen concentration system function is: (1) relatively sensitive to the values of the constants in the velocity and depth equations and to the water temperature, (2) somewhat less sensitive to the reoxygenation velocity constant, K_2 and the regulated flow, (3) sensitive to a low degree to the size of the waste loads, DO concentration in the waste loads and the deoxygenation velocity constant, K_1 and, (4) insensitive to the deoxygenation and reoxygenation error factors, r and s, and to evaporation loss in the reservoirs.

The results of simulating releases from Colebrook Reservoir (the one most likely to stratify), which are either low in dissolved oxygen or relatively high in BOD, show that the effects on the downstream water quality are localized and that below reach 7, recovery from these imposed conditions is essentially complete. Colebrook Dam is at mile 58 and reach 7 begins at mile 31.3.

REFERENCES

1. ____________, *Interim Report on Review of Survey, Farmington* River Basin, United States Army, Corps of Engineers, New England Division, Waltham, Massachusetts, December, 1958.

2. See Reference 1, pp. 8-9.

3. See Reference 1, p. 14.

4. Long, Michael J., Personal Communication.

5. Sprong, R. C., Personal Communication.

SECTION 9
APPLICATION OF OPTIMIZATION MODEL

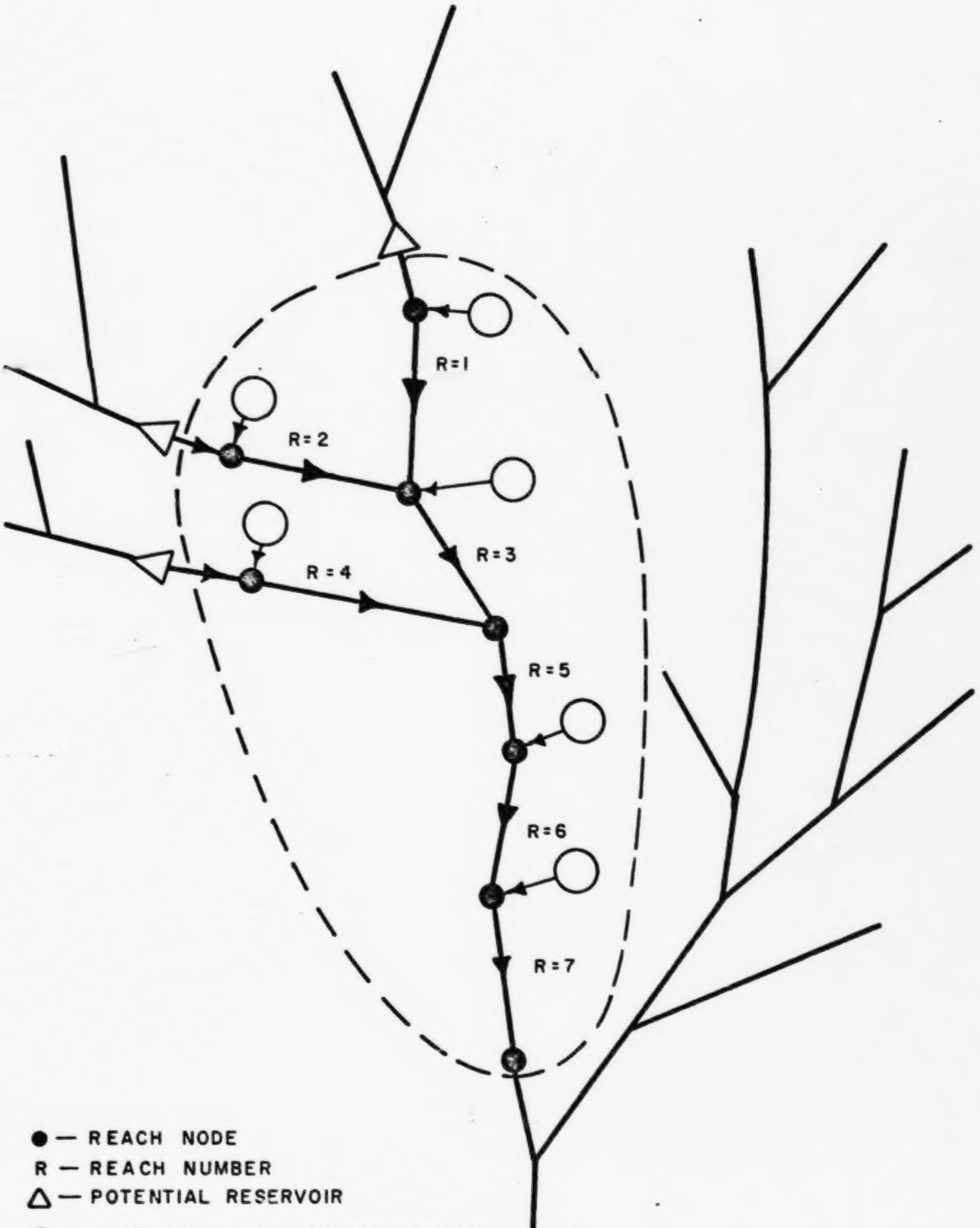
9.1 Introduction

The methodology developed for the optimization model was applied to a hypothetical watershed to permit testing of the model. Given a watershed, a region is then defined as a subset of the watershed. The hypothetical region used in this study consists of an area encompassing multiple reaches. Once delineated, the region may be analyzed as an independent part of the watershed. The critical period used in this case was a design flow, usually considered as the minimum average consecutive seven-day flow expected once in 10 years.

The hypothetical region was divided into headwater and interior reaches, with potential reservoir locations only at the beginning of the headwater reaches. The region used for testing the model is shown in Figure 9-1. It consists of six wastewater treatment facilities in the seven designated reaches, with a reservoir at any of the three headwater reaches. The reach nodes represent the beginning or end of a reach. Each wastewater treatment facility shown adjacent to a node affects the water quality in the region by the amount of BOD released into the reach. The population and industrial growth in the area is expected to increase the BOD loads so that additional treatment and/or low flow augmentation will be necessary. In order to prevent floating solids from entering the stream,each facility is required to remove at least 35% of its BOD load. For this case, a 90% maximum BOD removal was also assumed for the treatment facilities.

This generalized approach used in developing the model rendered the model applicable to a variety of watersheds with minimal alterations. The model was constructed in modular form so that any watershed can be examined by simply selecting the appropriate number of modules. The objective of the optimization model is to determine the combination of wastewater treatment facilities and flow augmentation which meet the water quality goals at the least cost to the region.

FIGURE 9-1
HYPOTHETICAL REGION



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9.2 Input Data for Model

The input data for the optimization model are output data from a simulation model for the critical period at the selected region in the watershed. The parameters used as input to the optimization model are summarized in Table 9-1. The hydrologic, wastewater, and economic data used in this study were taken from a 1967 article by Loucks, Revelle, and Lynn (1) and are shown in Tables 9-2 and 9-3. Upon accepting the data, the model internally generates all of the elements for the mathematical programming tableau. Information obtained from its solution may then trigger further analysis within the optimization model or call for new simulated data. The model is capable of handling a region with up to 50 reaches and generating a new matrix for any changes in the input data.

The costs and limiting bounds set on the variables in the objective function for the mathematical programming model were determined from predicted annual cost data of BOD removal. The nonlinear convex cost functions for the treatment facilities were divided into piecewise linear segments representing various percentages of BOD removal. The bounds on the variables were determined from the mg/l of BOD removed within each segment. The units of the BOD removed were transformed from mg/l to pounds, as shown in Table 9-4. From this datum, unit costs were determined for each of the treatment facilities and are listed in Table 9-5.

9.2.1 Changes in Hypothetical Data for Flow Augmentation

When considering flow augmentation from $\underline{S}_i^{(1)}$, the base (unaugmented) flow, to $\bar{S}_i^{(1)}$, the maximum flow, the following assumptions were made in obtaining new parameter values for the data in Table 9-2:

1. The reaches in the region under analysis are established with $t \leq t_c$, as described in the "Water Quality Constraints" part of Section 7.

2. Values of K_2 and T vary as a function of Q.

3. Values of K_1, K_3, M, and R remain constant.

TABLE 9-1
SUMMARY OF REACH INPUT PARAMETERS FOR OPTIMIZATION MODEL

Symbol	Description
T	Flow time through reach r, days
QW	Wastewater inflow, MGD
QT	Tributary inflow, MGD
CS	Saturation DO concentration, mg/l
$\bar{D}$	Maximum allowable DO deficit, mg/l
$\underline{D}$	Minimum allowable DO concentration, mg/l
CW	Wastewater DO concentration, mg/l
CT	Tributary DO concentration, mg/l
BW	Wastewater BOD concentration to treatment facility, mg/l
BT	Tributary BOD concentration, mg/l
K_1	Deoxygenation rate constant, days $^{-1}$
K_2	Reaeration rate constant, days $^{-1}$
K_3	Sedimentation and absorption rate constant, days $^{-1}$
M	Oxygen production ($M>0$) or reduction ($M<0$) due to plants and benthal deposits, mg/l/day
P	% BOD removal of 1980 load with existing wastewater treatment facility
R	BOD addition rate due to runoff and scour, mg/l/day

TABLE 9-2
REGIONAL STREAM AND WASTEWATER DATA FOR THE 7 REACHES (1)

Reach No.	T (days)	QW (MGD)	QT (MGD)	CS (mg/1)	$\bar{D}$ (mg/1)	$\underline{D}$ (mg/1)	CW (mg/1)	CT (mg/1)
1	.235	5.	1355.	10.20	3.20	7.0	1.0	9.5
2	1.330	37.	1290.	9.95	2.45	7.5	1.0	8.0
3	1.087	8.	0.	9.00	2.00	7.0	1.0	-
4	2.067	14.	296.	9.54	3.54	6.0	1.0	9.7
5	.306	0.	0.	9.00	2.50	6.5	-	-
6	1.050	26.	0.	8.35	2.35	6.0	1.0	-
7	6.130	41.	0.	8.17	4.17	4.0	1.0	-

Reach No.	BT (mg/1)	K_1 (days^{-1})	K_2 (days^{-1})	K_3 (days^{-1})	M (mg/1/day)	R (mg/1/day)
1	1.66	.31	1.02	.02	.85	.15
2	0.68	.41	.60	.03	.14	.14
3	-	.36	.63	.04	.18	.14
4	1.00	.35	.09	.04	.05	.11
5	-	.34	.72	.05	.39	.11
6	-	.35	.14	.06	.07	.13
7	-	.30	.02	.00	.00	.00

TABLE 9-3
REGIONAL WASTEWATER TREATMENT DATA (1)

Reach No.	BW (mg/1)	P %	Annual Costs of 1980 BOD Removal: Dollars					
			35%	50%	60%	75%	85%	90%
1	248	67	0	0	0	22,100	77,500	120,600
2	408	10	546,000	552,000	630,000	780,000	987,000	1,170.000
3	240	26	160,000	170,000	210,000	277,500	323,000	378,000
4	1440	24	324,000	339,000	413,000	523,000	626,000	698,000
5	0	0	0	0	0	0	0	0
6	2180	12	385,000	408,000	500,000	638,000	790,000	900,000
7	279	26	670,000	690,000	840,000	1,072,000	1,232,500	1,350,000

TABLE 9-4
REGIONAL WASTEWATER TREATMENT FOR 1980 BOD REMOVAL

Reach No.	Existing BOD Removal		Future BOD Removal in Indicated Treatment Segment: Pounds					
	%	Pounds	35%	50%	60%	75%	85%	90%
1	67	6900	0	0	0	830	1030	520
2	10	12600	31500	18900	12600	18900	12600	6300
3	26	4200	1400	2400	1600	2400	1600	800
4	24	40400	18500	25200	16800	25200	16800	8400
5	0	0	0	0	0	0	0	0
6	12	108700	108700	70900	47300	70900	47300	23600
7	26	8600	8600	14300	9500	14300	9500	4800

TABLE 9-5
REGIONAL WASTEWATER TREATMENT
UNIT COST DATA

Reach No.	Existing BOD Removal	Unit Cost of BOD Removal in Indicated Treatment Segment: Dollars/Pound of BOD Removed					
	%	35%	50%	60%	75%	85%	90%
1	67	0	0	0	26.7	53.6	83.4
2	10	17.4	0.3	6.2	7.9	16.4	29.1
3	26	111.0	4.2	25.0	28.1	28.4	68.7
4	24	17.5	0.6	4.4	4.4	6.1	8.6
5	0	0	0	0	0	0	0
6	12	3.5	0.3	2.0	2.0	3.2	4.6
7	26	78.0	1.4	15.7	16.2	16.8	24.6

The relationships used in calculating how these parameters vary with changes in Q were based on the stream studies of river systems by Leopold and Maddock (2). They detected that the following relationships hold for natural cross sections:

$$w = a\,Q^{l} \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.1]}$$

$$d = c\,Q^{f} \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.2]}$$

$$v = k\,Q^{m} \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.3]}$$

where:
Q = discharge,
w = water surface width,
d = mean depth,
v = mean velocity, and
a,c,f,k,l,m are numerical constants.
The relationship for calculating discharge in a stream is:

$$Q = Av \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.4]}$$

where A = cross-sectional area of the stream.
For rectangular cross sections,

$$A = wd \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.5]}$$

Substituting equations [9.1], [9.2], [9.3] and [9.5] into [9.4] indicates that

$$Q = a\,c\,k\,Q^{f+l+m} \quad \ldots\ldots \quad \text{[Eq. 9.6]}$$

iff

$$a\,c\,k = 1.0 \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. 9.7]}$$

and

$$l + f + m = 1.0 \quad \ldots\ldots\ldots \quad \text{[Eq. 9.8]}$$

Average values of the exponents l, f, and m were obtained from these empirical studies. The average values of the constants were found to be:

$$l = 0.26$$

$$f = 0.40$$

$$m = 0.34$$

Using the Leopold-Maddock article as a source, Langbein and Durum (3) stated that "As a river rises in response to an increase in discharge, it increases its depth and velocity, a condition causing the reaeration coefficients to decrease. In general, rivers increase in depth and velocity at about the 0.4 power of the discharge. Hence, at any given location, the coefficients of reaeration decrease at about the 0.13 power of the discharge." The above estimate will be used in analyzing the coefficient of reaeration, or

$$K_2 = g\ Q^{-0.13} \quad . . . . \text{[Eq. 9.9]}$$

With the values of $S_1^{(1)}$ as boundary conditions, the value of the constant, g, can be determined for each reach from equation [9.9]. The amount of time it takes the water to flow from the beginning of a reach to the end can be determined using equation [9.3] and noting that

$$v = L/T \quad . . . . \text{[Eq. 9.10]}$$

where L = length of the reach.

Therefore

$$L/T = k\ Q^{m}$$

or

$$T = (L/k)Q^{-m} \quad . . . . \text{[Eq. 9.11]}$$

The travel time in each reach was found by letting m = 0.34.

Similarly, as in equation [9.9], the value of the L/k can be determined from the known boundary conditions for the base flow, $\underline{S}_i^{(1)}$.

9.3 Treatment Model Formulation

The multi-commodity network format utilized in the optimization model is believed to simplify tracking commodities being transported down a region of a watershed. This network is essentially viewed as a unidirectional transportation system conveying three commodities: water, biochemical oxygen demand, and dissolved oxygen. The carrier commodity, water, is conservative while in transit through the network, but the two water quality constituents, BOD and DO, attenuate or amplify while in transit.

The methodology developed for the treatment model and the relationships incorporated into its network format are found in Section 7. Equation [7.31] is now used in matrix notation for application to this hypothetical problem. The objective function is as follows:

$$\text{Min } Z = [C^{(2T)}]' \, [Q^{(2T)}] \quad . \; . \; . \; . \; . \quad \text{[Eq. 9.12]}$$

where:

$[C^{(2T)}]$ = unit cost matrix for transporting a unit of commodity 2 (BOD) along treatment branch b in the plant in reach i, and

$[Q^{(2T)}]$ = matrix for the amount of BOD removed at the corresponding branches.

The unit cost matrix may be partitioned as follows:

$$[C^{(2T)}]' = \left[\begin{array}{c} C_{1b}^{(2T)} \\ \hline C_{2b}^{(2T)} \\ \hline \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hline C_{7b}^{(2T)} \end{array}\right] \quad . \; . \; . \; . \; . \; . \quad \text{[Eq. 9.13]}$$

where $[C_{ib}^{(2T)}]$ is the cost vector for branch b (b denotes upper bound on % BOD removal) in the treatment facility in the ith reach. In this problem reach five has no treatment facilities, therefore $[C_{5b}^{(2T)}]$ is a null vector. The required minimum of 35% BOD removal permits restricting the analysis to the convex portion of the cost function. Selecting reach two as a numerical example yields the following unit cost vector for up to 90% BOD removal:

$$[C_{2b}^{(2T)}]' = \begin{bmatrix} C_{2,10}^{(2T)} \\ C_{2,35}^{(2T)} \\ C_{2,50}^{(2T)} \\ C_{2,60}^{(2T)} \\ C_{2,75}^{(2T)} \\ C_{2,85}^{(2T)} \\ C_{2,90}^{(2T)} \end{bmatrix}' = \begin{bmatrix} 0 \\ 17.4 \\ 0.3 \\ 6.2 \\ 7.9 \\ 16.4 \\ 29.1 \end{bmatrix}' \quad \ldots\ldots [\text{Eq. } 9.14]$$

Likewise $[Q^{(2T)}]$ can be partitioned as follows:

$$[Q^{2T}] = \left[\begin{array}{c} Q_{1b}^{(2T)} \\ \hline Q_{2b}^{(2T)} \\ \hline \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hline Q_{7b}^{(2T)} \end{array}\right] \quad \ldots\ldots\ldots\ldots [\text{Eq. } 9.15]$$

where $[Q_{ib}^{(2T)}]$ is the BOD commodity vector for the treatment facility in the ith reach. The vector for reach two is as follows:

$$[Q_{2b}^{(2T)}] = \begin{bmatrix} Q_{2,10}^{(2T)} \\ Q_{2,35}^{(2T)} \\ Q_{2,50}^{(2T)} \\ Q_{2,60}^{(2T)} \\ Q_{2,75}^{(2T)} \\ Q_{2,85}^{(2T)} \\ Q_{2,90}^{(2T)} \end{bmatrix} = \begin{bmatrix} 12,600. \\ 31,500. \\ 18,900. \\ 12,600. \\ 18,900. \\ 12,600. \\ 6,300. \end{bmatrix} \quad \ldots \text{[Eq. 9.16]}$$

Given the objective function, the physical - technical constraints of the primal problem shown in equation [7.31] are restated below:

$$\begin{bmatrix} A^{(1)} & 0 & 0 & 0 \\ P^{(12)} & A^{(2)} & 0 & T^{(2)} \\ P^{(13)} & P^{(23)} & A^{(3)} & 0 \\ D^{(13)} & D^{(23)} & D^{(33)} & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} Q^{(1)} \\ \hline Q^{(2)} \\ \hline Q^{(3)} \\ \hline Q^{(2T)} \end{bmatrix} = \begin{bmatrix} S^{(1)} \\ \hline S^{(2)} \\ \hline S^{(3)} \\ \hline \underline{Y}^{(3)} \\ \hline \bar{Q}^{(2T)} \end{bmatrix} \text{[Eq. 9.17]}$$

$$[Q^{(1)}],\ [Q^{(2)}],\ [Q^{(3)}],\ [Q^{(2T)}] \geq 0$$

where the following vectors are defined in reach i, i=1,2,..,7

$[Q^{(1)}]$ = flow of commodity 1,
$[Q^{(2)}]$ = flow of commodity 2,
$[Q^{(3)}]$ = flow of commodity 3,
$[Q^{(2T)}]$= amount of BOD removed at the corresponding branches in the plant,
$[S^{(1)}]$ = additional amount of commodity 1 entering system,
$[S^{(2)}]$ = additional amount of commodity 2 entering system,
$[S^{(3)}]$ = additional amount of commodity 3 entering system,
$[\underline{Y}^{(3)}]$ = minimum allowable flow of commodity 3, and
$[\overline{Q}^{(2T)}]$= upper bound on $[(Q^{(2T)})]$

A description of the above matrices [A], [P], [D], [T], and [I] is presented below. $A^{(k)}$, k = 1,2,3, is the matrix of continuity coefficients for the three commodities: water, BOD and DO, respectively. The attenuation and amplification coefficients describe the change in BOD and DO between reach points. Using the network format developed for the mathematical programming model, the nodes are the rows and the branches connecting the nodes are the columns (variables) in the matrix. The general form of the $A^{(k)}$ matrix is shown below:

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$$
\begin{array}{cc|cccccccccccccc|l}
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 1 & 2 & 3 & 4 & 6 & 7 & 8 & \\
 & & P & P & P & P & P & P & P & B & B & B & B & B & B & P & \text{Source Node No.} \\
 & & 3 & 3 & 5 & 5 & 6 & 7 & 8 & 1 & 2 & 3 & 4 & 6 & 7 & 9 & \\
 & & P & P & P & P & P & P & P & P & P & P & P & P & P & P & \text{Sink Node No.} \\
 & & k & k & k & k & k & k & k & k & k & k & k & k & k & k & \text{Commodity} \\
\hline
 & k\ 1B & & & & & & & & 1 & & & & & & & \\
 & k\ 1P & 1 & & & & & & & -1 & & & & & & & \\
N & k\ 2B & & & & & & & & & 1 & & & & & & \\
 & k\ 2P & & 1 & & & & & & & -1 & & & & & & \\
O & k\ 3B & & & & & & & & & & 1 & & & & & \\
 & k\ 3P & \omega_1 & \omega_2 & 1 & & & & & & & -1 & & & & & =[A^{(k)}] \\
D & k\ 4B & & & & & & & & & & & 1 & & & & \\
 & k\ 4P & & & & 1 & & & & & & & -1 & & & & [\text{Eq. } 9.18] \\
E & k\ 5P & & & \omega_3 & \omega_4 & 1 & & & & & & & & & & \\
 & k\ 6B & & & & & & & & & & & & 1 & & & \\
S & k\ 6P & & & & & \omega_5 & 1 & & & & & & -1 & & & \\
 & k\ 7B & & & & & & & & & & & & & 1 & & \\
 & k\ 7P & & & & & & \omega_6 & 1 & & & & & & -1 & & \\
 & k\ 8P & & & & & & & \omega_7 & & & & & & & 1 & \\
\hline
\end{array}
$$

where
$\omega_i = -1$ for $k = 1$; $i = 1,2,\ldots,7$
$\omega_i = -\lambda_i$ for $k = 2$; $i = 1,2,\ldots,7$
$\omega_i = -\alpha_i$ for $k = 3$; $i = 1,2,\ldots,7$

Other coefficients describe the interrelationship between the commodities in transit through each reach. The interdependencies between any two of the three commodities can be represented in the following manner.

k',k	Commodity Interrelationship
1,2	Water, BOD
1,3	Water, DO
2,3	BOD, DO

The general form of $[P^{k',k}]$, representing the matrix of interdependency coefficients, is shown below:

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$$\begin{array}{cc|ccccccc|l}
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \\
 & & P & P & P & P & P & P & P & \text{Source Node No.} \\
 & & 3 & 3 & 5 & 5 & 6 & 7 & 8 & \\
 & & P & P & P & P & P & P & P & \text{Sink Node No.} \\
 & & k' & k' & k' & k' & k' & k' & k' & \text{Commodity } k' \\
\hline
 & k\ 1B & & & & & & & & \\
 & k\ 1P & & & & & & & & \\
N & k\ 2B & & & & & & & & \\
 & k\ 2P & & & & & & & & \\
O & k\ 3B & & & & & & & & \\
 & k\ 3P & \nu_1 & \nu_2 & & & & & & \\
D & k\ 4B & & & & & & & & \\
 & k\ 4P & & & & & & & & \\
E & k\ 5P & & & \nu_3 & \nu_4 & & & & \\
 & k\ 6B & & & & & & & & \\
S & k\ 6P & & & & & \nu_5 & & & \\
 & k\ 7B & & & & & & & & \\
 & k\ 7P & & & & & & \nu_6 & & \\
 & k\ 8P & & & & & & & \nu_7 & \\
\hline
\end{array} = [P^{k',k}] \quad \ldots [\text{Eq. } 9.19]$$

where
$\nu_i = -\mu_i$ for $k',k = 1,2$ For $i = 1,2,\ldots,7$
$\nu_i = -\Psi_i$ for $k',k = 1,3$ For $i = 1,2,\ldots,7$
$\nu_i = \gamma_i$ for $k',k = 2,3$ For $i = 1,2,\ldots,7$

Using similar notation and letting $[D^{k',k}]$ represent the matrix of coefficients for water quality DO standards, then k',k signifies in the following the relationship between each commodity and the DO in the reach.

k',k	Commodity Interrelationship
1,3	Water, DO
2,3	BOD, DO
3,3	DO, DO

Shown below is the general form of the $[D^{k',k}]$ diagonal matrix:

$$\begin{array}{cl|ccccccc|l}
 & & \multicolumn{7}{c}{\text{BRANCHES}} & \\
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \\
 & & P & P & P & P & P & P & P & \text{Source Node No.} \\
 & & 3 & 3 & 5 & 5 & 6 & 7 & 8 & \\
 & & P & P & P & P & P & P & P & \text{Sink Node No.} \\
 & & k' & k' & k' & k' & k' & k' & k' & \text{Commodity } k' \\
\hline
 & \text{kQUAL1P} & \eta_1 & & & & & & & \\
N & \text{kQUAL2P} & & \eta_2 & & & & & & \\
O & \text{kQUAL3P} & & & \eta_3 & & & & & \\
D & \text{kQUAL4P} & & & & \eta_4 & & & & = [D^{k',k}] \quad . \quad \text{[Eq. 9.20]} \\
E & \text{kQUAL5P} & & & & & \eta_5 & & & \\
S & \text{kQUAL6P} & & & & & & \eta_6 & & \\
 & \text{kQUAL7P} & & & & & & & \eta_7 & \\
\hline
\end{array}$$

where
$\eta_i = \psi_i$ for $k',k = 1,3$ For $i = 1,2,\ldots,7$
$\eta_i = -\gamma_i$ for $k',k = 2,3$ For $i = 1,2,\ldots,7$
$\eta_i = \alpha_i$ for $k',k = 3,3$ For $i = 1,2,\ldots,7$

Let $[T^{(2)}]$ represent the matrix of BOD treatment alternatives with the general form shown below:

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$$
\begin{array}{ll|cccccc|l}
 & & 1 & 2 & 3 & 4 & 6 & 7 & \text{Source Node No.} \\
 & & b & b & b & b & b & b & \text{Treatment Segment} \\
 & & 2 & 2 & 2 & 2 & 2 & 2 & \text{BOD Commodity No.} \\
\hline
N & 2\ 1B & \beta & & & & & & \\
O & 2\ 2B & & \beta & & & & & \\
D & 2\ 3B & & & \beta & & & & = [T^{(2)}] \quad . \quad . \ \text{[Eq. 9.21]} \\
E & 2\ 4B & & & & \beta & & & \\
S & 2\ 6B & & & & & \beta & & \\
 & 2\ 7B & & & & & & \beta &
\end{array}
$$

where

β = sum vector with n components

n = number of linear segments representing percent treatment at a facility.

The remaining matrix [I] in Equation [9.17] is the identity matrix of upper bounds on BOD removal.

9.3.1 Optimization Programs

The theoretical development and application of the optimization model necessitated the development of two main computer programs and one auxiliary program. These three routines make up the optimization package and were programmed for an IBM 360/65 computer. Below is a brief discussion of each routine. A thorough description and coding of the optimization package is contained in Sections A4.7, A4.8, and A4.9 of the appendices.

1. INTERF - Interface Program

The objective of this main routine is to accept specified stream and wastewater data from a simulation model and to generate the mathematical programming model in the format specified for the MPS/360 Processor. The program contains the flexibility of handling a region with up to 50 reaches and generating a new matrix for any changes in the simulated input data.

2. MPS Control Program
The objective of the auxiliary routine is to specify the optimization procedures to be used in providing the necessary outputs needed for analysis. Depending only on the statements used in this control program, post-optimal procedures may be applied with only minor changes in the matrix generated by INTERF.

3. LPLF - Linear Programming Model
The objective of this main routine is to allocate waste treatment and/or low flow augmentation to meet preset water quality standards and determine the optimal solution for a specified region in a watershed. The linear programming model, developed in a multi-commodity network format, is composed of the output from INTERF, the MPS Control Program, and predetermined treatment cost data along with waste treatment bounds.

These routines in the optimization package may be considered as components of a closed loop information feedback system, giving solutions to any changes in exogenous or endogenous data. Information obtained from its output, via control points, may trigger further analyses of the optimal solution or specify changes in the original data as shown in Figure 9-2.

9.3.2 Post-Optimal Analysis of Regional Waste Treatment Costs

9.3.2.1 Primary or Secondary Treatment Required at All Reaches

Given the water quality standards, each of the seven reaches seeks to find the least cost way to meet the standard. The optimization model provides the decision-makers with the information needed to achieve the above objective. The summary output for the seven reaches with a minimum of primary treatment required is tabulated in Table 9-6. The summary output for the case where secondary treatment is required at all waste sources is tabulated in Table 9-7. The regional authority now knows the "best" combination of wastewater treatment facilities. Note that the total cost to the region is higher if secondary treatment is required. If secondary treatment is not required at all reaches, then more intensive treatment is provided where it is most effective. The addition of a secondary treatment constraint cannot decrease the regional cost. However, the regional cost will be unchanged if this additional constraint is not binding, i.e., all waste sources are already removing more than 85% of

FIGURE 9-2
OPTIMIZATION PACKAGE

INPUT—STREAM AND WASTEWATER DATA
OUTPUT—OPTIMAL SOLUTION AND POSTOPTIMAL ANALYSIS
0 —CONTROL POINTS

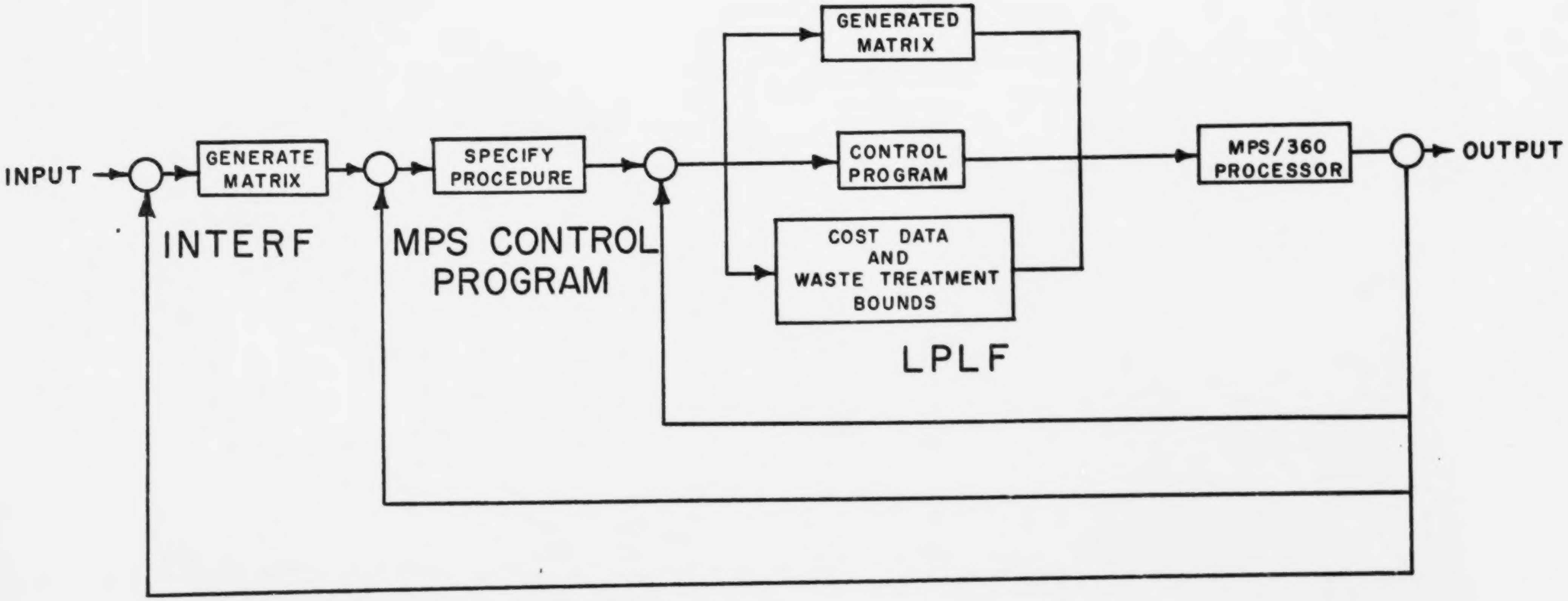

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TABLE 9-6
OPTIMAL SOLUTION WITH ONLY PRIMARY TREATMENT (35% BOD REMOVAL) REQUIRED

Reach	% BOD Removal		Future Annual Cost $\$ \times 10^3$	Future Minimum DO in Reach: mg/l	
	Present	Future		Actual	Allowable
1	67	67	0	9.7	7.0
2	10	69	715	7.9	7.5
3	26	50	170	8.5	7.0
4	24	89.9	696	6.0	6.0
5	--	--	--	8.3	6.5
6	12	90	900	7.3	6.0
7	26	50	690	4.0	4.0

Total Cost = 3171

TABLE 9-7
OPTIMAL SOLUTION WITH PRIMARY AND SECONDARY TREATMENT (85% BOD REMOVAL) REQUIRED

Reach	% BOD Removal		Annual Costs $\$ \times 10^3$	Future Minimum D.O. in Reach: mg/l	
	Present	Future		Actual	Allowable
1	67	85	77	9.7	7.0
2	10	85	987	8.4	7.5
3	26	85	323	8.8	7.0
4	24	89.9	696	6.0	6.0
5	--	--	--	8.6	6.5
6	12	85	790	7.4	6.0
7	26	85	1232	4.9	4.0

Total Cost = 4105

the BOD. From analyzing the primary treatment solution, it is apparent that requiring 85% treatment, i.e., secondary treatment, at all sources would increase regional costs. In fact, the costs are increased from \$3.17 x 10^6 to \$4.10 x 10^6 because of this secondary treatment constraint. The analysis will be extended to provide a methodology which is applicable to a wide variety of actual circumstances. For example, how should this total cost be apportioned among the decision-making units? How much would augmented flow be worth to any combination of these units? This will provide the requisite information to permit not only formulation, but also implementation of optimal regional waste management strategies.

9.3.2.2 Cost Allocation Among Reaches

Given the solution to the primary treatment case, how should the total annual cost be apportioned among the decision-making units? It would be inequitable to assess the costs as shown in Column four of Table 9-6. Comparison of columns five and six in this table tells which reaches are providing excess treatment. Reach six is overtreating - apparently to assist reach seven in meeting its constraint. The decision-maker in reach six would be irrational to participate in this plan since it would cost him more than he would have to pay to just meet his standard. Reaches two and three are also overtreating while reach four is just treating enough to meet exactly its water quality standard. Reach one would be indifferent as to participating since it is also overtreating but has zero unit cost, up to 67% removal. For this system, reach seven should be willing to pay the upstream reaches for providing the supplementary treatment to meet its standard.

The response of reach seven to any DO standard can be determined by parametric programming. Table 9-8 shows the system response as the dissolved oxygen standard at reach seven is varied from one to seven mg/l. At one mg/l the standard is being met by a combination of treatment at reaches one, six, and seven. Reach one is providing 67.0% removal (32.0% for reach seven) due to the fact that the cost is zero up to that level and is thereby the favored alternative. Reach six is providing 70.9% treatment of which 1.4% is attributable to assisting in meeting the reach seven constraint. All of the treatment in reach seven naturally goes to meet the standard in that reach since it is the last reach. Reaches two and four are providing exactly enough treatment to meet their local standard, while reach three is treating at the 35.0% minimum requirement.

As the water quality standard increases toward 4.0 mg/l, the amount of treatment continues to increase in reach six. All other reaches stay the same. This means that, over this range, treatment in reach six is the most economical. However, as the standard approaches 4.0 mg/l, reach six attains its maximum capacity and it becomes necessary to treat elsewhere. Examination of the basis changes in the range from 3.5 to 4.0 mg/l indicates that the most economical treatment sequence is to utilize 15.0% of the capacity at reach three; then the remaining capacity of reach four (0.1%); and then begin to utilize capacity at reach two (12.8%). Between 4.0 and 4.5 mg/l, the model initially continues to increase treatment at reach two and then switches to reach seven. The procedure continues in this manner until the parameterization stops at 5.81 mg/l - the point at which an infeasible solution is reached. This indicates that all of the facilities are treating at full capacity and it is impossible to achieve any higher level of water quality goal.

TABLE 9-8
SEQUENCE OF REGIONAL WASTE TREATMENT AS DO STANDARD
AT REACH SEVEN VARIES FROM 1 TO 7 mg/1.

D.O. Standard at Reach Seven mg/1	Incremental % BOD Removal in Each Reach to Meet Reach 7 Standard					
	1	2	3	4	6	7
1.0	32.0	0	0	0	1.4	50.0
1.5	32.0	0	0	0	5.0	50.0
2.0	"	0	0	0	8.6	50.0
2.5	"	0	0	0	12.2	"
3.0	"	0	0	0	16.1	"
3.5	"	0	0	0	19.4	"
4.0	"	12.8	15.0	0.1	20.5	"
4.5	"	19.9	15.0	0.1	20.5	62.8
5.0	"	19.9	15.0	0.1	20.5	80.1
5.5	32.0	29.9	15.9	0.1	20.5	90.0
5.81	55.0	34.9	55.0	0.1	20.5	90.0
>5.81	INFEASIBLE SOLUTION					

9.3.2.3 Equivalent Prices for Upstream BOD Removal

Next, the question of estimating the price that the decision-making unit in reach seven would be willing to pay for upstream BOD removal is examined. It might be conjectured that he would be willing to pay any amount less than his own cost for upstream BOD removal. Unfortunately, there is not a one to one (or even a trivial) correspondence between BOD removal at reach seven and at any other upstream reach due to the complex instream processes that occur. Thus, it is necessary to describe how the economic value of the product (waste treatment) varies according to the location of the facility.

Recall that in Section 7 the various states of the regional system were identified by examining the shadow price with respect to BOD removal, $\pi_i^{(2)}$, and the shadow price with respect to the DO standard, $\pi_i^{(4)}$. Table 9-9 shows these prices for the original set of water quality standards with only primary treatment required.

TABLE 9-9

SHADOW PRICES FOR BOD REMOVAL AND WATER QUALITY STANDARD IN EACH REACH

(PRIMARY TREATMENT REQUIRED)

Shadow Price	Reach 1	2	3	4	5	6	7
$\pi_i^{(2)}$:\$/#BOD	10.87	7.90	11.36	11.17	13.61	13.93	14.44
$\pi_i^{(4)}$:\$/#DO	0	0	0	0	0	0	18.58

Examination of Table 9-9 indicates that competition exists at the treatment facility in reach seven. This is indicated by non-zero shadow prices for BOD removal in all contributing reaches and a positive shadow price for the water quality standard only in reach seven. It is also seen from Table 9-9 that the BOD shadow price decreases in the upstream direction because of the diminishing effectiveness of upstream waste treatment.

The general recursive relationship, developed in Section 7, is now used to determine the rate at which upstream BOD removal can be substituted for removal at reach seven:

$$\Theta_{ij} = \pi_j^{(2)} / \pi_i^{(2)} \text{ For } j \varepsilon A_i \quad . \; . \; [\text{Eq. } 9.22]$$

where A_i = set of all contributing reaches above reach i.
For this river system, Θ_{ij} can be calculated as follows:

Reach 6 relative to reach 7

$$\rho_7^{(2,3)}(\pi_7^{(2)}) = [\rho_7^{(2,3)}\varepsilon_6^{(2)} + \rho_6^{(2,3)}\varepsilon_7^{(3)}]\pi_6^{(2)}$$

[Eq. 9.23]

Reach 4 relative to reach 7

$$\rho_7^{(2,3)}(\pi_7^{(2)}) = [\rho_7^{(2,3)}(\prod_{4,5,6}\varepsilon_j^{(2)}) +$$

$$\rho_6^{(2,3)}(\prod_{4,5}\varepsilon_j^{(2)})\varepsilon_7^{(3)} + \rho_5^{(2,3)}\varepsilon_4^{(2)}(\prod_{6,7}\varepsilon_j^{(3)})$$

$$+ \rho_4^{(2,3)}(\prod_{5,6,7}\varepsilon_j^{(3)})]\,\pi_4^{(2)} \quad \ldots\ldots \quad \text{[Eq. 9.24]}$$

Reach 3 relative to reach 7

$$\rho_7^{(2,3)}(\pi_7^{(2)}) = [\rho_7^{(2,3)}(\prod_{3,5,6}\varepsilon_j^{(2)}) + \rho_6^{(2,3)}(\prod_{3,5}\varepsilon_j^{(2)})\varepsilon_7^{(3)}$$

$$+ \rho_5^{(2,3)}\varepsilon_3^{(2)}(\prod_{6,7}\varepsilon_j^{(3)}) + \rho_3^{(2,3)}(\prod_{5,6,7}\varepsilon_j^{(3)})]\,\pi_3^{(2)}$$

[Eq. 9.25]

Reach 2 relative to reach 7

$$\rho_7^{(2,3)}(\pi_7^{(2)}) = [\rho_7^{(2,3)}(\prod_{2,3,5,6}\varepsilon_j^{(2)}) +$$

$$\rho_6^{(2,3)}(\prod_{2,3,5}\varepsilon_j^{(2)})\,\varepsilon_7^{(3)} + \rho_5^{(2,3)}(\prod_{2,3}\varepsilon_j^{(2)})(\prod_{6,7}\varepsilon_j^{(3)}) +$$

$$\rho_3^{(2,3)}\,\varepsilon_2^{(2)}\,(\prod_{5,6,7}\varepsilon_j^{(3)}) + \rho_2^{(2,3)}(\prod_{3,5,6,7}\varepsilon_j^{(3)}]\,\pi_2^{(2)}$$

[Eq. 9.26]

Reach 1 relative to reach 7

$$\rho_7^{(2,3)}(\pi_7^{(2)}) = [\rho_7^{(2,3)}(\prod_{1,3,5,6} \varepsilon_j^{(2)}) +$$

$$\rho_6^{(2,3)}(\prod_{1,3,5} \varepsilon_j^{(2)})\ \varepsilon_7^{(3)} + \rho_5^{(2,3)}(\prod_{1,3} \varepsilon_j^{(2)})\ (\prod_{6,7} \varepsilon_j^{(3)})$$

$$+ \rho_3^{(2,3)}\ \varepsilon_1^{(2)}\ (\prod_{5,6,7} \varepsilon_j^{(3)} + \rho_1^{(2,3)}(\prod_{3,5,6,7} \varepsilon_j^{(3)})]\ \pi_1^{(2)}$$

. . . . [Eq. 9.27]

Knowing the BOD shadow prices for all contributing reaches and consequently the Θ_{ij}, it is now possible to describe this regional system in a market context. The BOD shadow prices represent the marginal value of upstream waste treatment to reach seven under optimal conditions.

For the hypothetical region, the vector of Θ_{ij}'s are shown in Table 9-10.

TABLE 9-10
RATE OF SUBSTITUTION OF UPSTREAM WASTE TREATMENT FOR TREATMENT AT REACH SEVEN

Item	Reach 1	2	3	4	6	7
Θ_{ij}	.752	.546	.785	.771	.965	1.00

If, for example, the marginal cost of BOD removal at reach seven is $10/pound, then he would be willing to pay reach one up to $7.52/pound, reach two up to $5.46/pound, etc. for upstream BOD removal.

9.4 Post-Optimal Analysis to Determine Regional Flow Augmentation Benefits

This section describes how the more productive sources of augmented flow might be identified under the assumption that the activity coefficients are independent of flow. The flow dependency of the activity coefficients will be analyzed later.

The augmented flow may come from reservoirs located at any of the headwater reaches (one, two or four in this case). The assumed upper bounds on the availability of augmented flow are twice the base flow at reaches one and two, and four times the base flow in reach four. The relative effectiveness of augmented flow depends on its quality characteristics and the reaches in which it is of use in reducing waste treatment costs. For example, a unit release of augmented flow in reach one would be of potential value to reaches one, three, five, six, and seven.

9.4.1 Assumed Sources of Augmented Flow

A point of departure for the analysis might be to determine where one additional unit of water could be most effectively employed. But this information is already known from inspecting the solution to the dual problem. The shadow price with respect to the water is $\partial Z^*/\partial S_i^{(1)} = \pi_i^{(1)}$. The shadow price vector for the original run is shown in Table 9-11. As might be expected the value of the unit of water is higher in the upper reaches. Reach two is seen to be the most desirable reach to introduce an additional unit of water.

TABLE 9-11

SHADOW PRICE FOR WATER SUPPLY IN EACH REACH

(PRIMARY TREATMENT REQUIRED)

Item	Marginal Value of Water in Indicated Reach: $/MG						
	1	2	3	4	5	6	7
$\pi_i^{(1)}$	1045	1189	936	654	515	295	136

Because of the fixed costs involved in constructing storage facilities, it is likely that only a single source of augmented flow would be used if a capital expansion is required. However, if there are existing reservoirs on reaches one, two, and four, then it is possible that the desired augmented flow could be obtained from a combination of reservoir releases. Benefit functions are developed below for both these cases.

9.4.1.1 Selection of Most Effective Single Source

The selection of the single most effective source of augmented flow will be determined in this section using parametric programming. The procedure is as follows:

1. Obtain an optimal solution with no flow augmentation.

2. Using parametric programming find $\partial Z^*/\partial S_i^{(1)}$ over a range from the base flow, $\underline{S}_i^{(1)}$, to the maximum flow, $\bar{S}_i^{(1)}$.

3. Do this analysis for augmented flow at reach one, reach two, and lastly at reach four.

4. Plot the benefit functions for comparative purposes.

The results of this analysis are shown in Figure 9-3. Reach two is the most desirable reach for flow augmentation. Reach one is the second choice; reach four is the last choice. The most significant difference in the slope of the benefit function occurs at smaller levels of augmented flow. This difference is partially attributable to the differential value of water in the headwater reaches and the fact that augmentation in reach four does not help reach three. The diminishing marginal benefits result from the fact that higher levels of augmented flow substitute for the less costly waste treatment.

A description of the impact of augmented flow at reach two is presented below to assist the interpretation of the results. With no augmented flow, reach two is treating 69% of its waste. At 125 MG of augmentation (the first breakpoint in Figure 9-3), the required treatment at reach two has been reduced to less than 60% so that the unit costs are lower. The amount of waste treatment has also been reduced in downstream reaches. At the second breakpoint (268 MG of flow in reach two), the treatment at reach two has been reduced to 50%. Next, the treatment at reach four was reduced to less than 90% BOD removal. Reach four has been

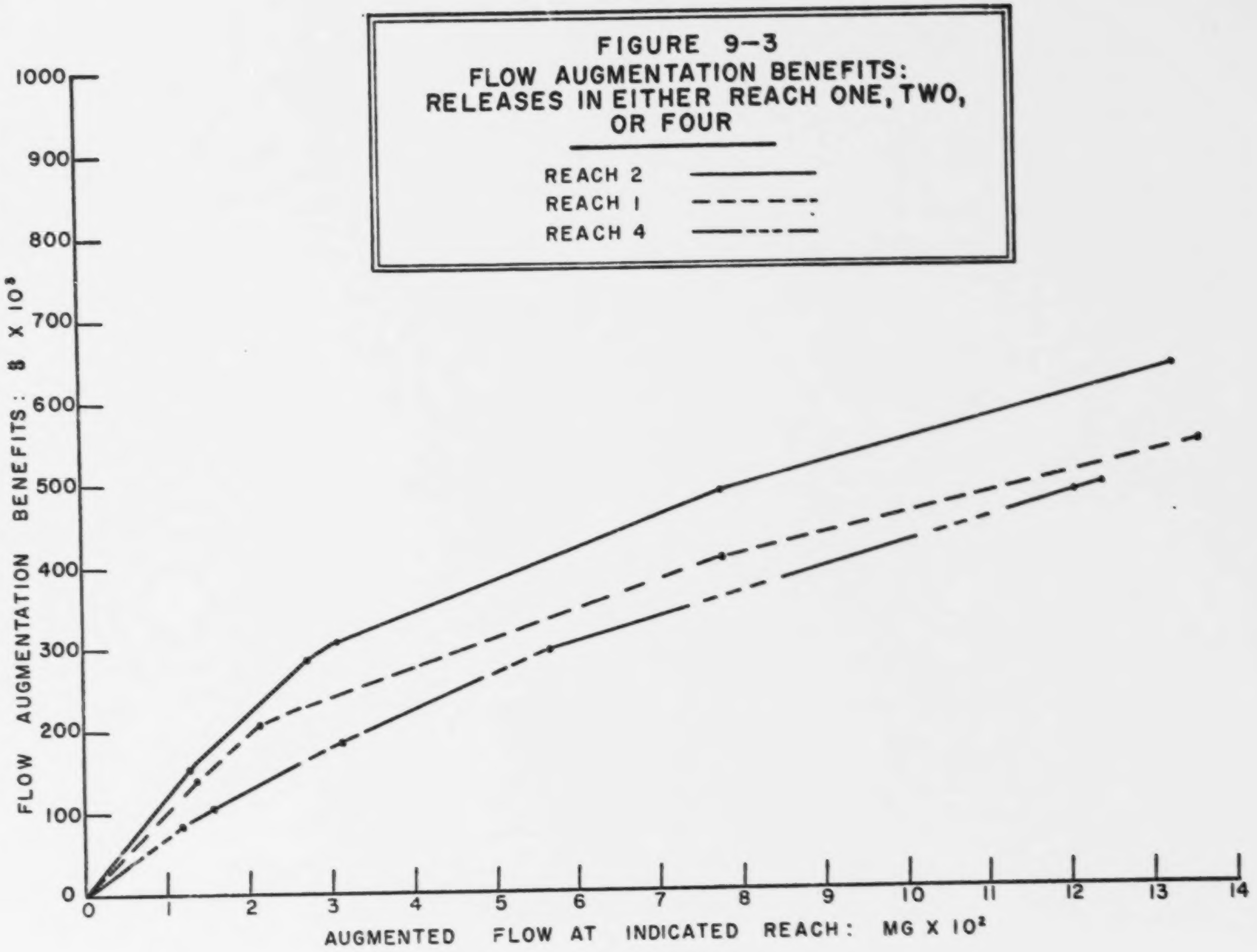


FIGURE 9-3
FLOW AUGMENTATION BENEFITS:
RELEASES IN EITHER REACH ONE, TWO, OR FOUR

overtreating to assist a downstream waste discharger. The third breakpoint (271 MG) occurs when reach four has reduced its treatment to the point at which it is just satisfying its own standard. The fourth breakpoint (311 MG) occurs when waste treatment is reduced from 50% to 35% at reach three. The fifth breakpoint (780 MG) occurs when waste treatment at reach six is reduced from 90.0% to 85.0%. Finally, for flows from 780 MG to the prespecified maximum augmentation (1325 MG) waste treatment at reach six is reduced from 85.0% to 79.2%.

The parameterization shows how flow augmentation substitutes for the most costly waste treatment which is being used at a given stage in the analysis. Discontinuities result from the original assumption of a separable piecewise linear convex cost function. If desired, it is possible to modify the original selection of these segments and obtain a more suitable approximation.

Naturally, the selection of the most effective headwater reach is also dependent on the relative cost of impounding water for flow augmentation. The analysis presented here does not deal with these cost differentials.

9.4.1.2 Selection of Most Effective Combination of Sources

Next the case where potential sources of augmented flow are available from reaches one, two, and four will be analyzed. It is desired to determine the optimal release sequence from these three sources. This water is assumed costless so that we are indifferent, from a cost standpoint, as to which source the water comes from. The selected procedure for this analysis is outlined below.

1. Obtain an optimal solution with no flow augmentation. Let $[Q^*]_0$ be the vector of the optimal activity levels and $[\pi_i^{(1)}]_0$ be the vector of shadow prices with respect to low flow augmentation. Let $[H_i]$ be the set of headwater reaches.

2. Find, $\underset{i \varepsilon H_i}{\text{Maximum}} [\pi_i^{(1)}]_0$.

3. Augment the flow in this headwater reach until a basis change occurs. For the next optimal basis, $[Q^{*}]_1$,

$$\text{find, } \underset{i\varepsilon H_i}{\text{Maximum}} \, [\pi_i{}^{(1)}]_1 .$$

4. Augment flow in this headwater reach.

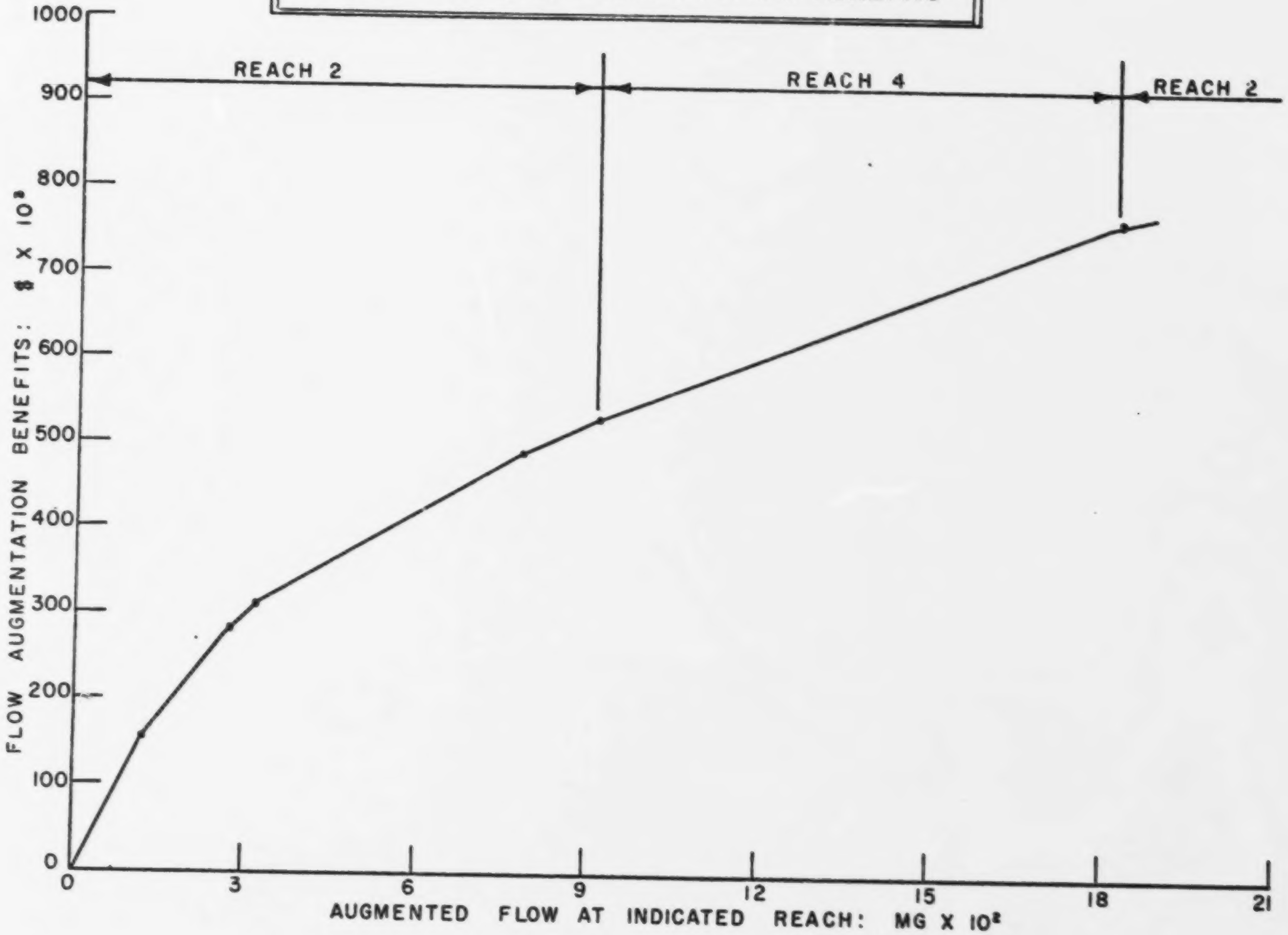
5. Continue this process up to a specified maximum augmentation or the flow at which $[\pi_i{}^{(1)}] = [\Phi]$, the null vector, which indicates that further augmentation is valueless.

Figure 9-4 shows the results of this analysis. Up to an augmented flow of 781 MG, reach two is the most effective source of augmented flow. At this point, only reaches four and seven are exactly meeting the water quality standard. Thus augmented flow from either reach one or two benefits only reach seven whereas augmentation at reach four benefits both reaches four and seven. Reach four remains the most effective source of augmented flow from 781 MG to 905 MG because low flow augmentation is reducing the required treatment at reach four in the 85-90% BOD removal range - the most costly range. Above a flow of 905 MG the optimal strategy dictates using water from reach two until a flow of 1825 MG. Then augmentation is most effective from reach four.

9.4.2 Effect of Variation in Quality of Augmented Flow

An important consideration in evaluating the benefits of flow augmentation is the quality of the augmented water. It is well established that the quality of the water in deeper reservoirs is not homogeneous. During summer months reservoirs usually stratify into a warmer upper layer (epilimnion) which overlies a cooler bottom layer (hypolimnion). The dissolved oxygen in these lower layers is often quite low. Because of the differing quality of water that would be delivered as a function of the portion of the reservoir from which it is removed, it is necessary to evaluate the effect of such changes on the optimal solution. The effect of releasing warmer water was described in Section 8. The related case where the DO of the augmented flow is zero, and where the augmented flow has a high BOD concentration, will be discussed below. The discussion will be restricted to augmentation from a single source. The multiple source case can be developed as a direct extension of the this analysis.

FIGURE 9-4
RELEASE SEQUENCE WHICH
MAXIMIZES FLOW AUGMENTATION BENEFITS



9.4.2.1 Single Source With Zero DO

The flow in reach two was increased from its original level to twice that level with the augmented flow containing zero DO. The results from this analysis are interesting. Figure 9-5 shows the regional treatment cost as a function of the augmentation at reach two. It is seen that a cost reduction (positive benefit) results for smaller levels of flow augmentation. However, a net increase in regional costs results if the augmentation exceeds about 435 MG. The minimum regional cost occurs when the augmented flow is approximately 133 MG.

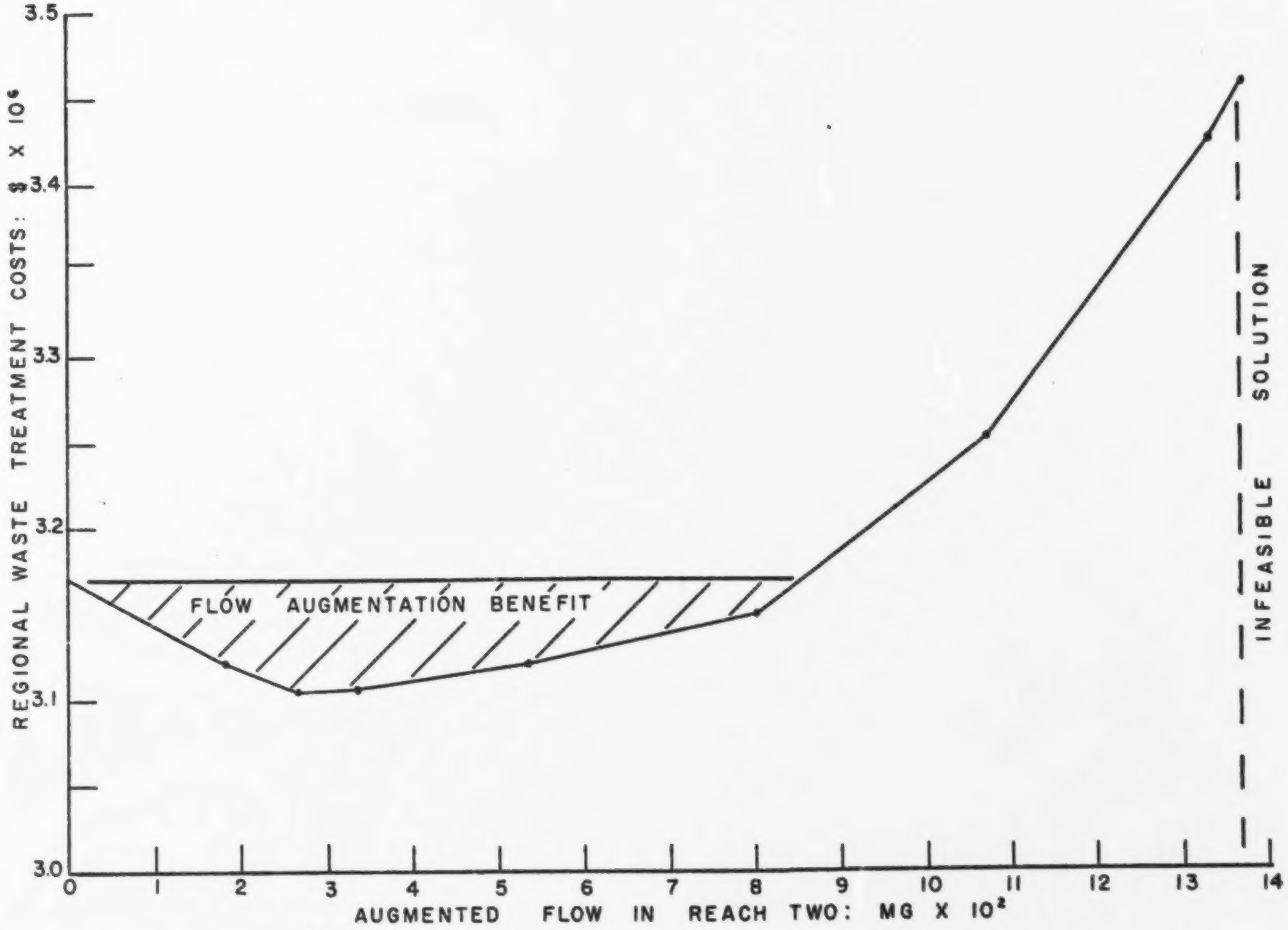
The shape of the cost function can be explained by examining the operation of the regional system under the assumed conditions. Table 9-12 shows the percentage treatment for selected levels of augmented flow. The initial impact of the augmented flow is to reduce the required treatment at reaches two and four. The reason why treatment could be reduced at reaches two and four was that they were overtreating to assist reach seven. As augmentation continues, reductions in treatment are realized at reaches three,

TABLE 9-12

TREATMENT REQUIRED WHEN AUGMENTED FLOW
ENTERING AT REACH TWO HAS ZERO DO

Augmented Flow	% BOD Removal at Each Reach					
At Reach Two:MG	1	2	3	4	6	7
0	67.0	68.7	50.0	90	90.0	50.0
133	67.0	61.8	50.0	89.9	90.0	50.0
266	67.0	68.6	35.0	89.8	88.3	50.0
400	67.0	75.4	35.0	88.6	86.3	50.0
533	67.0	82.3	35.0	88.6	84.2	50.0
666	67.0	89.0	35.0	88.6	82.1	50.0
680	67.0	90.0	35.0	88.6	81.8	50.0

FIGURE 9—5
REGIONAL WASTE TREATMENT COST
AUGMENTED FLOW WITH ZERO DO AT REACH TWO



four, and six. However, the augmented flow with zero DO utilizes the available excess oxygen in reach two and consequently more treatment is required. In fact an infeasible solution to the problem results if the augmentation exceeds 680 MG because the treatment plant at reach two is using its entire capacity.

For this hypothetical river system, augmented flow with zero DO has a positive benefit over a restricted flow range. However, it is impossible to generalize from these results regarding the benefit of this type of flow augmentation since it clearly depends on the particular regional system under study.

9.4.2.2 Single Source With High BOD

The next task is to examine the effect of augmented water with a high BOD entering reach two. In this case, the BOD concentration of the augmented flow was much higher than the BOD concentration of the base flow. The results of this analysis showed that the increased BOD loading reduced the effectiveness of the augmented flow to the system. Figure 9-6 shows the regional cost function for augmented flow at the original BOD, twice the original BOD, and five times the original BOD in the augmented flow. The added BOD load has a relatively minor effect on the regional cost function. Even raising the BOD concentration to five times the base level decreased benefits by only about 15%. Here again, the total impact depends on the specific regional configuration.

9.4.2.3 Varying Activity Coefficients Using Single Source

Section 7 discussed a technique for estimating low flow augmentation benefits which accounts for the changing coefficients in the activity matrix. This approach will be utilized here to determine the flow augmentation benefits for the limiting conditions described in Section 7. The two limiting cases for augmentation in reach two are as follows:

1. use activity coefficients for $\underline{S}_i^{(1)}$, the base flow; and
2. use activity coefficients for $\bar{S}_i^{(1)}$, the upper bound on the flow .

Figure 9-7 shows the regional benefit function for these two conditions. The regional benefits are seen to be larger if the

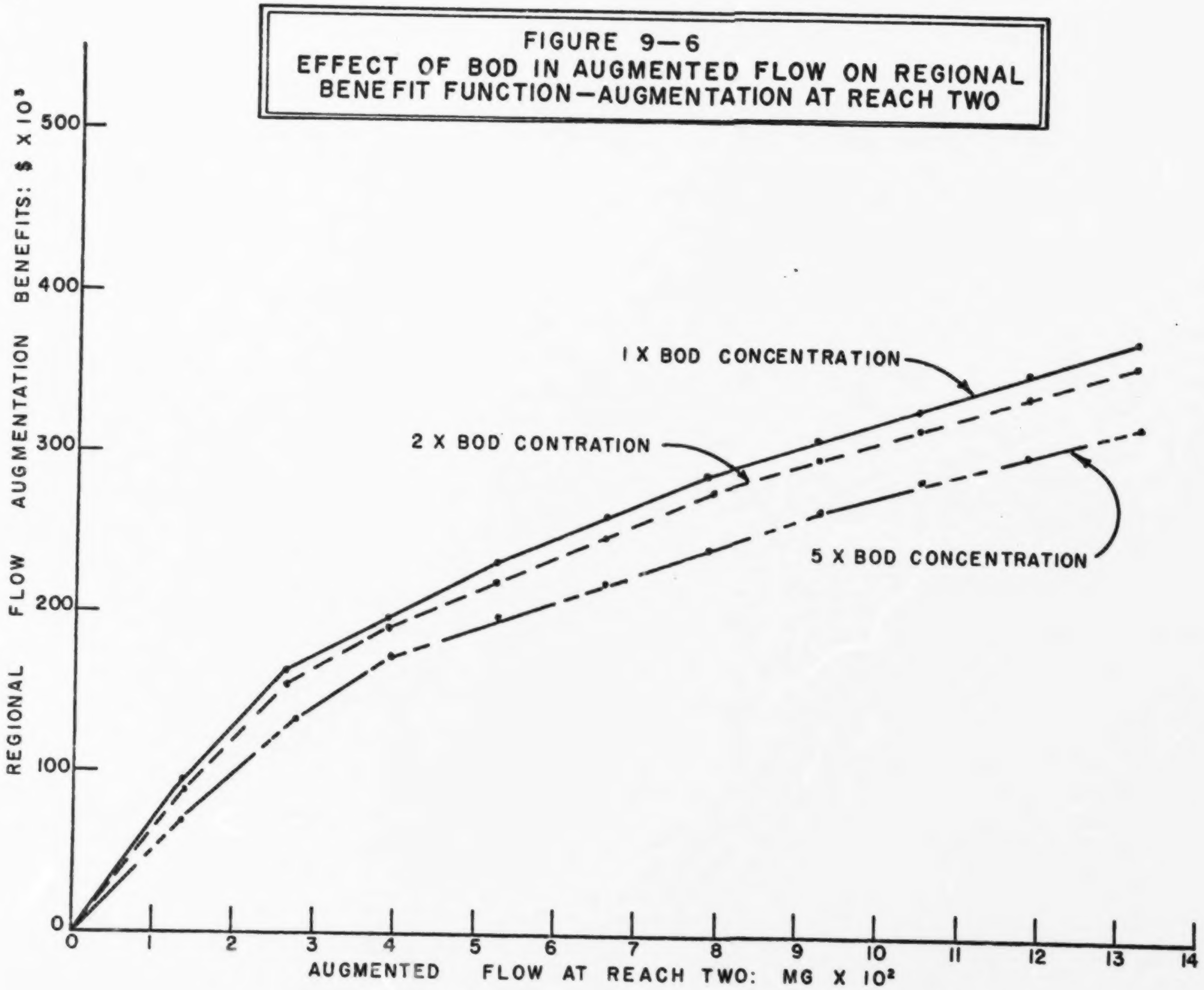


FIGURE 9—6
EFFECT OF BOD IN AUGMENTED FLOW ON REGIONAL BENEFIT FUNCTION—AUGMENTATION AT REACH TWO

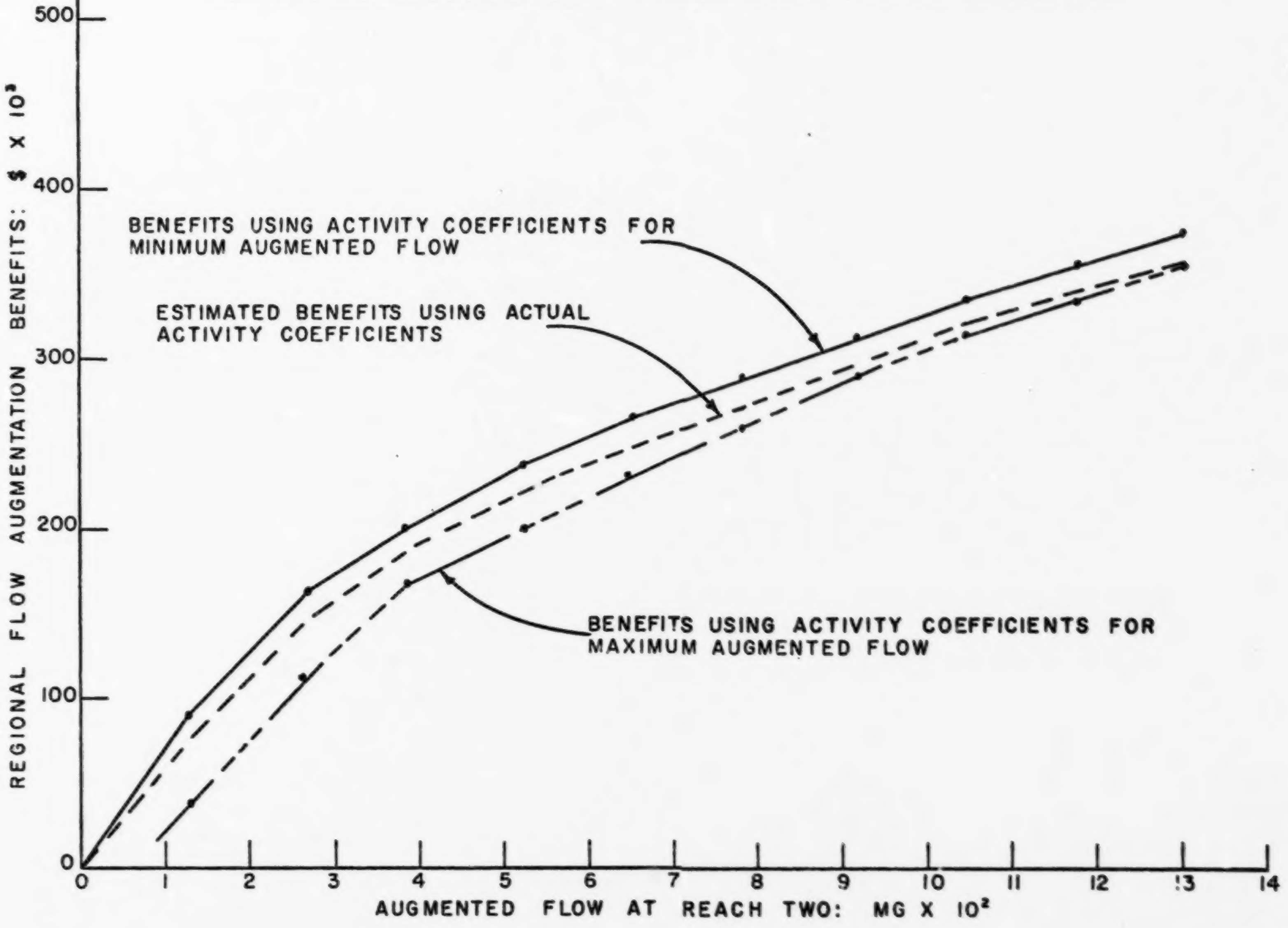
coefficients for the base flow are used. This result was expected. These two curves provide the analyst with the limiting cases of the influence of changing coefficients on the optimal solution. For this hypothetical river system the actual curve would lie between these two curves as shown in Figure 9-7. From the results of the parameterization, the limiting conditions for the flow augmentation benefit function can be derived as outlined in Section 7. Improved results can be obtained by adding more breakpoints to the original approximation of the cost function. It is necessary to revise only those cost functions which are affected by the parameterization. The ones which are in this category are known. For example, in the curve with base flow coefficients, flow augmentation in excess of 306 MG simply reduces the waste treatment at reach six from 90% on down to about 79%. Consequently, a better approximation can be obtained by increasing the number of breakpoints, in this treatment range, only for the reach six cost function.

9.5 Conclusions

Application of the optimization model to a hypothetical region has been presented. In the first part of the section, the important linkage between theory and application is provided by illustrating how the model is structured and operated. The implementation, shown in the latter part of the section, demonstrates the value of the model for quantifying the benefits of low flow augmentation, via post-optimal analysis for various assumed conditions.

The flexibility of the model is demonstrated for the deterministic case. However, its structure should permit formulation of the stochastic case using chance-constrained programming or a related method. Also, the scope of the analysis was restricted to finding the gross benefits of low flow augmentation. The net benefit

FIGURE 9-7
EFFECT OF VARIABLE ACTIVITY COEFFICIENTS ON REGIONAL BENEFIT FUNCTION AUGMENTATION AT REACH TWO



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function could be determined if the work is extended to include a comprehensive derivation of the flow augmentation cost function. This derivation should include not only direct costs, e.g., reservoir construction, but also imputed costs which reflect the value of water in alternative uses. These imputed costs may be important because of the relatively intensive demands for the water that could be expected during low flow conditions.

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SECTION 10

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The three-year study reported upon herein has been a cooperative project involving the inputs of many individuals, both from within and without the University of Florida.

Major contributions to the development of the hydrologic simulation model were made by Dr. John C. Schaake, Jr. during the period when he was an assistant professor at this institution (1966-1968). Another former faculty member, Dr. David H. Moreau (1966-1968), provided valuable insights concerning the performance of sensitivity analyses. We are appreciative, too, of the extensive programming vary capably handled by Research Associates E. J. Weiner and R. Aleman. Drafting of illustrations, key punching and general engineering calculations were the responsibility of Research Assistants R. A. Jandrucko and John Moore.

Two doctoral dissertations were closely tied to the project: A Methodology for Selecting Among Water Quality Alternatives, by Gilbert S. Nicolson (Ph.D. 1969), and A Water Quality Simulation Model, by G. R. Grantham (Ph.D. 1969), the latter of which served as a basis for Sections 6 and 8 of the present report. Two M.S.E. theses, Cost Curves of Sewage Treatment for Low Flow Augmentation, by R. David G. Pyne (1967), and An Investigation of Parameters Affecting the Cost of Impounding Water for Low Flow Augmentation, by A. I. Perez (1968) were modified by Messrs. Pyne and Perez to form Sections 5.3 and 5.4, respectively, of the present report.

We are grateful to our colleague, Dr. Kenneth C. Gibbs, Assistant Professor of Environmental Engineering and Assistant Professor of Agricultural Economics, for his careful review of major portions of the report - especially, Section 5, "Economic Analysis." And to Mrs. Kathy Weller and Miss Jackie Rimes, secretaries, we offer our thanks for typing and clerical assistance.

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The application of the methodology to a specific river basin - the Farmington River Basin in Connecticut and Massachusetts - was greatly facilitated through the interest and help of local organizations. We drew heavily upon the published report of the Travelers Research Center, Inc., Hartford, Connecticut, entitled Water Resources Planning Study of the Farmington Valley. The data contained in that report were extended and updated by making personal contact with Mr. Harold Peters, Executive Secretary of the Farmington River Watershed Association, Messrs. Gilbert U. Gustafson and Michael Long of the Metropolitan District Commission of Hartford County, Mr. Robert Taylor of the Connecticut State Water Resources Commission, Mr. Robert Tolles of the Stanley Company, Mr. George Sarandis of the New England Division, Corps of Engineers and, finally, Messrs. John Horton and Edward L. Burke of the Hartford District Office, U. S. Geological Survey.

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APPENDIX A1
DEFINITION OF TERMS
USED IN EACH SECTION

Section 4

K_1	deoxygenation velocity constant
K_2	regeneration velocity constant
K_3	rate factor, loss of BOD in reservoir

Section 5

Y	total annual waste treatment costs
a,b	coefficients
Q	design capacity of treatment plant, MGD
QMAX	maximum average monthly streamflow, MGD/square mile
QMIN	minimum average monthly streamflow, MGD/square mile
CMAX	maximum average monthly coefficient of variation of streamflows
CMIN	minimum average monthly coefficient of variation of streamflows
SMAX	maximum average monthly coefficient of serial correlation of streamflows
SMIN	minimum average monthly coefficients of serial correlation of streamflows
DMAX	maximum monthly demand rate, MGD/square
DMIN	minimum monthly demand rate, MGD/square mile
DLAG	time period elapsed between the largest flow and the largest demand, months
PROB	probability that the reservoir will not become empty in any given year
v	required storage volume per square mile of drainage area of the watershed, acre feet/square mile
V	total storage volume in acre feet
A	number of square miles in the drainage area
Z_1	represents DMAX, cubic feet per second
Z_2	represents DMIN, cubic feet per second
Z_3	represents QMIN, cubic feet per second
Z_4	represents QMAX, cubic feet per second
c	unit cost equation for region B
C	total capital cost of the reservoir

Section 6

Q	rate of flow, cubic feet per second
A	area, sq. ft.
V	velocity, feet per second
n	Manning's friction factor
R	hydraulic radius, feet
S	slope of hydraulic grade line
w	width of channel, ft.

d	mean depth of flow, ft.
a	constant, width-discharge equation
b	constant, width-discharge equation
c	constant, depth-discharge equation
f	constant, depth-discharge equation
K	constant, velocity-discharge equation
m	constant, velocity-discharge equation
μ	mean
α	standard deviation
r	random variable
x	data point
ρ	serial correlation coefficient
ε	standard normal deviate
γ	skewness
ξ	random component, gamma distributed
η	standard normal random deviate
e	natural logarithm base
t	dummy variable
α	constant, transformation equations
n	constant exponent
n	sample size
T	temperature
R	random number (0,1)
L	week index
t	time
K_2	reoxygenation velocity constant
c	dissolved oxygen concentration
K_1	deoxygenation velocity constant
$\bar{u}$	mean velocity
h	depth
g	gravitational constant
L	BOD concentration
γ	deoxygenation error term
D	dissolved oxygen concentration
K_1	deoxygenation velocity constant
s	reoxygenation error term
L_o	initial BOD concentration
D_o	initial DO concentration
X	data value
β	regression constant
x	data value, residual
r	correlation coefficient
S	covariance matrix
λ	root of correlation matrix

B	regression constant, generation eqn.
C	variance coefficient, generation eqn.
A	deterministic component, generation eqn.
τ	week of the year
n	number of basis gages
QO	flow at point 0
DAO	drainage area at point 0
α	weight coefficient
QS	flow at gage point
DAS	drainage area at gage point
s	transformed variable, laplace transforms
p	reoxygenation error term
L_a	initial BOD concentration
D_a	initial DO concentration
t_c	critical time
BOD_{in}	BOD at upstream point in reach
BODOUT	BOD at downstream point in reach
BOD_w	BOD of waste
Q_W	flow rate of waste
QIN	flow into reservoir
QREG	regulated flow
QOUT	flow out of reservoir
RREL	reservoir release rate
DIV	diversion rate
y	BOD concentration in reservoir
V	volume of water in reservoir
Yin	BOD conc. incoming to reservoir
K_3	rate factor, loss of BOD in reservoir
QVAP	evaporation loss in reservoir
Y_o	BOD concentration in reservoir, beginning of time interval
C_o	DO concentration in reservoir, beginning of time interval
W	equals QOUT/V
Z	equals QIN/V
A	equals $K_1 + W + K_3$
B	equals $K_2 + W$
$\bar{C}$	average DO conc. in reservoir
$\bar{y}$	average BOD conc. in reservoir
Cin	DO conc. incoming to reservoir
S_K^F	sensitivity function
α_{ij}	overall transfer function,
A_{ij}	constant, transfer function regression equation, BOD
r_{ij}	flow transfer factor
B_{ij}	regression coefficient, transfer function reg. eqn. BOD
C_{ij}	regression coefficient, transfer function reg. eqn. DO
D_{ij}	constant, transfer function regression equation, DO
d_{ij}	DO transfer coefficient
a_{ij}	BOD transfer coefficient

q	transformed flow value
ν_i	ith statistical moment about zero
μ_i	ith statistical moment about mean
α_3	skewness parameter
β_2	kurtosis parameter
Θ	angular constant
A_k	Fourier constant
B_k	Fourier constant
N	number of years of data

Section 7

A_i	set of all contributing reaches above reach i
F_i	set of all contributing reaches immediately above reach i
G_i	set of all recipient reaches downstream from reach i
V_i	set of all treatment plants in region (q elements)
$S_i(kT)$	flow of commodity k into treatment plant in reach i
$S_i(k)$	new supply of commodity k entering reach i
$Q_i(kT)$	flow of commodity k from treatment plant to reach i
$Q_i^{(k)}$	flow of commodity k at beginning of reach i
$Q_{i'}^{(k)}$	flow of commodity k at end of reach i
$\varepsilon_i^{(k)}$	attenuation or amplication of commodity k in transit through reach i
$\rho_i^{(k',k)}$	change in flow of commodity k' through reach i per unit of flow of commodity k through reach i
$C_{ib}^{(kT)}$	cost of transporting a unit of commodity k along treatment branch b in ith reach
$\underline{Q}_{ib}^{(kT)}$	flow of commodity k along treatment branch b in ith reach
$\overline{Q}_{ib}^{(kT)}$	upper nound on $Q_{ib}^{(kT)}$
$\underline{Q}_{ib}^{(kT)}$	lower bound on $Q_{ib}^{(kT)}$
$\underline{Y}_i$	minimum allowable flow of commodity k in reach i
Z	value of the objective function
n	number of treatment branches
p	number of reaches
q	number of treatment plants
K_1	rate constant for deoxygenation, day^{-1}
K_2	reaeration rate constant, day^{-1}
K_3	rate constant for sedimentation and adsorption, day^{-1}
B	BOD concentration, #/MG
B_o	BOD concentration at time 0, #/MG
B_t	BOD concentration time t, #/MG
R	rate of BOD addition due to runoff and scour, #/MG/day
D	DO deficit, #/MG
D_o	DO deficit at time 0, #/MG
D_t	DO deficit at time t, #/MG
M	oxygen production ($M>0$) or reduction ($M<0$) due to plants

$\bar{Q}_i^{(3)}$	saturation DO quantity at the beginning of the reach, lbs
$\bar{Q}_{i'}^{(3)}$	saturation DO quantity at the end of the reach, lbs
$\bar{k}_t^{(3,1)}$	saturation DO concentration, #/MG
$[A^{(1)}]$	(p+q)x(p+q) matrix of water continuity coefficients
$[A^{(2)}]$	(p+q)x(p+q) matrix of BOD continuity coefficients
$[A^{(3)}]$	(p+q)x(p+q) matrix of DO continuity coefficients
$[P^{(1,2)}]$	(p+q)x(p+q) matrix of water-BOD interdependency coefficients
$[P^{(2,3)}]$	(p+q)x(p+q) matrix of BOD-DO interdependency coefficients
$[P^{(1,3)}]$	(p+q)x(p+q) matrix of water-DO interdependency coefficients
$[D^{(1,3)}]$	(p)x(p+q) matrix of water-DO quality coefficients
$[D^{(2,3)}]$	(p)x(p+q) matrix of BOD-DO quality coefficients
$[D^{(3,3)}]$	(p)x(p+q) matrix of DO quality coefficients
$[T^{(2)}]$	(p+q)x(nxq) matrix of BOD treatment alternatives
$[I]$	(nxq)x(nxq) identity matrix of upper bounds on BOD removal
$[\pi^{(1)}]$	vector of dual variables associated with $[S^{(1)}]$
$[\pi^{(2)}]$	vector of dual variables associated with $[S^{(2)}]$
$[\pi^{(3)}]$	vector of dual variables associated with $[S^{(3)}]$
$[\pi^{(4)}]$	vector of dual variables associated with $[\underline{Y}^{(3)}]$
$[\pi^{(5)}]$	vector of dual variables associated with $[\bar{Q}^{(2T)}]$
Θ_{ij}	rate of substitution of upstream BOD removal at reach j relative to BOD removal at reach i
$\pi_j^{(2)}$	BOD removal shadow price for upstream reach j
$\pi_i^{(2)}$	marginal cost of BOD removal at reach i
$\pi_j^{(4)}$	water quality shadow price for upstream reach j
$\pi_i^{(1)}$	shadow price with respect to flow at reach i

Section 8

E	evaporation rate, inches per week
L	week of year
D	diversion rate, million cubic feet per week
K_1	deoxygenation velocity constant
K_2	reaeration velocity constant
K_3	rate factor, loss of BOD in reservoir
c	constant
f	constant
h	constant
m	constant
T	temperature
w	width of channel, ft.
d	mean depth of flow, ft.
v	velocity, ft. per second
Q	rate of flow, cubic feet per second
q	transformed flow values
μ_i	ith statistical moment about mean
$S_k^{F(k)}$	sensitivity function

Section 9

$\underline{S}_i^{(1)}$	base (unagumented) flow, MGD
$\bar{S}_i(1)$	maximum flow, MGD
T	flow time through reach r, days
QW	wastewater inflow, MGD
QT	tributary inflow, MGD
CS	saturation DO concentration, mg/l
$\bar{D}$	maximum allowable DO deficit, mg/l
$\underline{D}$	minimum allowable DO concentration, mg/l
CW	wastewater DO concentration, mg/l
CT	tributary DO concentration, mg/l
BW	wastewater BOD concentration to treatment facility, mg/l
BT	tributary BOD concentration, mg/l
K_1	deoxygenation rate constant, day^{-1}
K_2	reaeration rate constant, day^{-1}
K_3	sedimentation and adsorption rate constant, day^{-1}
M	oxygen production ($M>0$) or reduction ($M<0$) due to plants and benthal deposits, mg/l/day
P	% BOD removal 1980 load with existing wastewater treatment facility
R	BOD addition rate due to runoff and scour, mg/l/day
Q	discharge
w	water surface width
d	mean depth
v	mean velocity
a,c,f,k,l,m,	numerical constants
A	cross-sectional area of the stream
L	length of the reach
$[C^{(2T)}]$	unit cost matrix for transporting a unit of commodity 2 (BOD) along treatment branch b in the plant in reach i
$[Q^{(2T)}]$	matrix for the amount of BOD removed at the corresponding branches
$[C_{ib}^{(2T)}]$	cost vector for branch b in the treatment facility in the ith reach
$[Q_{ib}^{(2T)}]$	BOD commodity vector for the treatment facility in the ith reach
$[Q^{(1)}]$	flow of commodity 1
$[Q^{(2)}]$	flow of commodity 2
$[Q^{(3)}]$	flow of commodity 3
$[Q^{(2T)}]$	amount of BOD removed at the corresponding branches in the plant
$[S^{(1)}]$	additional amount of commodity 1 entering system
$[S^{(2)}]$	additional amount of commodity 2 entering system
$[S^{(3)}]$	additional amount of commodity 3 entering system
$[\underline{Y}^{(3)}]$	minimum allowable flow of commodity 3

$[\bar{Q}^{(2T)}]$	upper bound on $[Q^{(2T)}]$
$A^{(k)}$	matrix of continuity coefficients for the three commodities
$p^{k',k}$	matrix of interdependency coefficients
$D^{k',k}$	matrix of coefficients for water quality DO standard
$T^{(2)}$	matrix of BOD treatment alternatives
β	sum vector with n components
n	number of linear segments representing percent treatment at a facility
I	identity matrix of upper bounds on BOD removal
$\pi_i^{(2)}$	marginal cost of BOD removal at reach i
$\pi_i^{(4)}$	water quality shadow price at reach i
A_i	set of all contributing reaches above reach i
Θ_{ij}	rate of substitution of upstream BOD removal at reach j relative to BOD removal at reach i
$\pi_i^{(1)}$	shadow price with respect to flow at reach i
H_i	set of headwater reaches

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APPENDIX A-2
CURVE FITTING TECHNIQUES

The mathematical formulation of relationships often requires that a mathematical equation be fitted to a set of data. In general, data are in periodic form or in a form that can be fitted by a polynomial or an exponential formula. Fitting curves to data points is a time-consuming process and one that is readily programmed to the computer. This section contains coding and documentation for two programs, FITCRV and CRVFIT which can be used to place data points in mathematical form.

The program FITCRV is designed to fit data having a primary period of one year. The basic assumption is that the data are represented by the form:

$$Y = f(x+2\pi) = f(x) \quad . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. A2.1]}$$

where x represents an angle in radians. This can be represented by a trigonometric series:

$$Y = Y_m + C_i \cos(x-\Phi_i) + C_2 \cos(2x-\Phi_2) + \ldots$$

$$C_n \cos(nx-\Phi_n) + \ldots, \quad . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. A2.2]}$$

where Y_m is the mean Y for the period, C_1 are the amplitudes of the i^th^ harmonic and Φ_1 are the phases of the i^th^ harmonic (1).

The computation is carried out using the forms:

$$Y_m = \frac{1}{n} \sum_{i=0}^{n-1} (Y_i) \quad . \; . \; . \; . \; . \; . \quad \text{[Eq. A2.3]}$$

$$a_k = \frac{2}{n} \sum_{i=0}^{n-1} Y_i \cos k\, x_i \quad . \; . \; . \; . \; . \quad \text{[Eq. A2.4]}$$

$$k = 1, \ldots N$$

$$b_k = \frac{2}{n} \sum_{i=0}^{n-1} Y_i \sin x_i \quad . . . . . [\text{Eq. A2.5}]$$

$k = 1, \ldots N$

$$c_k = (a_k{}^2 + b_k{}^2)^{1/2} \quad . . . . . . [\text{Eq. A2.6}]$$

$k = 1, \ldots N$

$$\Phi_k = \tan^{-1} \frac{b_k}{a_k} \quad . . . . [\text{Eq. A2.7}]$$

$k = 1, \ldots N$

N is the number of harmonics desired.

In a set of data, the values (Y_i, X_i) are known and may be substituted into Equations [A2.4], through [A2.7] to obtain C_k and Φ_k. The program is set up for maximum N = 6. The program furnishes values of C_k and Φ_k for all values of $k < N$ for all harmonics 1,2...,N. This latter feature allows the use of fewer than N harmonics, without recomputing, if it is found that the C_k and Φ_k for the higher harmonics have little effect on the result.

The program CRVFIT utilizes the "least squares" criterion for fitting a polynomial of degree 1 to 7, $f(x) = Ax^B$ or $f(x) = Ae^{Bx}$, depending upon the control variable selected. The routine to fit the polynomial starts with degree 1, sets up and solves the normal equations, regenerates the data using the polynomial and compares the original and renerated data. If the comparison is not within the limits specified, the degree is augmented by one and the procedure is repeated. The maximum degree that can be used in this program is 7. The normal equations are solved using the method of Gaussian elimination, wherein the matrix of coefficients is triangularized and the variables solved for by starting at the bottom row and moving upward.

For the exponential forms, two normal equations are formed and solved for the "best fit" values of A and B.

The program coding for both programs follows:

---

(1) Mackey, Charles., Graphical Solutions, John Wiley and Sons, New York, 2nd ed., 1944, p. 142.

```
//FITCRV   JOB (1143,47,C09,05,1000),'WEINER                    ',MSGLE
                                                                 VEL=1
//         EXEC F4GDX    FORTRAN G, COMPILE, PUNCH OBJ. DECK, EXE
                                                                 CUTE
//FORT.SYSIN DD *
      DIMENSION Y(520),THETA(52),THETAC(52),A(6),B(6),C(6),TAU(6)
                                                                 ,
     1          YEST(52),YCALC(6)
     2          ,ALPHA(20)
   16 READ(5,18,END=17)ALPHA
   18 FORMAT(20A4)
      READ(5,1)KMAX,NIN,NPERO,NPERI,(Y(I),I=1,NIN)
    1 FORMAT(I3,3I4,/(10F8.2))
      WRITE(6,19)ALPHA
   19 FORMAT(1H1,20A4///)
      WRITE(6,3)KMAX,NIN,NPERI,NPERO,(Y(I),I=1,NIN)
    3 FORMAT(1H ,'NUMBER OF HARMONICS=',I3 ,/
     1' NUMBER OF RECORDS TO BE READ IN =',I4,/
     2' NUMBER OF INPUT RECORDS PER PERIOD=',I4,/
     3' NUMBER OF OUTPUT RECORDS PER PERIOD='I4,///
     51H0,'ORIGINAL INPUT DATA',/(1X,10F12.2))
C     CONVERT WEEKS TO RADIANS
      XM=(2*3.1416)/NPERI
      XMM=(2*3.1416)/NPERO
C     CALCULATE THETAS-CALCULATE YMEAN
      YSUM=0
      DO 2 I=1,NIN
    2 YSUM=YSUM+Y(I)
      DO 13 I=1,NPERI
   13 THETA(I)=XM*I
      YMEAN=YSUM/NIN
      WRITE(6,20)YMEAN
   20 FORMAT(1H ,'MEAN OF INPUT DATA = ',F14.4)
C     CALCULATE A(K),B(K),C(K),TAU(K)
C     FOR EACH OF KMAX HARMONICS
      DO 6 K=1,KMAX
      A(K)=0
      B(K)=0
      C(K)=0
      J=0
      DO 7 I=1,NIN
      J=J+1
      ARG=K*THETA(J)
      A(K)=A(K)+Y(I)*COS(ARG)
      B(K)=B(K)+Y(I)*SIN(ARG)
    7 IF(J.EQ.NPERI)J=0
      A(K)=A(K)*(2./NIN)
      B(K)=B(K)*(2./NIN)
      CARG=A(K)**2+B(K)**2
      C(K)=SQRT(CARG)
      BB=B(K)
      AA=A(K)
      TAU(K)=ATAN2(BB,AA)
    6 WRITE(6,9)K,A(K),B(K),C(K),TAU(K)
    9 FORMAT(1H0,'HARMONIC',I3,/' A(K)=',F14.4,/
     1        ' B(K) = ',F14.4,/' C(K) = ',F14.4,
     2        /' TAU(K) = ',F14.4)
      WRITE(6,19)ALPHA
      WRITE(6,10)
   10 FORMAT(1H ,'FOURIER APPROXIMATIONS TO ORIGINAL DATA FOR EAC
                                                               H OF K
     1HARMONICS',/1X,'  I',11X,'K=1',11X,'K=2',11X,'K=3',11X,'K=4
                                                               ',
     211X,'K=5',11X,'K=6')
C     CALCULATE YEST
      DO 12 I=1,NPERO
      THETAC(I)=XMM*I
   12 YEST(I)=YMEAN .
      DO 15 I=1,NPERO
      DO 8 K=1,KMAX
      ARG=K*THETAC(I)-TAU(K)
      YEST(I)=YEST(I)+C(K)*COS(ARG)
    8 YHOLD(K)=YEST(I)
      WRITE(6,11)I,(YHOLD(K),K=1,KMAX)
   11 FORMAT(1H ,I3,6F14.4)
   15 CONTINUE
      GO TO 16
   17 STOP
      END
```

```
//CRVFIT   JOB (1143,47,005,06,0000),'ALEMAN              ',CLASS
                                                          =S
/*PASSWORD     1,LOFLOJOB
// EXEC F4GCXS     FORT G COMPILE (NODECK), EXECUTE, CLASS S
//FORT.SYSIN DD *
      NSP=1
      CALL CRVFIT(NSP)
      CALL EXIT
      END
      SUBROUTINE SIMUL(N,A,B,C)
C     THIS SUBROUTINE SOLVES A SET OF SIMULTANEOUS EQUATIONS(LINE
                                                          AR)
C     USING GAUSS'S METHOD.
C     N=NO. OF EQUATIONS.
C     A=ARRAY OF COEFFICIENTS OF THE EQUATIONS.
C     B=VECTOR OF CONSTANT TERMS FOR THESE EQUATIONS.
C     C=VECTOR OF SOLUTIONS FOR THESE EQUATIONS.
C     ***********************************************************
                                                          *******
      DIMENSION A(20,20),B(20),C(20)
      NM1=N-1
      DO 60 K=1,NM1
      KP1=K+1
      L=K
C     NOW ALL THE ROWS ARE ARRANGED SO THAT  A(K,K) IS NOT ZERO.
C     AND ALL DIAGONAL TERMS ARE LARGEST IN ABSOLUTE VALUE.
      DO 20 I=KP1,N
      IF(ABS(A(I,K))-ABS(A(L,K)))20,20,21
   21 L=I
   20 CONTINUE
      IF(L-K)50,50,25
   25 DO 30 J=K,N
      TT=A(K,J)
      A(K,J)=A(L,J)
   30 A(L,J)=TT
      TTT=B(K)
      B(K)=B(L)
      B(L)=TTT
C     ***********************************************************
                                                          *******
C     PROCEED WITH GAUSS'S ELIMINATION.
   50 DO 60 I=KP1,N
      FX=A(I,K)/A(K,K)
      A(I,K)=0.0
      DO 56 J=KP1,N
   56 A(I,J)=A(I,J)-FX*A(K,J)
   60 B(I)=B(I)-FX*B(K)
C     MATRIX  A  IS NOW A TRIANGULAR MATRIX.
C     BACK SOLUTION METHOD PROCEEDS.
C     ***********************************************************
                                                          *******
      C(N)=B(N)/A(N,N)
      I=NM1
   61 IP1=I+1
      S=0.0
      DO 70 J=IP1,N
   70 S=S+A(I,J)*C(J)
      C(I)=(B(I)-S)/A(I,I)
      I=I-1
      IF(I)80,80,61
   80 RETURN
      END
      FUNCTION F(N,X,C)
      DIMENSIONC(20)
      TERM=0.0
      DO 1 I=1,N
      J=N-I+1
      K=J-1
      TERM=TERM+(C(J)*X**K)
      IF(J.EQ.1) GO TO 2
    1 CONTINUE
    2 F=TERM
      RETURN
      END
      SUBROUTINE CRVFIT(NSP)
C     CURVE-FITTING ROUTINE USING THE METHOD OF LEAST SQUARES.
C     NSP=1 CORRESPONDS TO A POLYNOMIAL FIT.
C     NSP=2 CORRESPONDS TO AN EXPONENTIAL FIT.
C     NSP=3 CORRESPONDS TO A NATURAL EXPONENTIAL FIT.
      DIMENSION X(200),Y(200),X1(200),Y1(200),A(11,11),B(11),C(11
                                                                )

      DIMENSION P(20),X11(20),X22(20),CC(11),XX(20)
      DATA X11/20*1H+/,X22/20*1H-/
C     N$= NUMBER OF DATA POINTS.
      READ(5,1) N$
    1 FORMAT(I5)
      READ(5,2)(X(I),Y(I),I=1,N$)
    2 FORMAT(10F8.0)
      GO TO (100,200,300),NSP
  100 DO 199 M=1,7
      MX2=M*2
C     M=DEGREE OF POLYNOMIAL.
C     FOR SINGLE APROXIMATION MAKE STATEMENT 100 OF THE TYPE M=NU
                                                             MBER.

C     POWERS OF X ARE NOW FORMED.
      DO 13 I=1,MX2
      P(I)=0.0
      DO 13 J=1,N$
   13 P(I)=P(I)+X(J)**I
C     N= NO. OF EQUATIONS.
C     A=ARRAY OF COEFFICIENTS.
C     B=VECTOR OF CONSTANT TERMS.
```

```
      N=M+1
      DO 30 I=1,N
      DO 30 J=1,M
      K=I+J-2
      IF(K) 29,29,28
   28 A(I,J)=P(K)
      GO TO 30
   29 A(1,1)=N$
   30 CONTINUE
      B(1)=0.0
      DO 31 J=1,N$
   31 B(1)=B(1)+Y(J)
      DO 32 I=2,N
      B(I)=0.0
      DO 32 J=1,N$
   32 B(I)=B(I)+Y(J)*X(J)**(I-1)
C     TO SOLVE THE SIMULTANEOUS EQUATIONS WE CALL ON SIMUL.
      CALL SIMUL(N,A,B,C)
C     WE PRINT RESULTS IN DESIRED FORM.
   80 DO 85 I=1,N
      IF(C(I).LT.0.0) GO TO 82
   81 XX(I)=X11(I)
      GO TO 83
   82 XX(I)=X22(I)
   83 CC(I)=ABS(C(I))
   85 CONTINUE
      WRITE(6,86)(XX(I),CC(I),I=1,N)
   86 FORMAT('1',/////T37,'1',T52,'2',T67,'3',T82,'4',T97,'5',T11
                                                           2,'6',
     1T127,'7',/T6,'F(X)=',A2,F10.6,A2,F11.6,'X',6(A2,F12.6,'X'))
C     LETS CHECK FOR 90  ACCURACY.
      DO 90 I=1,N$
      X1(I)=X(I)
      X$=X(I)
      Y1(I)=F(N,X$,C)
   90 CONTINUE
      NN=1
      DO 98 I=1,N$
      IF(ABS(Y(I)-Y1(I)).LE.1.E-04) GO TO 95
      GO TO 98
   95 NN=NN+1
      IF(NN.LE.(0.1*N$)) GO TO 98
      WRITE(6,96)
   96 FORMAT(1H0,T10,'THIS POLYNOMIAL FIT DOES NOT CONVERGE TO TH
                                                          E DESIR
     1ED ACCURACY')
      NE=0
      GO TO 99
   98 CONTINUE
      WRITE(6,109)
  109 FORMAT('0',T10,'THIS POLYNOMIAL FIT SATISFIES ERROR CRITERI
                                                                A',/T10
     1,'IT IS CORRECT TO FOUR DECIMAL PLACES FOR 90  OF DATA.')
            NE=1
      GO TO 99
   99 CALL XYPLOT(X,Y,N$)
      CALL XYPLOT(X1,Y1,N$)
      IF(NE.EQ.1) RETURN
  199 CONTINUE
      DO 170 J=1,3
      WRITE(6,189)
  189 FORMAT('**************POLYNOMIAL FIT FAILED TO MEET ERROR CR
                                                                ITERIA*

     1**********')
      WRITE(6,189)
      WRITE(6,189)
  170 WRITE(6,189)
      RETURN
  200 T=0.0
      S=0.0
      ST=0.0
      SS=0.0
      TT=0.0
      DO 202 I=1,N$
C     THE SUMS NECCESSARY FOR EXPONENTIAL FIT ARE NOW FORMED'
      T=T+LOG(X(I))
      TT=TT+LOG(Y(I))
      SS=SS+(LOG(X(I)))**2
  202 ST=ST+(LOG(X(I))*LOG(Y(I)))
      NE=0
      A(1,1)=N$
      A(1,2)=T
      B(1)=TT
      A(2,1)=T
      A(2,2)=SS
  208 B(2)=ST
      CALL SIMUL(2,A,B,C)
      BEXP=C(2)
      A1=EXP(C(1))
      IF(NE.EQ.1) GO TO 350
      WRITE(6,250)BEXP,A1
  250 FORMAT('1',/////T22,F6.4,/T6,'F(X)=',F10.4,'X')
      DO 260 I=1,N$
      X1(I)=X(I)
  260 Y1(I)=A1*(X(I)**BEXP)
  280 CALL XYPLOT(X,Y,N$)
      CALL XYPLOT(X1,Y1,N$)
      RETURN
  300 S=0.0
      T=0.0
      ST=0.0
      SXQ=0.0
      DO 310 I=1,N$
      T=T+X(I)
      S=S+LOG(Y(I))
      ST=ST+(X(I)*LOG(Y(I)))
  310 SXQ=SXQ+X(I)*X(I)
      A(1,1)=N$
      A(1,2)=T
      A(2,1)=T
      A(2,2)=SXQ
      B(1)=S
      NE=1
      GO TO 208
  350 WRITE(6,360)BEXP,A1
  360 FORMAT('1',/////T22,F6.4,'X',/T6,'F(X)=',F10.4,'E')
      DO 370 I=1,N$
      X1(I)=X(I)
      Y1(I)=A1*(EXP(X(I)*BEXP))
  370 CONTINUE
      GO TO 280
11111 STOP
      END
      SUBROUTINE XYPLOT(X,Y,NN)
      REAL LOWX, LOWY, NINE, MINUS
      DATA ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,NINE/
     1     '0','1','2','3','4','5','6','7','8','9'  /
      DATA BLANK,VDASH,MINUS,CENTER,VAXIS,HAXIS/
     1     ' ',' ','-','0','I',' ' /
      DATA PLUS, ASTRIX/'+','*'/
      DIMENSION CHAR(81,81)
      DIMENSION X(NN), Y(NN)
      HIGHX = X(1)
      HIGHY = Y(1)
      LOWX = X(1)
      LOWY = Y(1)
      DO 1 I = 1,NN
      IF(X(I) .GT. HIGHX) HIGHX = X(I)
      IF(Y(I) .GT. HIGHY) HIGHY = Y(I)
      IF(X(I) .LT. LOWX) LOWX = X(I)
    1 IF(Y(I) .LT. LOWY) LOWY = Y(I)
      IF(ABS(HIGHX)-ABS(LOWX)) 2,3,3
    2 DIVX = ABS(LOWX)
      GO TO 4
    3 DIVX = ABS(HIGHX)
    4 CONTINUE
      IF(ABS(HIGHY) - ABS(LOWY)) 5;6;6
    5 DIVY = ABS(LOWY)
      GO TO 8
    6 DIVY = ABS(HIGHY)
    8 CONTINUE
      SIZEX = DIVX/5
      SIZEY = DIVY/5
    DO 100 IX = 1,81
    DO 100 IY = 1,51
100 CHAR(IX,IY) = BLANK
    CHAR(IX,1) = MINUS
    CHAR(IX,26) = HAXIS
101 CHAR(IX,51) = MINUS
    DO 102 IX = 1,81,8
    CHAR(IX,1) = PLUS
    CHAR(IX,26) = PLUS
102 CHAR(IX,51) = PLUS
    DO 103 IY = 1,51
    CHAR(1,IY) = VDASH
    CHAR(41,IY) = VAXIS
103 CHAR(81,IY) = VDASH
    DO 104 IY = 1,51,5
    CHAR(1,IY) = PLUS
    CHAR(41,IY) = PLUS
104 CHAR(81,IY) = PLUS
    CHAR(41,26) = CENTER
    DO 200 JJ = 1,NN
    I = (X(JJ)/DIVX)*40.0 + 41.5
    J = (-Y(JJ)/DIVY)*25.0 + 26.5
    IF(CHAR(I,J) .EQ.PLUS) GO TO 200
    CHAR(I,J) = ASTRIX
200 CONTINUE
    N = 0
    DO 201 JJ = 1,NN
    I = (X(JJ)/DIVX)*40.0 + 41.5
    J = (-Y(JJ)/DIVY)*25.0 + 26.5
    IF(CHAR(I,J) .NE. ASTRIX) GO TO 201
    IF(N .EQ.0) CHAR(I,J) = ZERO
    IF(N .EQ.1) CHAR(I,J) = ONE
    IF(N .EQ.2) CHAR(I,J) = TWO
    IF(N .EQ.3) CHAR(I,J) = THREE
    IF(N .EQ.4) CHAR(I,J) = FOUR
    IF(N .EQ.5) CHAR(I,J) = FIVE
    IF(N .EQ.6) CHAR(I,J) = SIX
    IF(N .EQ.7) CHAR(I,J) = SEVEN
    IF(N .EQ.8) CHAR(I,J) = EIGHT
    IF(N .EQ.9) CHAR(I,J) = NINE
    N = N+1
     IF(N .EQ. 10) GO TO 202
201 CONTINUE
202 CONTINUE
    WRITE(6,300)
300 FORMAT('1')
    WRITE(6,301) HIGHX,HIGHY,LOWX,LOWY
301 FORMAT(' MAXIMUM X VALUE =' G15.7,T40,' MAXIMUM Y VALUE ='
   1 G15.7,/' MINIMUM X VALUE = ' G15.7,T40, ' MINIMUM Y VALUE
                                                          ='

    2G15.7,/ )
```

```
      WRITE(6,303) SIZEX,SIZEY
  303 FORMAT(  '  X SCALE'/G15.7,' PER MAJOR HORIZONTAL DIVISION'
     1/'  Y SCALE'/G15.7,' PER MAJOR VERTICAL DIVISION '/ )
      WRITE(6,302)((CHAR(I,J), I= 1,81), J=1,51)
  302 FORMAT(1X,81A1)
      RETURN
      END
```

APPENDIX A3
FARMINGTON RIVER BASIN DATA

This Appendix contains the basic data for the Farmington River Basin, Connecticut and Massachusetts, which were gathered for application of the water quality simulation model to that basin. The data are compiled in their basic form. The preparation of these data for use in the simulation model is described in Section 8.2 of this report.

A3.1 Maps and Geographical Data

The maps used to obtain basic watershed data were U. S. Geological Survey quadrangle maps covering 7 1/2 minutes of longitude and latitude at a scale of 1:24000 (1 inch = 2000 feet). These maps were available for the entire Farmington River watershed and were purchased through the Map Information Office, Geological Survey, Washington, D. C. 20242. The cost is $ 0.50 per sheet.

The quadrangle maps are too large to include in the report. Twenty-one quadrangles are required to cover the watershed and each map is 27" x 22". The coverage of the watershed by the quadrangles is indicated on Figure A3-1.

The quadrangle maps were used to determine the watershed boundaries and the ridge lines within the watershed. This made it possible to measure the area of the watershed and its component parts, as needed to generate synthetic flow data. Gages were located on the maps and the areas tributary to the gage points were measured.

The various reaches of the watershed were determined partly from a study of the maps, partly from reconnaissance of the watershed and partly from the location of reservoirs, waste loads and other features which require a change in reach designation. Upstream areas from reach points were determined by measurement on the quadrangle maps. The length and average slope of the reaches were also determined from the maps.

A tabulation of reach numbers descriptions, lengths and tributary areas is contained in Table A3-1.

The gage locations in the watershed and their upstream areas are tabulated in Table A3-2.

TABLE A3-1
REACH INFORMATION

Reach Number	Reach Description Upper End	Lower End	Length Feet	Area Upstream* of Upper End	Area in* Reach	Area* Tributary at Upper End	
1	Rainbow Dam	Connecticut R.	39,200	583.35	19.69	0	Main Stem
2	Rainbow Reservoir	Rainbow Dam	Res.	578.65	4.70	0	Main Stem
3	Spoonville Br.	Rainbow Res.	6,700	576.00	2.65	0	Main Stem
4	Tariffville	Spoonville Br.	11,000	574.32	1.68	66.74	Main Stem
5	Simsbury	Tariffville	27,400	486.35	21.23	0	Main Stem
6	Avon	Simsbury	25,100	470.38	15.97	0	Main Stem
7	Pequabuck R.	Avon	36,900	452.32	18.06	57.64	Main Stem
8	Unionville	Pequabuck R.	22,400	389.83	4.85	7.41	Main Stem
9	Burlington Brk.	Unionville	15,900	374.33	8.09	9.21	Main Stem
10	Nepaug R.	Burlington Brk.	18,400	358.27	6.85	46.74	Main Stem
11	East Branch	Nepaug R.	20,000	306.37	5.16	67.05	Main Stem
12	Riverton	East Branch	44,200	217.89	21.43	90.81	Main Stem
13	Goodwin Dam	Riverton	12,500	123.56	3.52	0	Main Stem
14	Colebrook Dam	Goodwin Dam	Res.	117.07	6.49	0	Main Stem
15	Colebrook Res.	Colebrook Dam	Res.	93.86	23.21	0	Main Stem
16	Roosterville	Colebrook Res.	14,100	90.52	3.34	29.09	Main Stem
17	Otis	Roosterville	32,900	47.67	13.76	13.76	Main Stem
18	Source	Otis	37,200	0	29.21	0	Main Stem
19	Gage 1895	Tariffville	17,200	33.57	33.17	0	Salmon Brk.
20	Source	Gage 1895	17,000	0	33.57	0	Salmon Brk.
21	Gage 1890	Pequabuck R.	32,400	45.61	12.03	0	Pequabuck R.

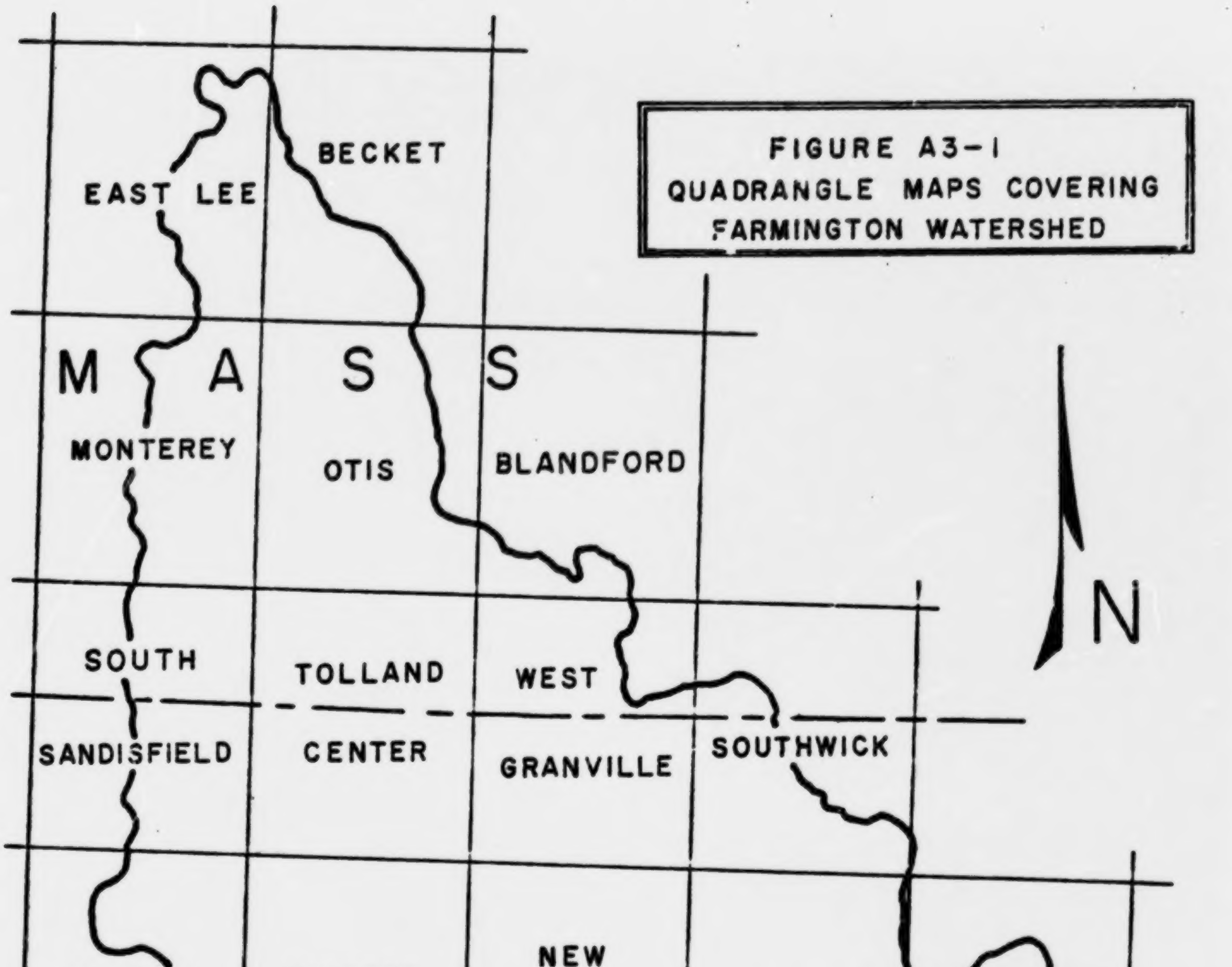

FIGURE A3-1
QUADRANGLE MAPS COVERING
FARMINGTON WATERSHED

TABLE A3-1 (Continued)

22	Source	Gage 1890	22,000	0	45.61	0	Pequabuck R.
23	Gage 1880	Burlington	15,100	4.09	5.12	0	Burlington Brk.
24	Source	Gage 1880	12,000	0	4.09	0	Burlington Brk.
25	Gage 1878	Nepaug R.	Res.	23.62	22.54	0.58	Nepaug R.
26	Source	Gage 1878	25,000	0	23.62	0	Nepaug R.
27	Source	Gage 1878	5,000	0	0.58	0	Nepaug R.
28	Compens. Dam	East Branch	5,000	63.35	3.35	0	East Branch
29	Barkhamsted Dam	Comp. Dam	Res.	53.32	10.03	0	East Branch
30	Barkhamsted Res.	Barkham. Dam	Res.	27.94	25.38	27.29	East Branch
31	Source	Barkham.Res.	20,000	0	20.65	0	Hubbard R.
32	Source	Barkham. Res.	12,000	0	7.29	0	Valley Brk.
33	Sandy Brk.	Riverton	7,400	89.33	1.48	39.80	Still R.
34	Winsted	Sandy Brk.	23,000	43.40	6.13	10.17	Still R.
35	Mad River Dam	Winsted	8,600	7.14	6.49	19.60	Mad R.
36	Highland Lake	Mad R. Dam	Res.	3.40	0.14	3.60	Highland Lake
37	Sucker Brk.	Highland Lake	Res.	3.20	0.20	0	S. Brk. Res.
38	Source	Sucker Brk.	6,000	0	3.20	0	Sucker Brk.
39	Source	Highland Lake	5,000	0	3.60	0	Taylor Brk.
40	Mad R. Res.	Mad R. Dam	Res	19.52	0.08	0	Mad R. Res.
41	Source	Mad R. Res.	15,000	0	19.52	0	Mad River
42	Otis Res.	Otis	Res.	17.75	0.91	0	Otis Res.
43	Source	Otis Res.	15,000	0	17.75	0	Big Pond

*Areas in square miles

TABLE A3-2
GAGE LOCATION AND AREA

Gage NO.**	Reach No.	Area Upstream*	Gage Location
1855	16	90.52	Main Stem, Roosterville
1860	13	127.08	Main Ste,. Riverton
1861	35	19.52	Mad River, Winsted
1865	33	89.61	Still River, Riverton
1873	31	20.40	Hubbard River
1874	32	7.18	Valley Brk.
1878	26	23.62	Nepaug River
1878.5	27	0.58	Collins Brk.
1880	24	4.09	Burlington Brk.
1890	22	45.61	Pequabuck R.
1895	20	33.57	Salmon Brk.
1900	1	585.47	Main Stem, Windsor

*Areas in square miles
**U. S. Geological Survey designation.

A3.2 Gages and Gage Data

The basic historical streamflow data for the thirteen Farmington River watershed gaging stations were made available on 9 track magnetic tape by the U. S. Geological Survey, Water Resources Division, Washington, D. C. The gage numbers, locations and periods of record are listed in Table A3-3.

It is noted that the flows at gage 1855 are regulated by the releases at Otis Reservoir, flows at gage 1860 are regulated by releases from Otis and Goodwin Reservoirs, flows at gage 1861 are regulated by Mad River Dam, flows at gage 1865 are regulated by releases from Mad River and Highland Lake, flows at gage 1890 are regulated by the Whigville and Copper Mine Brook Reservoirs and flows at gage 1900 are regulated by all releases made in the watershed.

Data for use in computation of the constants in equations [5.6], [5.7] and [5.8] were supplied by the U. S. Geological Survey, Water Resources Division, District Office, Hartford, Connecticut. These data consist of measurements of width, area, mean velocity, gage height and discharge at gage locations. The measurements were made at various times of the year. Data were provided for gages 1-1860, 1-1878, 1-1890 and 1-1895 in the Farmington Basin. The data are tabulated in Table A3-4.

TABLE A3-3
GAGE LOCATION AND PERIOD OF RECORD

Gage Number	Gage Location	Period of Record	Years of Record
01-1855	W. Br. Farmington R., New Boston	June 1913-Sept. 1963	50
01-1860	W. Br. Farmington R., Riverton	Oct. 1955-Sept. 1963	8
01-1861	Mad River, Winsted, Conn.	Oct. 1956-Sept. 1963	7
01-1865	Still River, Riverton, Conn.	Oct. 1948-Sept. 1963	15
01-1870	W. Br. Farmington R., Riverton	Oct. 1929-Sept. 1955	26
01-1873	Hubbard R., W. Hartland, Conn.	Oct. 1938-Oct. 1955 Oct. 1956-Sept. 1963	26 24*
01-1874	Valley Brk, W. Hartland, Conn.	Oct. 1940-Sept. 1963	23
01-1878	Nepaug R., Nepaug, Conn.	Oct. 1921-Oct. 1955 Oct. 1957-Sept. 1963	40*
01-1878.5	Clear Brk., Collinsville	Oct. 1921-Sept. 1963	42
01-1880	Burlington Brk., Burlington	Oct. 1931-Sept. 1963	32
01-1890	Pequabuck R., Forestville	Oct. 1941-Sept. 1963	22
01-1895	Salmon Brk., Granby, Conn.	Oct. 1946-Sept. 1963	17
01-1900	Main Stem, Farmington R., Windsor	Oct. 1939-Sept. 1963	24

*Record not consecutive
Source: Water Resources Division, U. S. Geological Survey, Hartford, Conn.

A3.3 Evaporation and Temperature Data

Evaporation data were obtained from the files of Metropolitan District Commission, Hartford, Connecticut. The data, listed in Table A3-5, are those used by MDC for predicting evaporation from the Nepaug and Barkhamsted Reservoirs.

Table A3-4
Cross Section Data

Date	Width ft.	Area	Mean Vel. ft./sec.	Gage Height ft.	Discharge cfs.
For Gage 1-1860, W. Farmington, Riverton					
7/16/65	19	1.74	0.80	2.68	14.0
11/ 4/65	122	64.1	0.33	2.77	21.2
11/ 9/65	130	152	0.97	3.52	147
12/ 3/65	128	114	0.82	3.52	93.4
1/21/66	128	127	1.09	3.87	138
5/26/66	134	195	0.91	3.60	177
7/14/66	40	48.9	0.40	2.77	19.4
8/11/66	42	47.9	0.22	2.67	10.4
9/15/66	81	98.5	0.38	2.89	37.3

TABLE A3-4 (Continued)

10/ 4/66	135	241	1.00	3.95	341
11/ 4/66	130	364	1.93	5.23	704
12/ 8/66	97	195	1.18	3.88	230
1/ 9/67	100	229	1.05	4.21	241
2/10/67	98	158	0.96	3.96	151
3/21/67	130	169	1.22	3.75	207
4/13/67	134	264	1.44	4.37	381
5/17/67	133	247	1.35	4.25	333
6/29/67	96	146	0.76	3.33	111
10/11/67	130	179	1.11	3.80	198
12/ 6/67	130	232	1.11	4.10	257
12/27/67	129	206	1.04	3.96	240
2/ 8/68	121	119	1.69	4.56	201
3/ 8/68	125	76.1	1.13	4.05	86
3/28/68	146	491	2.40	5.98	1,177
4/ 9/68	100	129	0.85	3.29	110
5/ 9/68	128	127	1.13	3.42	143
6/ 5/68	135	278	1.56	4.52	435
7/29/68	103	92	0.56	2.98	52.2
9/16/68	100	81	0.50	2.90	40.3
10/21/68	100	100	0.44	2.88	32.5
11/19/68	135	295	1.59	4.62	468
12/13/68	130	209	1.16	3.95	243
1/ 7/69	130	125	1.11	4.02	139
2/ 7/69	100	94.5	1.97	4.37	186
3/ 6/69	125	159	0.90	3.73	143
4/ 9/69	130	163	1.10	3.65	179
5/16/69	135	452	1.58	4.61	452

For Gage 1-1878, Nepaug, Conn.

8/23/65	32	17.6	0.13	0.10	2.26
11/ 5/65	33	21.0	0.26	0.17	5.40
12/ 7/65	13.5	9.98	1.11	0.26	11.1
12/16/65	30	30.7	0.76	0.50	23.2
4/27/66	33	33.6	0.82	0.56	27.4
6/16/66	32	28.0	0.80	0.48	22.3
7/15/66	13.2	7.55	0.29	0.11	2.2
8/17/66	32	22.6	0.48	0.28	10.8
9/20/66	32	15.6	0.27	0.16	4.2
9/22/66	34	57.8	1.83	1.42	106
10/11/66	28	17.1	0.40	0.22	6.8
11/ 7/66	35	44.3	1.37	0.93	60.6
12/12/66	32.2	31.1	0.88	0.55	27.4
1/11/67	33.5	42.4	0.84	0.80	35.5
1/12/67	33	37.4	1.04	0.69	38.8
2/27/67	32.2	28.4	0.81	0.46	38.8

TABLE A3-4(Continued)

3/23/67	33	38.5	1.12	0.74	22.9
4/19/67	35.5	71.2	2.04	1.71	32.0
4/ 8/67	29	41.6	2.93	1.45	145
6/28/67	32	28.6	0.75	0.49	21.4
2/ 9/68	31	42.2	1.07	0.75	45.2
2/28/68	32	28.3	0.58	0.39	16.3
3/ 8/68	30	27.5	0.71	0.44	19.5
5/ 8/68	30	29.6	0.81	0.51	24.0
6/17/68	34	44.2	1.12	0.84	49.6
8/ 1/68	31.5	22.1	0.38	0.23	8.5
9/17/68	29	22.4	0.34	0.24	7.5
10/ 3/68	14	11.0	0.44	0.17	4.9
11/21/68	30.5	38.6	0.99	0.69	38.2

For Gage 1-1890, Pequabuck River, Forestville, Conn.

9/14/65	58	27.4	0.63	0.88	17.3
11/ 9/65	55	25.0	0.78	0.90	19.5
11/19/65	51.5	22.8	0.99	0.90	22.6
12/ 7/65	54	22.8	0.78	0.88	17.9
12/16/65	54	27.9	1.10	0.09	30.8
1/24/66	52	29.0	1.04	0.98	20.0
4/27/66	58	29.0	1.04	0.98	30.2
6/ 8/66	58	28.2	0.96	0.96	27.2
8/15/66	57	41.1	1.63	1.22	67.1
10/17/66	56	26.0	0.96	0.95	24.9
11/21/66	57	28.0	1.02	0.98	28.5
1/11/67	65	48.5	0.76	1.08	36.9
2/14/67	57	30.5	1.04	1.03	31.7
3/10/67	57	42.2	1.21	1.20	50.9
4/11/67	58	59.0	2.10	1.51	124
5/10/67	58	59.1	1.96	1.49	116
6/ 8/67	58	36.0	1.18	1.11	42.6
8/29/67	56	28.6	1.07	1.01	30.7
10/16/67	55	24.3	0.86	0.91	20.8
12/13/67	60	82.0	2.93	1.96	240
2/ 5/68	60	63.0	1.98	1.55	250
4/ 8/68	57	46.8	1.40	1.27	65.7
6/19/68	58	49.6	1.86	1.38	92.5
8/ 2/68	56	32.6	1.13	1.09	37.0
10/14/68	55	27.8	1.06	0.98	29.4
11/ 6/68	55	24.5	0.89	0.92	21.9
1/20/69	56	35.8	1.33	1.15	27.5
3/14/69	56	34.0	1.32	1.14	45.0
6/ 9/69	58	35.3	1.25	1.16	44.1
6/ 9/69	58	34.7	1.21	1.14	42.1

TABLE A3-4 (Continued)

For Gage No. 1-1895 Salmon Brook, Granby, Conn.

9/ 9/59	50	49.6	0.38	2.41	19.0
12/16/59	56.5	101	1.66	3.43	168
1/ 1/60	54	69.0	1.40	2.76	96.8
2/19/60	52	99.7	3.30	3.90	329
2/19/60	52	100	3.26	3.95	326
3/ 3/60	57.5	73.2	1.49	2.88	109
4/ 5/60	111	542	3.14	7.55	1,700
5/16/60	58	59.1	1.69	2.96	99.9
6/ 8/60	61	80.6	1.68	3.16	135
7/18/60	60	58.3	1.15	2.73	67.4
8/29/60	62	49.8	0.86	2.66	42.7
9/21/60	77	121	2.10	3.96	254
9/26/60	62	69.7	1.38	3.10	96.5
11/16/60	61	77.2	1.28	3.06	99.0
12.27.60	61	72.3	1.21	2.94	87.5
2/ 7/61	32	36.2	1.96	3.40	70.9
2/17/61	34	37.3	2.16	2.80	80.7
2/26/61	62	296	3.58	6.46	1.060
3/ 9/61	44	96.2	3.34	4.08	322
4/14/61	48	107	3.30	4.06	342
4/28/61	40	65.2	3.16	3.50	206
5/15/61	39	92.1	2.39	3.53	222
5/23/61	39	69.6	2.02	2.12	141
7/27/61	31	20.8	1.62	2.22	33.7
8/24/61	44	51.2	0.90	2.41	46.0
10/ 9/61	40	41.4	0.68	2.16	28.3
11/16/61	37	50.9	0.90	2.37	46.0
1/17/62	72	95.0	1.84	3.14	175
2/14/62	32	42.0	0.98	2.21	41.1
3/ 6/62	34	32.5	1.65	2.30	53.5
4/17/62	41	60.0	2.87	3.16	172
5/31/62	33	32.0	1.30	2.21	41.5
6/20/62	48	47.5	0.67	2.06	31.6
7/11/62	50	59.8	0.52	2.37	31.2
8/24/62	47	64.0	0.33	2.47	21.3
10/12/62	39	67.4	0.78	2.56	52.4
1/ 9/03	41	58.1	0.90	2.42	52.2
2/21/63	45	46.6	1.57	2.46	73.0
2/26/63	43	43.4	1.32	2.33	57.3
4/ 2/63	79	154	2.75	4.18	424
4/16/63	59	78.3	1.45	2.85	113
5/ 8/63	57	66.0	1.24	2.60	82.0
6/ 4/63	58	60.4	0.90	2.52	54.6
7/2/63	48	51.6	0.69	2.61	35.8
8/13/63	40	32.9	0.88	1.69	29.1
9/10/63	20	11.2	1.28	3.70	14.4
10/ 8/63	42	24.8	0.83	1.75	20.5
10/21/63	26	14.7	1.05	3.82	15.5

Source: Water Resources Division, U. S. Geological Survey, Hartford, Conn.

TABLE A3-5
EVAPORATION DATA

Month	Average Evaporation, inches
January	0.96
February	1.05
March	1.70
April	2.97
May	4.46
June	5.54
July	5.98
August	5.50
September	4.12
October	3.16
November	2.25
December	1.51
Total	39.20
Mean, Monthly	3.27

Source: MDC, Hartford, Connecticut.

TABLE A3-6
TEMPERATURE DATA

Week of Year	Mean Temperature °C	Week of Year	Mean Temperature, °C	
	1966		1966	1967
1	1.07	27		21.99
2	0.92	28		24.01
3	1.31	29		23.55
4	1.94	30		24.10
5	0.46	31		24.08
6	0.02	32		22.98
7	0.36	33		23.09
8	0.85	34	23.28	20.60
9	0.74	35	24.28	20.79
10	0.69	36	20.76	21.01
11	1.00	37	18.85	18.61
12	1.11	38	16.38	19.43
13	4.16	39	14.70	17.00
14	4.34	40	13.86	16.71
15	5.86	41	12.73	13.97
16	7.14	42	11.24	13.64
17	8.68	43	10.73	9.94
18	13.07	44	9.35	
19	9.73	45	8.21	
20	15.13	46		4.43
21	16.91	47	5.05	3.33
22	17.67	48	6.30	
23	21.54	49	2.75	
24	22.73	50	3.66	
25	19.94	51	1.45	
26	21.86	52	1.05	

Mean of the weekly means - 11.86°C

Source: Mr. David Bennett, Cheshire, Connecticut.

Temperature data were obtained from Mr. David Bennett, who established a continuous temperature recorder in the Farmington River at Farmington, Connecticut to measure the river temperature. The data provided by Mr. Bennett were contained in his M.S. Thesis, 1968, University of Connecticut. The data are tabulated in Table A3-6.

A3.4 Reservoir Data

A3.4.1 Otis Reservoir

Otis Reservoir is located in Berkshire and Tolland Counties, Massachusetts. The present ownership is the State of Massachusetts, although it was formerly owned by The Collins Company, Collinsville, Connecticut. It was used to augment low flow for the Collins Mill from 1865, when it was built, until 1966. The reservoir is now used primarily for recreation, but the level is drawn down in the fall to afford a measure of flood control protection in the early spring. Otis Reservoir is a Type II reservoir according to the classification set in Appendix A4.5.

The reservoir capacity-gage height data for Otis Reservoir were obtained from the U. S. Geological Survey, Hartford, Connecticut. These data are listed in Table A3-7. No data exist relating the depth and surface area of Otis Reservoir. The area data, contained in Table A3-7 were computed by equating the area to the volume between adjacent depth readings. The method of computation is approximate but for the reservoir full or nearly full, as it would be in the summer when the evaporation constant is high, the approximation is very close. The planimetered area for a full reservoir was 46.68×10^6 square feet, compared to the calculated area of 46×10^6 square feet.

The operation of Otis Reservoir is simply to: only discharge overflow between April 1 and October 1, draw the level down to 17.0 feet during the month of October and discharge the overflow at 17.0 feet between November 1 and March 31.

A3.4.2 Colebrook Reservoir

Colebrook Reservoir is located on the upper main stem of the West Branch of the Farmington River in Litchfield County, Connecticut. The dam and reservoir were built by the Corps of Engineers with completion of construction occurring in early 1969. Approximately one-third of the cost of the project was financed by the Metropolitan District Commission

TABLE A3-7
CAPACITY-AREA-DEPTH DATA-OTIS RESERVOIR

Gage Height feet	Total Capacity 10^6 cubic feet	Surface Area 10^6 square feet
0	0	0
1	8	8
2	17	9
3	27	10
4	38	11
5	51	13
6	66	15
7	83	17
8	102	19
9	122	20
10	144	22
11	167	23
12	192	25
13	220	28
14	250	30
15	282	32
16	316	34
17	352	36 - Winter level
18	389	37
19	428	39
20	468	40
21	510	42
22	553	43
23	597	44
24	642	45
25	688	46 - Spillway level
26	780	46

Source: Water Resources Division, U. S. Geological Survey, Hartford, Connecticut.

which controls a water supply and replacement pool of 41,700 acre-feet capacity.

Colebrook Reservoir is a multipurpose project. The various pools are:

Dead Storage and Sedimentation		1,000 Acre-feet
Replacement Water Supply, Goodwin Res.		11,000 Acre-feet
Water Supply		30,700 Acre-feet
Fishery, for brown trout		5,000 Acre-feet
Joint, Fishery and Flood Control		5,000 Acre-feet
Flood Control		45,000 Acre-feet
	Total	98,500 Acre-feet

Colebrook Reservoir is a Type I reservoir. The spillway is an uncontrolled chute having a crest width of 205 feet.

The reservoir capacity-depth and area-depth data were obtained from curves furnished by the New England Division, Corps of Engineers, Waltham, Massachusetts. Data read from the curves are tabulated in Table A3-8.

The operation of Colebrook Reservoir is described as follows:

(1) The flood control capacity will be kept empty for storage of floods and will be emptied as rapidly as possible after flood control operations.
(2) The water supply storage will be used in part to provide a minimum release of 50 cfs for downstream riparian owners and in part (future) for diversion into the Metropolitan District Commission (MDC) system for water supply.
(3) The 11,000 acre-feet replacement pool is also controlled by MDC, for flow augmentation, being volume that was originally in the Goodwin Reservoir.
(4) The fishery pool of 5,000 acre-feet is for release in late April and May for enhancement of the spring shad fishery. The release will be prior to the hurricane season when this volume is needed for flood control.
(5) The fishery pool of 5,000 acre-feet for enhancement of the sea-run brown trout fishery will contain water stored in the spring for release in August, September and October. This pool also provides additional water for summer recreation activities.

A3.4.3 Barkhamsted Reservoir.

Barkhamsted Reservoir is located on the East Branch of the Farmington River in Litchfield and Hartford counties, Connecticut. It was built by the MDS, Hartford and is owned and operated by this body for water supply purposes only. Water is diverted to the MDS distribution system, which lies outside the Farmington watershed. Excess water is released downstream to the Farmington. The reservoir is Type IV.

The reservoir capacity-area-depth information, as well as data on diversions, was provided by the MDS. The data are listed in Tables A3-9 and A3-10.

The operation of the Barkhamsted Reservoir is to provide the demands for diversion out of storage and inflow, and to release any spillway overflow to the Farmington.

A3.4.4 Sucker Brook Reservoir.

The Sucker Brook project is a Corps of Engineers flood control project now under construction. The dam and reservoir are located adjacent to Highland Lake in the Town of Winchester, Connecticut. The discharge from the reservoir is through an ungated 30 inch diameter conduit. When the flows exceed the capacity of this outlet, the excess is stored in the reservoir. An overflow spillway is provided. There is no permanent or recreation pool.

Reservoir capacity-area-depth data, in the form of curves were obtained from the Corps of Engimeers, New England Division. The outlet rating curve was also furnished by The Corps. These data are listed in Tables A3-11 and A3-12.

TABLE A3-8
CAPACITY-AREA-DEPTH DATA-COLEBROOK RESERVOIR

Pool Elevation, Feet above MSL	Depth feet	Capacity 10^6 cubic feet	Area 10^6 square feet
567	0	0	0
580	13	12.3	1.32
590	23	21.9	2.83
600	33	52.3	5.14
610	43	117.6	8.02
620	53	200.4	10.32
630	63	309.3	12.68
640	73	448.7	15.25
650	83	618.6	17.95
660	93	801.5	20.39
670	103	1006.2	22.87
680	113	1237.1	25.57
690	123	1524.6	28.01
700	133	1812.1	30.71
710	143	2130.1	33.24
720	153	2474.2	36.11
730	163	2866.2	39.60
740	173	3275.7	43.34
750	183	3733.1	47.13
760	193	4238.4	51.84
761	194	4290.7	52.71 - Spillway Elev.

Source: New England Division, Corps of Engineers, Waltham, Massachusetts.

TABLE A3-9
CAPACITY-AREA-DEPTH DATA-BARKHAMSTED RESERVOIR
(Data are for top 24 feet only*.)

Pool Elevation Feet above MSL	Depth, feet above el. 506	Capacity 10^6 cubic ft.	Area 10^6 sq. ft.
506	0	2015	
507	1	2085	70
508	2	2158	72
509	3	2231	73
510	4	2305	74
511	5	2381	76
512	6	2458	77
513	7	2536	78
514	8	2616	80
515	8	2696	80
516	10	2778	82
517	11	2862	84
518	12	2946	84
519	13	3032	86
520	14	3118	86
521	15	3207	89
522	16	3296	89
523	17	3387	91
534	18	3479	92
525	19	3572	93
526	20	3666	94
527	21	3762	96
528	22	3858	96
529	23	3956	98
530	24	4054	98 - Spillway Level

Source: MDC, Hartford, Connecticut

* Data are available for the entire range of the depth in the reservoir but are not used because the operating range is included within the depths tabulated.

TABLE A3-10
DIVERSION DATA-BARKHAMSTED RESERVOIR

(Data are weekly diverted volumes computed from daily data furnished by MDC).

Week	Weekly Diverted Volume 10^6 cubic feet			Week	Weekly Diverted Volume 10^6 cubic feet		
	1963	1964	1967		1963	1964	1967
1	31.0	16.8	33.7	27	19.5	28.6	27.4
2	28.2	15.0	25.5	28	16.8	21.5	23.8
3	23.1	13.6	20.7	29	19.1	23.3	19.5
4	22.5	14.0	25.8	30	19.4	24.3	20.5
5	21.7	14.0	16.3	31	24.1	25.8	19.9
6	21.5	14.0	20.6	32	18.3	17.1	20.2
7	19.5	14.0	20.6	33	18.7	17.6	23.4
8	18.7	14.0	20.6	34	17.5	20.3	27.5
9	18.7	14.0	17.6	35	19.4	18.7	42.1
10	15.0	9.2	5.6	36	25.8	23.0	42.5
11	14.0	4.7	0.9	37	25.8	26.9	39.6
12	10.4	4.8	1.9	38	22.3	27.1	42.1
13	2.3	4.7	3.6	39	19.7	27.1	34.6
14	0	4.7	3.5	40	19.7	25.3	33.3
15	0	4.7	3.7	41	28.7	23.4	33.6
16	0.4	4.7	3.7	42	33.6	23.4	34.0
17	8.6	4.7	3.7	43	34.4	21.4	33.8
18	12.2	5.1	3.2	44	37.2	23.4	35.4
19	21.4	10.3	3.7	45	34.9	23.0	36.1
20	23.5	17.0	3.7	46	35.4	27.5	35.8
21	28.1	34.2	3.7	47	36.5	28.2	35.8
22	25.9	33.6	3.7	48	35.6	32.8	35.7
23	25.1	28.7	3.1	49	37.2	6.8	28.7
24	27.5	23.0	25.3	50	23.8	28.6	24.2
25	25.4	19.0	40.0	51	18.7	32.8	28.5
26	27.8	28.5	36.4	52	17.8	32.8	28.5

Source: MDC, Hartford, Connecticut

TABLE A3-11
CAPACITY-AREA-DEPTH DATA-SUCKER BROOK RESERVOIR

Pool Elev. Ft. Above MSL	Depth ft.	Capacity 10^3 cubic ft.	Area 10^3 square ft.
881	0	0	0
885	4	174	
890	9	1437	510
895	14	1574	784
900	19	8930	923
905	24	13939	1076
910	29	19863	1342
915	34	26572	1564
920	39	34979	1777
925	44	43996	1978
930	49	53797	2161
935	54	64556	2330

Spillway crest is at Elev. 926 feet

Source: New England Division, Corps of Engineers, Waltham, Mass.

TABLE A3-12
OUTLET RATING DATA
SUCKER BROOK RESERVOIR

Pool Elev. Ft. above MSL	Depth ft.	Discharge cfs
881	0	0
885	4	32
890	9	48
895	14	58.5
900	19	67.5
905	24	76
910	29	83.5
915	34	90
920	39	97
925	44	103
930	49	108.5
935	54	115

Source: New England Division, Corps of Engineers, Waltham, Mass.

A3.4.5 Rainbow Reservoir.

Rainbow Reservoir is an impoundment used to store water for hydroelectric power generation. The generating plant is used to provide peaking capacity and thus the use of water and consequent releases are intermittant. The storage capacity is relatively small and for the time averaging interval of one "week" used in this work, the plant may be considered "run of the river." Rainbow Reservoir is classified type V.

The reservoir is owned and operated by the Farmington River Power Company. Data were furnished through the courtesy of Mr. Robert Tolles, The Stanley Works, New Britain, Connecticut.

There is no complete information which allows the direct computation of capacity-depth and area-depth equations. The information supplied is:

maximum depth	50	feet
average depth	18.6	feet
surface area at spillway crest elevation	235	acres
volume at spillway crest elevation	4370	acre feet
spillway crest elevation	132.6	

Data were furnished from which the capacity-depth relationship can be determined in the range from 9 feet below to 3 feet above the crest. Because the operating range is within these limits, a capacity-depth equation can be developed. The area-depth relationship has been approximated using volume-incremental depth information within the above-mentioned range. The data, which are approximate but the best available, are contained in Table A3-13.

A3.4.6 Goodwin Reservoir.

Goodwin Reservoir is also called the West Branch Reservoir or Hogback Reservoir. This reservoir was built and is operated by the MDC. It is located on the main stem one mile downstream of Colebrook Dam and about 2 1/2 miles upstream of the confluence of the Still River and the West Branch of the Farmington. The newer Colebrook Dam was built in the pool of Goodwin Reservoir as noted in A3.4.2, above.

The Goodwin project was built to provide low flow augmentation for riparian owners downstream and is required by law to discharge a minimum of 150 cfs. The plan is to build an aqueduct from Goodwin to Barkhamsted to divert water from the West Branch into the MDC system in the future. Much of the function of Goodwin has been expanded by the larger storage available in The Colebrook Reservoir.

TABLE A3-13
CAPACITY-AREA-DEPTH DATA-RAINBOW RESERVOIR

Pool Elev. Ft. above MSL	Depth Dam, ft.	Storage Available KWH*	Capacity 10^6 cubic ft.	Area 10^6 sq. ft.
82.6	0		0	0
123.6	41	7,500	118	
124.6	42	15,500	125	7.4
125.6	43	24,100	132	7.7
126.6	44	33,000	140	8.0
127.6	45	42,100	148	8.2
128.6	46	51,500	156	8.5
129.6	47	61,200	164	8.8
130.6	48	71,200	172	9.1
131.6	49	81.500	181	9.4
132.6	50	92,100	190	9.7
133.6	51	103,100	200	10.1
134.6	52	114,600	210	10.4
135.6	53	126,600	220	10.7

* 1 KWH = 6400 gallons

Source: Farmington River Power Company through Mr. R. C. Sprong, Manager, Utilities and Services, The Stanley Company, New Britain, Connecticut.

Goodwin is classified type IV.

The capacity-area-depth data were provided by the MDC in the form that allowed separation of the volume into that included and that not included in the Colebrook Reservoir. The data shown in Table A3-14 are for the portion not included in Colebrook. Facilities for diversion are not completely constructed at the present time.

A3.4.7 Nepaug Reservoir.

Nepaug Reservoir is the first of the Farmington River Basin reservoirs built by MDC for its water supply. The project was completed in 1916. The reservoir is on the Nepaug River a short distance upstream of its confluence with the Farmington River near Collinsville, Connecticut. The capacity-area-depth and diversion data needed in this work were supplied by the MDC.

Nepaug Reservoir is operated in the same manner as Barkhamsted Reservoir; that is, water is diverted into the MDC water supply system according to demand and any excess after the reservoir is full is released downstream at the dam. The reservoir is used for water supply only and no recreation is allowed. Nepaug Reservoir is classified type IV.

Capacity-area-depth data and diversion data are tabulated in Tables A3-15 and A3-16, respectively.

A3.4.8 Compensating Reservoir.

Compensating Reservoir was originally built by MDC for the benefit of downstream riparian owners as compensation for the right to divert water out of the Farmington Basin. Later, when Barkhamsted Reservoir was built immediately upstream, the volume of water available for compensating was no longer sufficient. The construction of Goodwin Reservoir relieved Compensating Reservoir of this use. Compensating Reservoir is now used for reserve storage and recreation. It is classified as a type V reservoir with no scheduled diversion.

The capacity-area-depth data for Compensating Reservoir were furnished by MDC and are tabulated in Table A3-17.

TABLE A3-14
CAPACITY-AREA-DEPTH DATA-GOODWIN RESERVOIR

Pool Elev. ft. above MSL	Depth ft.	Capacity 10^6 cubic ft.	Area 10^6 sq. ft.
540	0	0	0
560	20	9.64	0.39
565	25	14.46	0.48
570	30	23.41	0.96
575	35	33.05	1.79
580	40	45.44	1.93
585	45	63.34	2.48
590	50	85.37	3.58
595	55	110.16	4.41
600	60	134.95	4.96
605	65	162.49	4.96
610	70	190.03	5.52
615	75	220.32	5.50
620	80	249.24	5.78
625	85	279.53	6.06
630	90	313.96	6.60
635	95	347.01	6.90
640	100	388.32	8.26 - Spillway Crest

Source: MDC, Hartford, Connecticut

TABLE A3-15
CAPACITY-AREA-DEPTH DATA-NEPAUG RESERVOIR
(For top 24 feet in Reservoir)

Pool Elev. ft. above MSL	Depth above EL 458	Capacity 10^6 cubic ft.	Area 10^6 sq. ft.
458	0	497	25
459	1	523	26
460	2	549	26
461	3	576	27
462	4	604	28
463	5	632	28
464	6	660	28
465	7	690	29
466	8	719	30
467	9	749	30
468	10	780	30
469	11	810	31
470	12	842	31
471	13	873	32
472	14	905	32
473	15	938	33
474	16	971	33
475	17	1004	33
476	18	1038	34
477	19	1072	34
478	20	1107	35
479	21	1142	35
480	22	1178	36
481	23	1214	36
482	24	1251	37 - Spillway Crest

Source: MDC, Hartford, Connecticut

TABLE A3-16
DIVERSION DATA-NEPAUG RESERVOIR
(Weekly averages computed from daily values)

Week	Average Weekly Diversion 10^6 cubic feet				Average Weekly Diversion 10^6 cubic feet		
	1963	1964	1967		1963	1964	1967
1	8.6	19.0	7.5	27	27.4	28.1	30.3
2	13.0	26.7	16.7	28	31.3	30.5	36.1
3	16.3	28.1	22.3	29	31.7	30.5	38.1
4	19.5	15.0	18.7	30	28.2	28.1	39.3
5	21.9	14.0	26.9	31	23.0	29.1	38.9
6	22.9	14.0	21.1	32	30.3	29.1	35.6
7	22.1	18.7	21.7	33	24.9	28.1	32.8
8	20.4	25.3	23.4	34	30.9	28.1	32.6
9	20.4	25.9	26.5	35	29.5	32.5	18.7
10	14.3	28.7	37.7	36	27.9	28.7	18.7
11	17.8	19.4	34.1	37	27.8	27.4	18.7
12	21.7	26.9	29.0	38	27.9	29.1	21.1
13	23.4	31.0	52.5	39	27.9	28.6	20.6
14	24.7	31.8	40.6	40	28.3	25.7	19.7
15	36.6	31.8	43.9	41	17.8	23.4	19.7
16	39.4	31.6	36.8	42	13.4	29.5	19.7
17	36.1	29.5	38.6	43	11.6	30.2	19.7
18	32.5	35.8	46.9	44	7.2	30.6	19.7
19	28.7	34.5	48.7	45	7.5	31.0	19.7
20	21.9	39.1	47.6	46	8.3	36.4	19.7
21	17.2	11.8	43.7	47	5.2	24.6	19.7
22	16.8	15.8	40.8	48	0	13.9	14.4
23	17.2	25.4	49.6	49	0	27.4	15.8
24	18.7	26.9	34.1	50	13.8	22.2	12.2
25	17.8	23.1	23.3	51	20.1	14.0	14.6
26	27.9	28.1	41.6	52	18.7	15.0	25.3

Source: MDC, Hartford, Connecticut.

TABLE A3-17
CAPACITY-AREA-DEPTH DATA-COMPENSATING RESERVOIR
(For top 24 feet in Reservoir)

Pool Elev. ft. above MSL	Depth above EL. 396	Capacity 10^6 cubic ft.	Area 10^6 sq. ft.
396	0	91.18	7.49
397	1	98.67	7.76
398	2	106.43	8.28
399	3	114.71	8.56
400	4	123.27	8.83
401	5	132.10	9.09
402	6	141.19	9.62
403	7	150.81	10.30
404	8	161.11	10.69
405	9	171.80	11.10
406	10	182.90	11.77
407	11	194.67	12.16
408	12	206.83	12.44
409	13	219.97	12.70
410	14	231.97	13.10
411	15	245.07	13.24
412	16	258.31	14.57
413	17	272.88	14.71
414	18	287.59	15.11
415	19	302.70	15.51
416	20	318.21	15.77
417	21	333.98	16.31
418	22	350.29	16.85
419	23	367.14	16.85
420*	24	383.99	

* Spillway Crest is at Elev. 420.5

Source: MDC, Hartford, Connecticut

A3.4.9 Highland Lake.

The following information relative to Highland Lake was furnished by the Corps of Engineers, New England Division, Waltham, Mass.

Highland Lake is used primarily for recreational purposes with year-around residences and cottages distributed all around the periphery. An industry, Union Pin Company, located near the outlet end of the lake, has water rights and controls the discharges made, excepting the spillway overflow. The industry uses the water for power and processing with the used water being discharged into the outlet stream. No rating curve exists for the sluice gate control device.

The agreement whereby Union Pin Company regulates the level of the lake is unwritten. For many years, water has been stored in the spring so that the water surface is at or near the spillway crest at elevation 882.5 MSL by June 1. The level is lowered not to exceed one foot per month during July, August and September. After October 1, the level is lowered to 877.5, five feet below the spillway crest. This fall drawdown provides a limited flood protection while the higher summer level contributes to the recreational benefit to the owners around the lake. Highland Lake is classified type II in this work.

The capacity-depth data for Highland Lake were furnished by the U. S. Geological Survey, Water Resources Division, Hartford. The data are tabulated in Table A3-18. No area-depth are available excepting that the area at spillway crest elevation is 444 acres and at the level at 6.5 feet below crest elevation the area is planimetered as 350 acres. It is assumed the area-depth relationship is linear between these two levels and may be extrapolated to the level of the outlet structure.

A3.4.10 Mad River Reservoir

Mad River Reservoir is, like the Sucker Brook Reservoir, a single purpose flood control project with an ungated outlet structure. The reservoir is owned and operated by the Corps of Engineers. The Mad River project was completed in 1962. Although the project is for flood control only, there is a small pool below the level of the outlet structure which is used for recreation.

The ungated structure is a circular conduit having a diameter of 45 inches. The outlet discharge rate is given by a rating curve furnished by the Corps of Engineers. Data read from the rating curve are listed in Table A3-19. Capacity-area-depth data, also obtained from Corps-furnished curves, are tabulated in Table A3-20.

TABLE A3-18
CAPACITY-DEPTH DATA-HIGHLAND LAKE

Pool Elev. ft. above MSL	Depth, ft. above 873.5	Capacity 10^6 cubic feet
873.5	0	233
874.5	1	247
875.5	2	262
876.5	3	277
877.5	4	292
878.5	5	309
879.5	6	325
880.5	7	343
881.5	8	361
882.5	9	380

Source: Water Resources Division, U. S. Geological Survey, Hartford Connecticut

A3.5 Population and Waste Load Projections.

The following data have been taken from a Report, Water Resources Planning Study of the Farmington Valley, made in 1965 by The Travelers Research Center, Inc. to the Water Resources Commission, State of Connecticut.

The projected populations for the various Towns in the watershed are listed in Table A3-21. The figures are corrected to reflect only the population in the Farmington Watershed for those towns which do not lie wholly within the watershed.

The projected waste discharge rates, municipal and industrial, are listed by Towns in Table A3-22.

A3.6 Irrigation Requirements

The Farmington Valley tobacco growers plant 3000 acres to tobacco in an average year. This crop in grown in the river lowlands between Farmington and Simsbury (reaches 5,6 and 7). Water for irrigation is pumped from the river. The normal irrigation demand is 4 to 6 inches per year. Assuming the use is 6 inches over a 2-month period, (9 weeks) for 3000 acres, the average weekly demand is:

$$\frac{6}{12} \times 3000 \times 43560 \times \frac{1}{9} = 7.26 \times 10^6 \text{ cubic feet}$$

or,

$$\begin{aligned} 7.26 \times 1.52 &= 11.04 \text{ cubic feet per second} \\ &= 3.68 \text{ cubic feet per second from each of three reaches.} \end{aligned}$$

TABLE A3-19
OUTLET DISCHARGE DATA-MAD RIVER RESERVOIR

Pool Elev. ft. above MSL	Depth ft.	Discharge Weir Gate open cfs	Discharge Weir Gate closed cfs
855	0	0	0
860	5	75	0
865	10	112	0
870	15	136	0
874	19	150	0
880	25	197	197
890	35	232	232
900	45	263	263
910	55	289	289
920	65	312	312
930	75	335	335
940	85	356	356
950	95	375	375
960	105	394	394
970	115	412	412
980	125	430	430
983	128	435	435

Source: New England Division, Corps of Engineers, Waltham, Mass.

TABLE A3-20
CAPACITY-AREA-DEPTH DATA-MAD RIVER RESERVOIR

Pool Elev. ft. above MSL	Depth ft.	Capacity 10^6 cubic ft.	Area 10^3 sq. ft.	
825	0	0	0	
840	15	0.44	44	
850	25	2.18	166	
860	35	3.92	261	
870	45	6.97	392	
872	47	7.84	418	- Recreation Pool Level
880	55	12.20	653	
890	65	19.17	1198	
900	75	32.23	1721	
910	85	53.50	2483	
920	95	80.15	3027	
930	105	113.26	3681	
940	115	153.31	4400	
950	125	203.86	5205	
960	135	259.18	6142	
970	145	322.78	6926	
980	155	397.27	7928	
983	158	424.71	8233	- Spillway Level

Source: New England Division, Corps of Engineers, Waltham, Mass.

In addition, two golf courses in the Farmington area use irrigation water pumped from the river. The rate of usage is not known.

Assume the total use is as shown in Table A3-23.

A3.7 Existing Sewage Treatment Plants.

Waste treatment plants having significant size are located at Plainville, Collinsville, Farmington, Bristol, Plymouth, Windsor Locks (2 plants), Tariffville and Winsted. Plant size, treatment type and available effluent BOD values are listed below. Data were furnished by the Water Resources Commission, State of Connecticut. The locations of these treatment plants are shown in Figure A3-2.

(1) Plainville. Municipal plant, secondary treatment, 1.4 mgd capacity, on line in late 1967. Effluent BOD 11/15/67 was 47 mg/l.

(2) Collinsville. Municipal plant, secondary treatment, 0.4 mgd capacity, on line late 1967. Effluent BOD 3/12/68, 110 mg/l; 7/2/68, 27 mg/l.

FIGURE A3-2
LOCATION OF WASTE
TREATMENT PLANTS

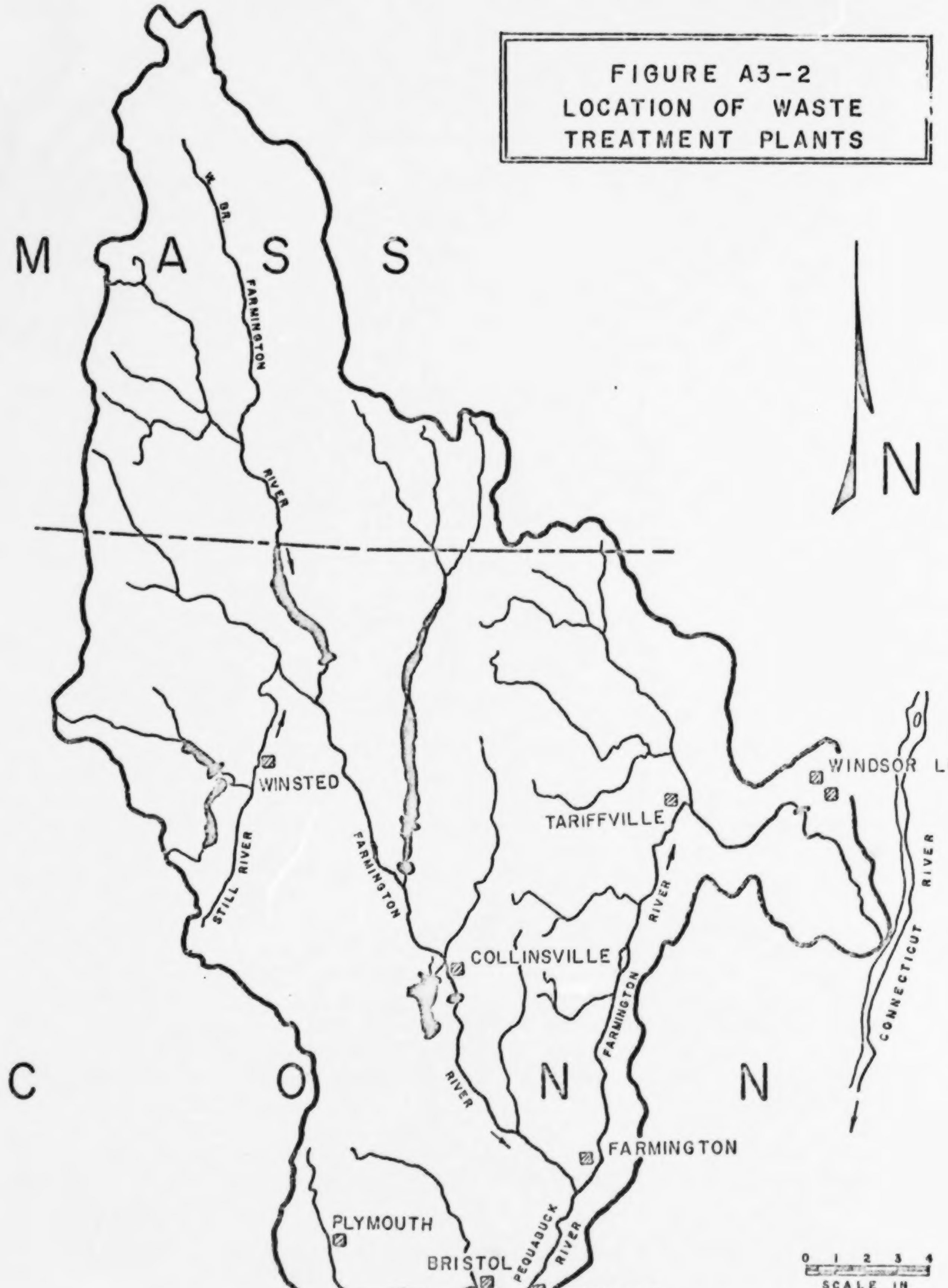

TABLE A3-21
PROJECTED POPULATION-FARMINGTON WATERSHED

Town	1970	1980	1990	2000	2015
Connecticut					
Windsor	4,088	5,400	6,800	5,880	4,950
Windsor Locks	1,838	2,700	3,553	4,520	6,368
Bloomfield	4,840	5,760	5,880	5,200	3,250
East Granby	4,500	7,500	11,100	16,000	24,500
Granby	8,200	14,000	23,500	35,000	50,000
Simsbury	12,819	15,819	19,419	23,869	30,069
Avon	6,886	9,186	11,936	15,336	20,136
Farmington	15,406	19,406	22,906	25,406	27,906
Plainville	5,363	7,000	8,813	10,800	14,025
Bristol	55,000	65,000	74,400	84,300	97,000
Plymouth	7,685	8,190	8,683	9,150	9,350
Burlington	3,290	3,890	4,590	5,390	6,890
New Hartford	3,533	4,132	4,832	5,632	7,117
Canton	5,892	7,242	8,992	11,142	16,142
Barkhamsted	1,870	2,370	2,870	3,870	5,370
Winchester	11,800	13,600	16,200	18,800	21,000
Hartland	1,340	1,640	2,140	2,640	3,650
Colebrook	990	1,190	1,490	1,790	2,290
Massachusetts					
Granville	1,070	1,270	1,570	1,870	2,370
Tolland	150	200	280	350	450
Sandisfield	690	840	1,040	1,240	1,740
Otis	620	770	970	1,170	1,670
Becket	970	1,170	1,470	1,770	2,270
Total	158 ,840	198,275	243,434	291,125	358,513

Source: Water Resources Planning Study of the Farmington River Valley, The Travelers Research Center, Hartford, Connecticut, 1965

TABLE A3-22
PROJECTED WASTE DISCHARGES-FARMINGTON WATERSHED
(Data in million gallons daily)

Town		1970			1980	
Connecticut	Munic.	Indust.	Total	Munic.	Indust.	Total
Windsor	0.33	0.95	1.28	0.49	1.15	1.64
Windsor Lake	0.15	0.93	1.08	0.24	0.24	1.12
Bloomfield	0.39	0.08	0.47	0.52	0.09	0.61
East Granby	0.29		0.29	0.56		0.56
Granby	0.53		0.53	1.05		1.05
Simsbury	1.03	0.08	1.11	1.42	0.10	1.52
Avon	0.55	0.04	0.59	0.83	0.05	
Farmington	1.23	1.18	2.41	1.75	1.28	3.03
Plainville	0.43	0.10	0.53	0.63	0.11	0.74
Bristol	4.40	4.60	9.00	5.85	5.00	10.85
Plymouth	0.62	0.20	0.82	0.74	0.22	0.96
Burlington	0.21		0.21	0.29		0.29
New Hartford	0.28		0.28	0.37		0.37
Canton	0.47		0.47	0.65		0.65
Barkhamsted	0.12		0.12	0.18		0.18
Winchester	1.24	0.50	1.74	1.50	0.60	2.10
Hartland	0.09		0.09	0.12		0.12
Colebrook	0.06		0.06	0.09		0.09
Massachusetts						
Granville	0.07		0.07	0.10		0.10
Tolland	0.01		0.01	0.02		0.02
Sandisfield	0.04		0.04	0.06		0.06
Otis	0.04		0.04	0.06		0.06
Becket	0.06		0.06	0.09		0.09
Sub Total	12.64	8.66		17.61	9.72	
Total			21.30			27.33

Source: Water Resources Planning Study of the Farmington River Valley, The Travelers Research Center, Hartford, Connecticut, 1965.

TABLE A3-22
(Cont.)
PROJECTED WASTE DISCHARGE-FARMINGTON WATERSHED
(Data in million gallons daily)

1990			2000			2015		
Munic.	Indust.	Total	Munic.	Indust.	Total	Munic.	Indust.	Total
0.65	1.35	2.00	0.59	1.50	2.09	0.52	1.60	2.12
0.34	1.30	1.64	0.45	1.50	1.95	0.67	1.60	2.27
0.56	0.10	0.66	0.52	0.10	0.62	0.34	0.10	0.44
0.95		0.95	1.52		1.52	2.45		2.45
2.00		2.00	3.32		3.32	5.00		5.00
1.85	0.13	1.98	2.39	0.15	2.54	3.16	0.18	3.34
1.13	0.06	1.19	1.53	0.07	1.60	2.11	0.08	2.19
2.18	1.39	3.57	2.54	1.50	4.04	2.93	1.60	4.53
0.84	0.12	0.96	1.08	0.13	0.21	1.47	1.14	1.61
7.06	5.50	12.56	8.43	6.00	14.43	10.17	6.50	16.67
0.83	0.24	1.07	0.92	0.26	1.18	0.98	0.28	1.26
0.39		0.39	0.51		0.51	0.69		0.69
0.46		0.46	0.56		0.56	0.75		0.75
0.85		0.85	1.11		1.11	1.69		1.69
0.24		0.24	0.37		0.37	0.54		0.54
1.86	0.70	2.56	2.16	0.80	2.96	2.42	0.90	3.32
0.18		0.18	0.25		0.25	0.37		0.37
0.13		0.13	0.17		0.17	0.23		0.23
0.13		0.13	0.18		0.18	0.24		0.24
0.02		0.02	0.03		0.03	0.05		0.05
0.09		0.09	0.12		0.12	0.17		0.17
0.08		0.08	0.11		0.11	0.17		0.17
0.13		0.13	0.17		0.17	0.23		0.23
22.95	10.89		29.03	12.01		37.35	12.98	
		33.84			41.04			50.33

Source: Water Resources Planning Study of the Farmington Valley, The Travelers Research Center, Hartford, Connecticut, 1965.

(3) Farmington. Municipal plant, secondary treatment, 1.20 mgd capacity, built in 1962 . Effluent BOD data:

1/20/64	15 mg/l	1/31/67	26 mg/l
1/26/65	44 mg/l	10/19/67	37 mg/l
12/13/65	30/mg/l	5/ 1/68	47/mg/l

(4) Bristol. Municipal plant, recently increased in size to 10 mgd capacity. Effluent BOD data:

3/19/64	50 mg/l	1/30/67	40 mg/l
11/ 9/64	68 mg/l	12/ 5/67	65 mg/l
9/20/65	35 mg/l	7/ 1/68	25 mg/l

(5) Plymouth. Municipal plant, secondary treatment, 1.7 mgd capacity. No BOD data.

(6) Windsor Locks (Bradley Field) - two plants, both afford secondary treatment. Plant No. 1 has 2.0 mgd capacity and Plant No. 2 has 0.5 mgd capacity. Effluent BOD data are:

Plant No. 1	5/11/64	30 mg/l	8/8/67	16 mg/l
	4/20/66	18/mg/l	4/4/68	20 mg/l
Plant No. 2	6/17/68	85 mg/l		

(7) Tariffville. Municipal plant, 0.2 mgd capacity, primary treatment. Effluent BOD data:

9/16/64	190 mg/l	3/15/67	91 mg/l

(8) Winsted. Municipal plant, secondary treatment, 1.0 mgd capacity. Effluent BOD data:

8/22/66	55 mg/l	5/1/67	40 mg/l
7/31/67	31 mg/l	4/2/68	54 mg/l

A 1.25 mgd plant at Windsor for the Combustion Engineering Corporation also discharges into the Farmington River below Rainbow Dam (near the mouth of the River). The present loading is 0.2 mgd. The plant affords secondary treatment. The effluent BOD in a sample taken 4/27/66 was 17 mg/l.

Four industrial plants in the Simsbury-Avon area discharge a total of 50,000 gallons daily. Their effect is neglected. Also the Simsbury Sewer Association plant of 85,000 gallons daily capacity is neglected.

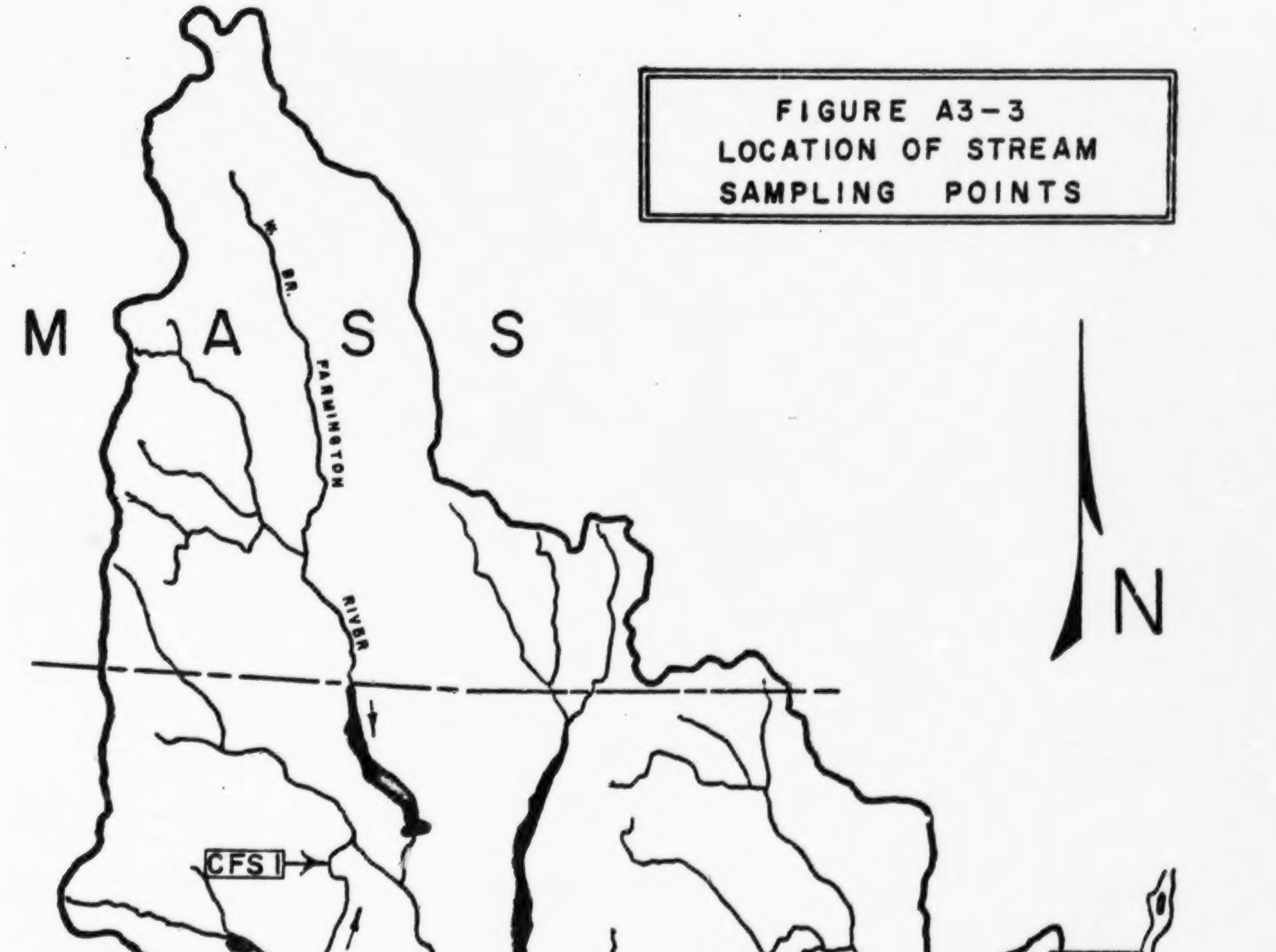
FIGURE A3-3
LOCATION OF STREAM
SAMPLING POINTS
MASS
CONN
N
W. BR.
FARMINGTON
RIVER
CFS 1
STILL RIVER
FARMINGTON
RIVER
CFS 2
CFS 3
RIVER
FARMINGTON
CFS 4
RIVER
CFS 5
CONNECTICUT
CFS 6
PEQUABUCK
RIVER
0 1 2 3 4
SCALE IN
MILES

A3.8 Stream Quality Data.

The Water Resources Commission, State of Connecticut, has established six sampling locations in the Farmington Basin and it samples the river at these locations at infrequent intervals to obtain water quality data. The location of each station is described below and is shown in Figure A3-3. The data, with dates of sampling, are tabulated in Table A3-24.

Station CFS-1, Still River between the Winsted Sewage Treatment Plant and the confluence of Still River and Sandy Brook.

Station CFS-2 Main stem, Farmington River at Collinsville.

Station CFS-3 Main stem, Farmington River at the Highway US-44 bridge at Avon.

Station CFS-4 Main stem, Farmington River at the Rainbow Dam.

Station CFS-5 Main stem, Farmington River at Mill Brook, about 2 1/2 miles above the confluence of the Farmington and Connecticut Rivers.

Station CFS-6 Pequabuck River a short distance upstream of its confluence with the Farmington River, near Farmington, Connecticut.

A3.9 MDC Water Demand.

The daily demand for water from the Barkhamsted and Nepaug Reservoirs was 51.5 million gallons in 1967, 52.5 million gallons in 1968 and 54 million gallons in 1968. It is expected that the 82 million gallons per day firm capacity will supply MDC until 1978. The plan then is to augment this supply by diversion from the Colebrook-Goodwin system, estimated to meet the demands until the year 2000.

TABLE A3-23
Assumed Irrigation Demand
(in cubic feet per second)

Week	Reach 5	Reach 6	Reach 7
22	0	0	1.0
23	0	0	1.0
24	3.7	3.7	5.0
25	3.7	3.7	5.0
26	3.7	3.7	5.0
27	3.7	3.7	5.0
28	3.7	3.7	5.0
29	3.7	3.7	5.0
30	3.7	3.7	5.0
31	3.7	3.7	5.0
32	3.7	3.7	5.0
33	0	0	1.3
34	0	0	1.3
35	0	0	1.0
36	0	0	1.0

TABLE A3-24
Water Quality Data

Station	Date	BOD, mg/l	DO, mg/l	Temp.°C
CFS-1	8/ 9/67	5.8	4.25	24.8
	10/ 4/67	2.05	4.45	18.5
	6/10/68	3.0	6.4	19.0
CFS-2	8 9/67	2.5	8.6	23
	10/ 4/67	0.7	9.1	17.8
	5/27/68	1.0	9.3	15.5
CFS-3	8/17/67	2.8	8.1	25
	10/ 2/67	2.1	7.6	15.5
	5/27/68	1.6	8.3	15.3
CFS-4	8/16/67	3.7	12.7	28
	9/20/67	1.6	13.0	22.3
	6/17/68	1.0	8.4	19
CFS-5	8/16/67	1.15	7.9	24.5
	9/20/67	1.0	8.2	20.8
	6/17/68	1.2	8.0	18.5
CFS-6	8/ 9/67	9.7	1.9	23.3
	10/ 4/67	4.05	1.8	18.3
	5/27/68	5.0	4.5	14.3

Source: Water Resources Commission, State of Connecticut, Hartford, Connecticut.

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APPENDIX A4
USER'S INSTRUCTIONS

The material contained in this Appendix is intended to be used by those making the detailed preparations for use of the water quality simulation model and the optimization model. This Appendix has been prepared in sufficient detail to allow its removal from the report to serve as a manual for guidance in the preparation of program inputs and control statements. Output content and format are also described in detail. The information is separated into program components and each component is described in the following pages. Figure A4-1 is an overview of the various programs showing the relationship between them.

A4.1 CHKDATA Streamflow Data Edit Program

A4.1.1 Purpose.

The CHKDATA program is designed to read daily raw streamflow data of the type available from the U. S. Geological Survey (USGS) for their stream gaging stations and to prepare these data for use in a synthetic streamflow data generator. The program reads the raw data either from punched cards or from magnetic tape, checking as it does so to assure that the data being read are in the proper sequence, for the proper station and for the years of record desired.

When all desired raw data have been read in the proper order, the program searches for missing data. If more than one month of consecutive data are missing, the program calls exit and the operator must reschedule the data sequence to remove that period from use. If fewer than one month of data are missing, the program fills the missing data so that the output is a complete set of daily streamflow records.

Missing data are filled by noting the day of the year of the missing data item and its station number. Then a search is made of all other years for the data item corresponding to the one which is missing. The mean and standard deviation are computed from these data items and the missing item is computed according to the formula:

$$Q_i = \mu_i + \sigma_i \, r_i \; . \; . \; . \; . \; . \; . \; . \; . \; [\text{Eq. A4.1}]$$

where:

Q_i = the computed missing data item.
μ_i = the mean of all data available corresponding to the day.
σ_i = the standard deviation of all data available corresponding to the day and station of the missing data item.
r_i = a standard normal random deviate.

Having read, edited and filled the data traces, the program then: (1) outputs daily flow data, (2) computes and outputs the mean flow for a weekly interval, or (3) computes and outputs the mean flow for a monthly interval, depending upon the value given the control variable, ITIME.

A4.1.2 Program Components

The CHKDATA program consists of the subroutines, listed with their lengths in bytes, as follows:

CHKDATA MAIN	93510	AVM	706
INPUT	1288	AVW	1418
INCARD	1640	RAN	832
FILL	1332	RANDU	448

The program length for functions is 19,776 bytes. The total program length is 122,750 bytes.

A4.1.2.1 CHKDATA MAIN

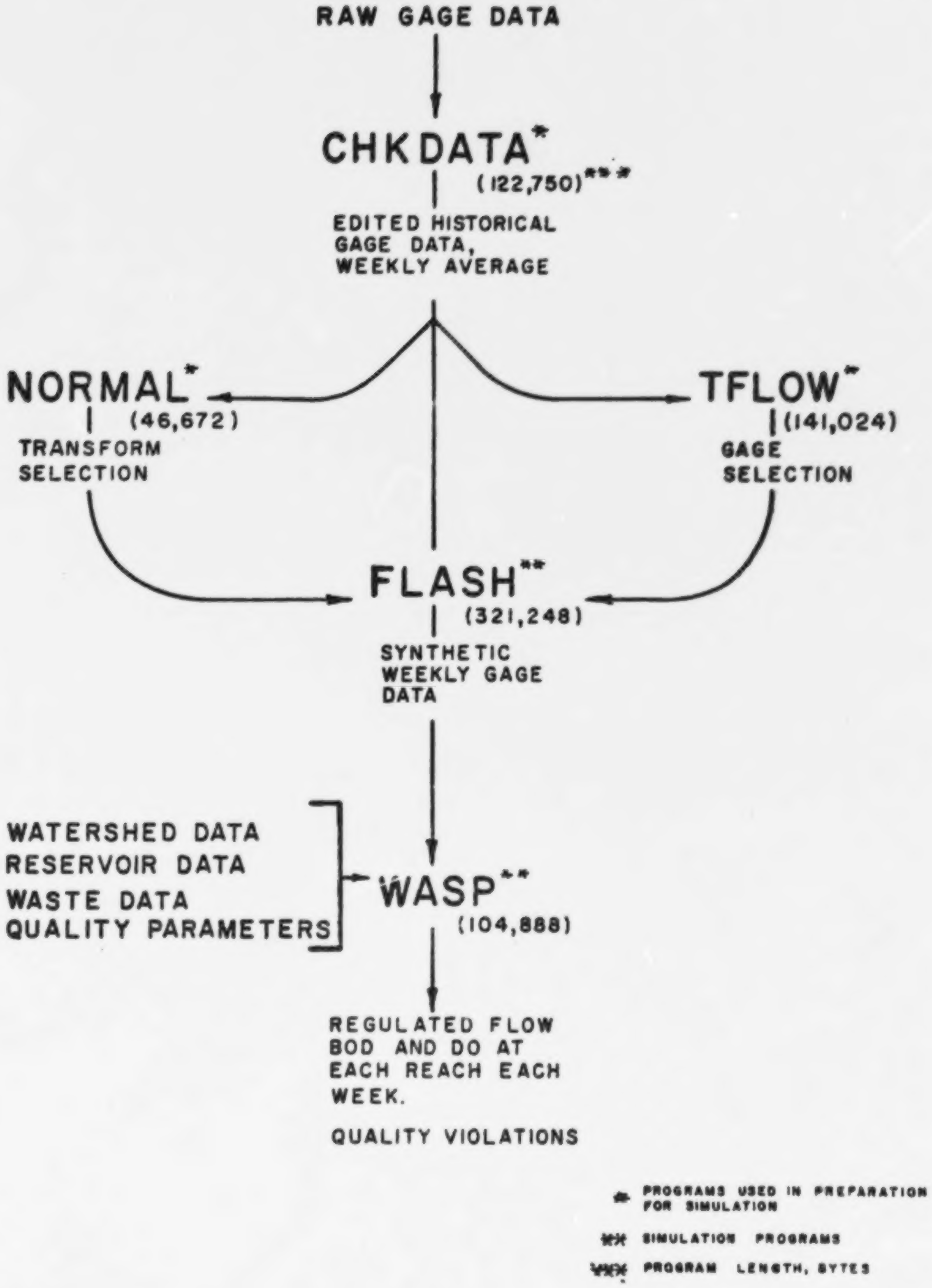
This program component reads in the program controlling information, coordinates the work of the other subroutines, makes certain checks and outputs the edited data. The program controlling information is supplied on two cards (see A4.1.3, Program Input and Output, which follows). The controlling information establishes the number of stations and each station number, the years in which the data for each station begin and end, the averaging interval and the mode of input and output.

After indexing the stations, the subroutines INPUT and INCARD are called. These subroutines read the data one month at a time, perform certain checks described below, and return for one month to CHKDATA MAIN. Checks for the station number and beginning and ending years then are made on the month's data just read. Thus, the data are checked for proper order by station, week, month and year. Notice of deviation from proper order is printed or the program is exited.

When the reading of data is completed, the subroutine FILL is called to fill in missing data. FILL is described below.

Finally, depending upon the controlling information supplied, CHKDATA MAIN outputs the edited streamflow data. The data are supplied in printed form, on punched cards or on magnetic tape. Daily flows are outputed without further change. Weekly flows are outputed as the average of the daily flows for the given week and monthly flows are the average of the daily flows for one month.

FIGURE A4-1
PROGRAM RELATIONSHIP



Normally, USGS data are in units of cubic feet per second and if this unit is used, the output will be in cubic feet per second units for average flow rate for the day, week or month, whichever is appropriate.

A4.1.2.2 Subroutine INPUT

This subroutine transmits the call to read data from CHKDATA MAIN to subroutine INCARD and the data read back to CHKDATA MAIN. Before the data are sent to CHKDATA MAIN, this subroutine checks to determine that the data read are of the proper station, month and year. The data are transmitted in a 1 dimensional array, one month's data at a time.

A4.1.2.3 Subroutine INCARD

INCARD actually reads the supplied raw historical data. The subroutine checks to determine if the cards (or tape data) are in the proper weekly sequence. Data for one month at a time are read, checked, and transmitted to subroutine INPUT.

A4.1.2.4 Subroutine FILL

After the data are all read and checked, subroutine FILL is called to determine if there are any missing data points. If 30 or more consecutive daily data points are missing for any station, exit is called and the operator must make an adjustment in the data years used. If scattered data points are missing, the subroutine fills the missing points one at a time, as described above.

A4.1.2.5 Subroutines RAN and RANDU

These subroutines generate standard normal deviates (mean = 0 and variance = 1) for use in subroutine FILL.

A4.1.2.6 Subroutines AVW and AVM

Subroutines AVW and AVM compute the weekly average and monthly average flows from the edited and filled daily data, according to the control number entered for the variable ITIME.

A4.1.3 Program Input

Input is required as follows:

A4.1.3.1 For CHKDATA MAIN

Card # 1 (9I5)* IYRI(I) = The starting year for station (I).
IYR2(I) = The ending year for station (I).
NNSTA = The number of stations.
ITAPE = 4 for data input on tape.
= 5 for data input on cards.
ISTART = A starting random number.
ITIME = 1 for daily flow output.

*Data fields on card.

= 2 for average weekly flow output.
= 3 for average monthly flow output.
IPRINT = 0 if output is not to be printed.
= 1 if output is to be printed.
IPUNCH = 0 if output is not to be punched.
= 1 if output is to be punched.
NTAPE = 0 if output is not to be taped.
= 1 if output is to be taped.

Card # 2 (10I8) (ISTA(I), I = 1, NNSTA) indicates the set of station numbers for the stations for which data are to be read. These station numbers must correspond to the station numbers of the raw data set. The number of stations, NNSTA, may be fewer than the number of stations contained in the raw data but the order in which stations are read must be the same as in the raw data set.

A4.1.3.2 For Subroutine INCARD

A set of input streamflow data cards or card images on a 7-trace tape is required. The data card format is:

(I8, I4, I2, I1, 8F6.2)

where,
I8 = the station number.
I4 = the calendar year.
I2 = an index, 1, 2, 3, or 4, which identifies the number of the card in the month.
8F6.2= The daily streamflow values for 8 consecutive days. The first card contains flow data for the first 8 days, the second for the 9th through the 16th days, the third for the 17th through the 24th days and the fourth for the 25th day through the last day of the month, 28, 29, 30 or 31, as appropriate.

A 4.1.4 Program Output

All output is produced by CHKDATA MAIN and is described as follows: (1) Data output may be printed and/or punched on cards, depending upon the control information furnished. The format of the output depends upon the control information supplied for variable ITIME. For daily streamflow data - a set of 4 cards or lines per month is produced and the format is the same as the input data format. For average weekly streamflow data - one card or line is produced for each month. The format is:

(I8, I4, I2, 4F8.2)

where,
I8 = the station number.
I4 = the calendar year.
I2 = the month.
4F8.2= four average weekly flows for the month.

For monthly streamflow data - a set of 2 cards or lines for each year is produced. The format is:

(I8, I4, 6F8.2/12X, 6F8.2)

where,
I8 = the station number.
I4 = the calendar year.
6F8.2= the six monthly average flows.

(2) Data output may be written on magnetic tape, depending upon the control information furnished as variable ITAPE. The format of the output depends upon the control information supplied for variable ITIME. For daily streamflow data - the format will be the same as that of the raw data tape, as described previously (A4.1.3.2). For average weekly streamflow data - a binary tape is written for each station. Each record contains ISTA, IYR1, IYR2, (AV(I,J), I = 1, 48), J = 1, NYR) where:

ISTA = the station number.
IYR1 = the beginning calendar year of data.
IYR2 = the ending calendar year of data.
AV(I,J) = the weekly average flow for week I and year J.
NYR = number of years of data = IYR2-IYR1+1.

For monthly streamflow data - a binary tape is written for each year at each station. Each record contains ISTA, IYR, (AV(I), I = 1, 12) where:

ISTA = the station number.
IYR = the calendar year.
AV(I)= the monthly average flow for month I.

A 4.1.5 Definition of Program Variables

Following is a list of variables used in CHKDATA and a brief definition of each:

AV(I,J)	Average flow during Ith week of Jth yesr.
AV2(I,J)	Average flow during Ith month of Jth year.
FN	Number of data items.
ICARD(I)	Card sequence number.
ICOUNT	Convenience Index.

IDAY	Day counting variable.
IISTA	Convenience station number, for checking.
IIYR1	Convenience starting year, for checking.
IIYR2	Convenience final year, for checking.
IPUNCH	Control variable-output, = 0 for no data punched. = 1 for data to be punched.
ISTART	Starting random number.
ISTA(I)	Identifying number, Ith station.
ITAPE	Control variable-input, = 5 data on cards. = 4 data on tape.
ITIME	Control variable-input, = 1 for daily flows. = 2 for weekly average flows. = 3 for monthly average flows.
IWEEK	Week counting variable.
IWEEK2	Month counting variable.
IYEAR	Year counting variable.
IYR1(I)	Starting year for data,Ith station.
IYR2(I)	Final year for data, Ith station.
JPREV	Convenience station number, for checking.
JYR	Convenience number of years for checking.
KMO(I)	Identifying number, Ith month.
KSTA(I)	Equals ISTA (I).
KYR(I)	Equals IYR (I).
L	Week index.
LMO	Number of months of data, computed.
MONTH	The current month.
ND(I)	Number of days in month I.
NEVEN	Convenience number, random number generator.
NMO	Number of months of data counted.
NNSTA	The number of stations.
NNYR	Number of years of data.
WODD	Convenience number, random number generator.
NSTA	Current station number.
NTAPE	Control variable-output, = 0 for no data on tape. = 1 for data output on tape.
Q(I, J, K)	Gage flow Ith month, Jth day, Kth year.
QMEAN	Mean flow.
QNEW	Computed flow to fill missing data point.
QSTD	Standard deviation of flow.
R1	Convenience variable, random number generator.
R2	Convenience variable, random number generator.
RN(I)	Random number, Ith time frame.

SUM	Sum of second term.
S(I,J)	Flow for Jth day of Ith week, current month.
YFL	Convenience number, random number generator.
Z(I)	Temporary storage, flow on Ith day in current month.
Z(L)	Temporary linear storage for one month's flow.

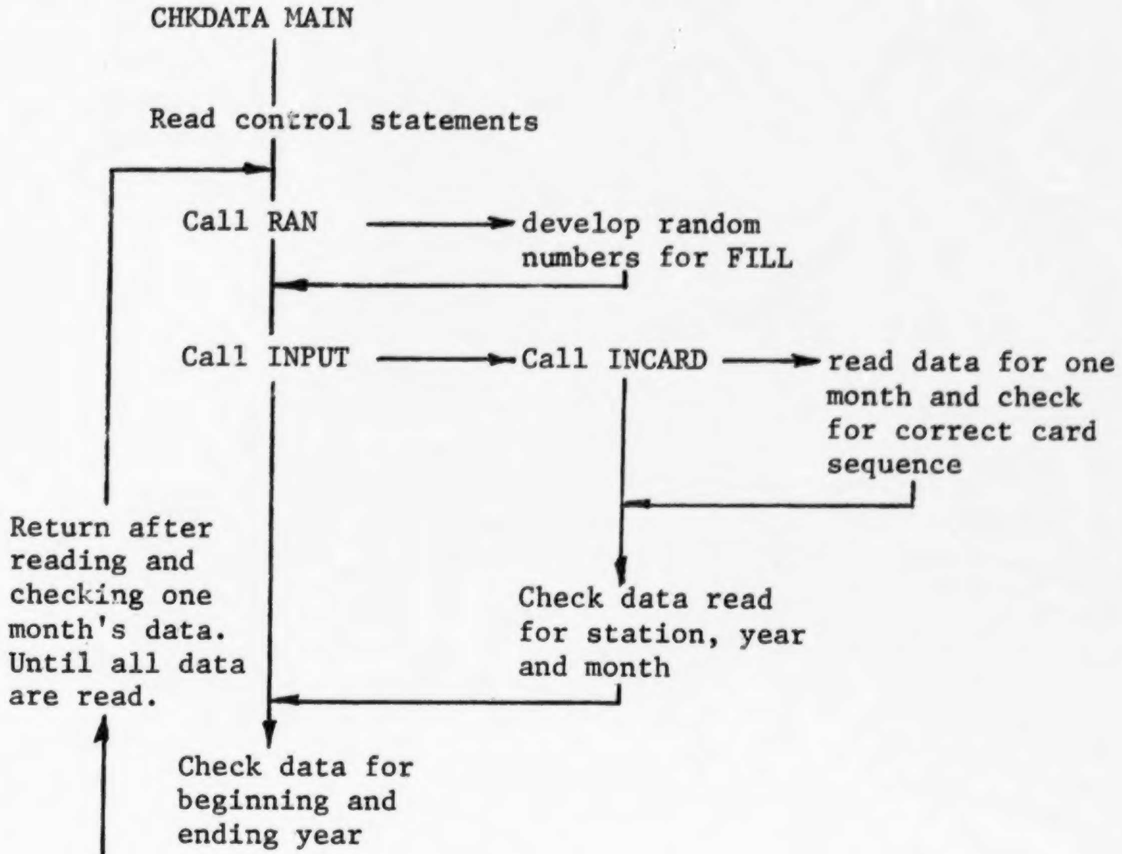
A4.1.6 Program Logic

Figure A4-2 is a diagram showing program logic for CHKDATA.

A4.1.7 Program Coding

The program coding for CHKDATA follows.

FIGURE A4-2
PROGRAM LOGIC - CHKDATA


CHKDATA MAIN
Read control statements
Call RAN
develop random numbers for FILL
Call INPUT
Call INCARD
read data for one month and check for correct card sequence
Return after reading and checking one month's data. Until all data are read.
Check data read for station, year and month
Check data for beginning and ending year
Call FILL
Check for missing data and compute filling data.
−
If more than 30 consecutive data points are missing
+
call exit
Output edited and filled daily flows, printed, on cards or on tape.
Call AVW
Compute average weekly flows and output, printed, on cards or on tape
Call AVM
Compute average monthly flows and output, printed, on cards or on tape
Check to determine if all data called for have been read
END

```
//CHKDATA  JOB (1143,47,009,09,9999),'ALEMAN                  ',CLASS
                                                                 =M
// EXEC F4GCXM     FORT G COMPILE (NODECK), EXECUTE, CLASS M
//FORT.SYSIN DD *
C     ND(I)=NUMBER OF DAYS IN MONTH I
C     Q(I,J,K)=FLOW IN ITH MONTH, JTH DAY, KTH YEAR
C     ISTA(I)=IDENTIFYING NUMBER OF ITH STATION
C     Z(I)=TEMPORARY STORAGE FOR FLOW ON ITH DAY OF A GIVEN MONTH
                                                                OF
C           INPUTTED DATA
C     AV(I,J)=AVERAGE FLOW DURING ITH WEEK OF JTH YEAR
C     AV2(I,J)=AVERAGE FLOW DURING ITH MONTH OF JTH YEAR
C     IYR1(I)=STARTING YEAR OF DATA FOR ITH STATION
C     IYR2(I)=FINAL YEAR OF DATA FOR ITH STATION
C     NNSTA=NUMBER OF STATIONS
C     ITAPE=TAPE NUMBER FOR INPUT DATA
C          =5 IF DATA ARE PUNCHED ON CARDS
C          =4 IF DATA ARE ON TAPE
C     ISTART=STARTING RANDOM NUMBER
C     ITIME INDICATES WHETHER DAILY,WEEKLY OR MONTHLY FLOWS ARE
C           DESIRED
C          =1 IF DAILY FLOWS ARE DESIRED
C          =2 IF WEEKLY FLOWS ARE DESIRED
C          =3 IF MONTHLY FLOWS ARE DESIRED
C     IPRINT=0 IF CHECKED DATA ARE NOT TO BE PRINTED
C           =1 IF CHECKED DATA ARE TO BE PRINTED
C     IPUNCH=0 IF CHECKED DATA ARE NOT TO BE PUNCHED
C           =1 IF CHECKED DATA ARE TO BE PUNCHED
C     NTAPE=0 IF NO OUTPUT DATA TAPE IS DESIRED
C          =1 IF AN OUTPUT DATA TAPE IS DESIRED
      DIMENSION ND(12),Q(12,32,50),ISTA(100),Z(32)
     1          ,AV(48,50),AV2(12,50),IYR1(100),IYR2(100)
      DIMENSION IISTA(100)
      ICOUNT=0
      ND(1)=31
      ND(2)=28
      ND(3)=31
      ND(4)=30
      ND(5)=31
      ND(6)=30
      ND(7)=31
      ND(8)=31
      ND(9)=30
      ND(10)=31
      ND(11)=30
      ND(12)=31
      READ(5,5001)NNSTA,ITAPE,ISTART,ITIME,IPRINT,IPUNCH,NTAPE
      IF (NTAPE.NE.0)NTAPE=1
      IF(ITAPE.NE.5)ITAPE=4
      IF(ITAPE.EQ.4)REWIND 4
      DO 1 I=1,10
      CALL RAN(ISTART,32,Z)
    1 CONTINUE
 5001 FORMAT(7I5)
      READ (5,5002) (ISTA(I),I=1,NNSTA)
 5002 FORMAT (10I8)
      READ(5,5003)(IYR1(I),IYR2(I),I=1,NNSTA)
 5003 FORMAT(16I5)
      WRITE(6,6001) NNSTA,ITAPE,NTAPE,(I,ISTA(I),IYR1(I),IYR2(I),
                                                              I=1,
     1NNSTA)
 6001 FORMAT(1H1,'STREAMFLOW DATA EDIT PROGRAM'/I3,' STATIONS'/
     1        ' INPUT DATA ON TAPE ',
     2        I2/' OUTPUT DATA ON TAPE ',I2//
     3        (' STATION(',I2,') = ',I8,'    STARTING YEAR= ',I4,
     4        '     FINAL YEAR= ',I4))
      ISTART = ISTART + 3
C
C  READ DATA FOR ONE MONTH
C
    5 CALL INPUT (NSTA,NYR,MONTH,Z,ITAPE)
    6 IF (NSTA) 1000,1000,10
   10 I = 1
   11 IF(NSTA.NE.ISTA(I))GO TO 200
      ICOUNT=ICOUNT+1
      IISTA(ICOUNT)=ISTA(I)
      NNYR=IYR2(I)-IYR1(I)+1
      LMO = 12*NNYR
      IIYR1=IYR1(I)
      IIYR2=IYR2(I)
      GO TO 20
  200 IF(I.GE.NNSTA)GO TO 5
      I=I+1
      GO TO 11
   20 JPREV = NSTA
      NMO=0
      GO TO 32
   21 JYR=NYR-IIYR1+1
   25 DO 30 I=1,32
   30 Q(MONTH,I,JYR) = Z(I)
   31 CALL INPUT(NSTA,NYR,MONTH,Z,ITAPE)
      IF (NSTA - JPREV) 40,32,40
   32 IF(NYR-IIYR1)31,33,33
   33 IF(NYR-IIYR2)34,34,31
   34 NMO = NMO + 1
      GO TO 21
 1000 IF(ITAPE.EQ.4)REWIND 4
      IF(NTAPE.EQ.0)RETURN
      END FILE 1
      REWIND 1
      RETURN
C
```

```
C   CHECK FOR MISSING DATA
C
    40 IF(NMO-LMO)41,45,45
    41 WRITE (6,6041) NMO,LMO,JPREV
  6041 FORMAT (' ONLY',I4,' OUT OF',I4,' MONTHS OF DATA ARE PRESEN
                                                             T FOR S
      1TATION',I9)
    45 CALL FILL (NNYR,ND,Q,ISTART,JPREV)
C
C   PUNCH DATA CARDS FOR STATION JPREV
C
       GO TO (60,61,62),ITIME
    60 WRITE(6,8002)
  8002 FORMAT(1H0,'DAILY FLOWS')
       DO 50 I=1,NNYR
       NYR=IIYR1+I-1
       DO 50 J=1,12
       L = 1
       M = 8
       DO 51 K=1,4
       IF(IPRINT.EQ.0.AND.IPUNCH.EQ.0.AND.NTAPE.EQ.0)GO TO 300
       IF(IPRINT.EQ.0)GO TO 52
    54 WRITE(6,6051)JPREV,NYR,J,K,(Q(J,N,I),N=L,M)
  6051 FORMAT (I8,I4,I2,I1,8F8.2)
    52 IF(IPUNCH.EQ.0)GO TO 53
       WRITE (7,6051) JPREV,NYR,J,K,(Q(J,N,I),N=L,M)
    53 IF(NTAPE)301,300,301
   301 WRITE(NTAPE,6051)JPREV,NYR,J,K,(Q(J,N,I),N=L,M)
   300 L=L+8
    51 M = M + 8
    50 CONTINUE
       GO TO 110
C      COMPUTE WEEKLY AVERAGES
    61 CALL AVW(Q,NNYR,AV)
       WRITE(6,8001)
  8001 FORMAT(1H0,'WEEKLY FLOWS')
       IF(IPRINT.EQ.0.AND.IPUNCH.EQ.0.AND.NTAPE.EQ.0)GO TO 110
       DO 400 I=1,NNYR
       IWEEK=1
       IWEEK2=4
       NYR=IIYR1+I-1
       DO 400 J=1,12
       IF(IPRINT.EQ.0)GO TO 401
       WRITE(6,6052)JPREV,NYR,J,(AV(II,I),II=IWEEK,IWEEK2)
  6052 FORMAT(1H ,I8,I4,I2,4F8.2)
   401 IF(IPUNCH.EQ.0)GO TO 402
       WRITE(7,6053)JPREV,NYR,J,(AV(II,I),II=IWEEK,IWEEK2)
  6053 FORMAT(I8,I4,I2,4F8.2)
   402 IWEEK=IWEEK+4
   400 IWEEK2=IWEEK2+4
       NYR=IIYR2-IIYR1+1
      IF(NTAPE.EQ.0)GO TO 110
      WRITE(NTAPE)JPREV,IIYR1,IIYR2,((AV(II,I),II=1,48),I=1,NYR)
      GO TO 110
C     COMPUTE MONTHLY AVERAGES
   62 CALL AVM(Q,NNYR,AV2 )
      WRITE(6,8000)
 8000 FORMAT(1H0,'MONTHLY FLOWS')
      IF(IPRINT.EQ.0.AND.IPUNCH.EQ.0.AND.NTAPE.EQ.0)GO TO 110
      DO 500 I=1,NNYR
      NYR=IIYR1+I-1
      IF(IPRINT.EQ.0)GO TO 501
      WRITE(6,6054)JPREV,NYR,(AV2(II,I),II=1,12)
 6054 FORMAT(1H ,I8,I4,6F8.2/1H ,12X,6F8.2)
  501 IF (IPUNCH.EQ.0)GO TO 502
      WRITE(7,6055)JPREV,NYR,(AV2(II,I),II=1,12)
 6055 FORMAT(I8,I4,6F8.2/12X,6F8.2)
  502 IF(NTAPE.EQ.0)GO TO 500
      WRITE(NTAPE)JPREV,NYR,(AV2(II,I),II=1,12)
  500 CONTINUE
  110 IF(NSTA)100,100,6
  100 DO 901 I=1,NNSTA
      DO 900 II=1,ICOUNT
      IF(ISTA(I).EQ.IISTA(II))GO TO 901
  900 CONTINUE
      WRITE(6,903)ISTA(I)
  903 FORMAT(1H ,'LOOKED FOR DATA FOR STATION',I8,' BUT COULDN T
                                                              FIND IT
     1')
  901 CONTINUE
      WRITE(6,101)
  101 FORMAT('     ********   THE END   ********     ')
      GO TO 1000
      END
      SUBROUTINE FILL (NNYR,ND,Q,ISTART,NSTA)
      DIMENSION ND(12),Q(12,32,50),RN(2)
      DO 20 J=1,12
      NDAY = ND(J)
      DO 20 K=1,NDAY
      DO 20 I=1,NNYR
      IF (Q(J,K,I)) 25,25,20
   25 N = 0
      QMEAN = 0.
      QSTD = 0.
      DO 30 L=1,NNYR
      IF (Q(J,K,L)) 30,30,31
   31 N = N + 1
      QMEAN = QMEAN + Q(J,K,L)
      QSTD = QSTD + Q(J,K,L) **2
   30 CONTINUE
      IF (N-2) 33,34,34
   33 WRITE (6,6033) NSTA,J    ,K
```

```
6033 FORMAT (' INSUFFICIENT DATA FOR STATION',I9,'  MONTH',I3,'
                                                              DAY',
    1        I3)
  34 FN = N
     QMEAN = QMEAN/FN
     QSTD = SQRT((QSTD- FN*QMEAN**2)/(FN-1.))
     CALL RAN (ISTART,2,RN)
   7 FORMAT(1H ,'RN(1)= ',F10.4/
    11H ,'QMEAN= ',F14.2,'    QSTD= ',F14.2)
     QNEW =      QMEAN + RN(1)*QSTD
     WRITE (6,6020) QNEW,J,K,I,NSTA,Q(J,K,I)
6020 FORMAT(' NEW FLOW (',F8.2,')  GENERATED FOR ',I2,'/',I2,'/'
                                                           ,I2,' F
    1OR STATION',I9,' IN PLACE OF ORIGINAL VALUE (',F8.2,')')
     WRITE(6,7)RN(1),QMEAN,QSTD
     Q(J,K,I) = QNEW
  20 CONTINUE
     RETURN
     END
     SUBROUTINE INPUT (NSTA,NYR,MONTH,Z,ITAPE)
     DIMENSION  Z(32),KSTA(4),KYR(4),KMO(4),ICARD(4),X(4,8)
C
C  READ DAILY STREAMFLOW DATA FOR ONE MONTH
C
  34 CALL INCARD(KSTA,KYR,KMO,ICARD,X,ITAPE)
  10 DO 11 I=1,4
  12 IF (KYR(I) - KYR(1)) 23,13,23
  13 IF (KMO(I) - KMO(1)) 24,14,24
  14 IF (KSTA(I) - KSTA(1)) 21,11,21
  21 WRITE (6,6021)
     GO TO 25
  23 WRITE (6,6023)
     GO TO 25
  24 WRITE (6,6024)
  25 DO 26 K=1,4
  26 WRITE(6,6026) KSTA(K),KYR(K),KMO(K),ICARD(K),(X(K,J),J=1,8)
     GO TO 34
6026 FORMAT(1X,I9,I5,I3,I2,8F9.2)
6021 FORMAT (' STATION IDENTIFICATION NOT CONSISTENT')
6023 FORMAT (' YEAR NOT CONSTANT')
6024 FORMAT (' MONTH NOT CONSTANT')
  11 CONTINUE
  33 NSTA=KSTA(1)
     NYR = KYR(1)
     MONTH = KMO(1)
     K = 0
     DO 30 I=1,4
     DO 31 J=1,8
     L = J + K
  31 Z(L) = X(I,J)
  30 K = K + 8
```

```
      RETURN
      END
      SUBROUTINE RAN(IX,N,RN)
      DIMENSION RN(2000)
      NEVEN=N
      NODD=(N/2)*2
      IF(N-NODD)100,101,100
100   NEVEN=NEVEN+1
101   IF(NEVEN-2000)102,102,103
103   NEVEN=2000
102   DO 104 I=1,NEVEN
        CALL RANDU(IX,IY,YFL)
104   RN(I)=YFL
      DO 105 I=1,NEVEN,2
      R1=RN(I)
      R2=RN(I+1)
      RN(I)=SQRT ((-2.)*ALOG(R1))*COS (6.28*R2)
105   RN(I+1)=SQRT ((-2.0)*ALOG(R1))*SIN (6.28*R2)
C     FOR TRIAL PURPOSES
      N=NEVEN
      RETURN
      END
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      IX=IY
      RETURN
      END
      SUBROUTINE INCARD(KSTA,KYR,KMO,ICARD,X,ITAPE)
      DIMENSION KSTA(4),KYR(4),KMO(4),ICARD(4),X(4,8)
      DO 1 I=1,4
      READ(ITAPE,5002) KSTA(I),KYR(I),KMO(I),ICARD(I),(X(I,J),J=1
                                                               ,8)
 5002 FORMAT(I8,I4,I2,I1,8F8.2)
      IF(ICARD(I)-I)3,1,3
    3 WRITE(6,6022)
 6022 FORMAT (' CARD SEQUENCE NOT CORRECT')
      WRITE(6,6000)I,KSTA(I),KYR(I),KMO(I),ICARD(I),(X(I,J),J=1,8
                                                               )
 6000 FORMAT(1H ,'I= ',I5/1X,I9,I5,I3,I2,8F9.2)
      IF(ICARD(I).NE.1)GO TO  1
    4 DO 5 II=2,4
    8 KSTA(1)=KSTA(I)
      KYR(1)=KYR(I)
      KMO(1)=KMO(I)
      ICARD(1)=ICARD(I)
      DO 9 J=1,8
      X(1,J)=X(I,J)
    9 CONTINUE
      READ(ITAPE,5002)KSTA(II),KYR(II),KMO(II),ICARD(II),(X(II,J)
                                                              ,J=1,8)
      IF(ICARD(II)-II)6,5,6
    6 WRITE(6,6000)II,KSTA(II),KYR(II),KMO(II),
     1ICARD(II),(X(II,J),J=1,8)
      GO TO 1
    5 CONTINUE
      GO TO 7
    1 CONTINUE
    7 RETURN
      END
      SUBROUTINE AVW(Q,NYRS,AV)
C     NDAY=NUMBER OF DAYS IN A GIVEN MONTH
C     NYRS=NUMBER OF YEARS OF DATA TO BE READ IN
C     ICOUNT COUNTS 48 WEEKS IN A YEAR
      DIMENSION AV(48,50),Q(12,32,50),ND(12)
      ND(1)=31
      ND(2)=28
      ND(3)=31
      ND(4)=30
      ND(5)=31
      ND(6)=30
      ND(7)=31
      ND(8)=31
      ND(9)=30
      ND(10)=31
      ND(11)=30
      ND(12)=31
      DO 1 IYEAR=1,NYRS
      ICOUNT=0
      DO 1 MONTH=1,12
      NDAY=ND(MONTH)
   20 IF(NDAY-30)28,30,31
C     IT IS FEBRUARY
   28 L=0
      DO 13 IWEEK=1,4
      ICOUNT=ICOUNT+1
      SUM=0
      DO 14 IDAY=1,7
      L=L+1
   14 SUM=SUM+Q(MONTH,L,IYEAR)
   13 AV(ICOUNT,IYEAR)=SUM/7.
      GO TO 1
C     IT IS A THIRTY DAY MONTH
   30 L=0
      DO 15 IWEEK=1,2
      ICOUNT=ICOUNT+1
      SUM=0
      DO 16 IDAY=1,8
      L=L+1
   16 SUM=SUM+Q(MONTH,L,IYEAR)
   15 AV(ICOUNT,IYEAR)=SUM/8.
      DO 17 IWEEK=1,2
      ICOUNT=ICOUNT+1
      SUM=0
      DO 18 IDAY=1,7
      L=L+1
   18 SUM=SUM+Q(MONTH,L,IYEAR)
   17 AV(ICOUNT,IYEAR)=SUM/7.
      GO TO 1
   31 L=0
      DO 19 IWEEK=1,3
      ICOUNT=ICOUNT+1
      SUM=0
      DO 50 IDAY=1,8
      L=L+1
   50 SUM=SUM+Q(MONTH,L,IYEAR)
   19 AV(ICOUNT,IYEAR)=SUM/8.
      ICOUNT=ICOUNT+1
      SUM=0
      DO 21 IDAY=1,7
      L=L+1
   21 SUM=SUM+Q(MONTH,L,IYEAR)
      AV(ICOUNT,IYEAR)=SUM/7.
    1 CONTINUE
      RETURN
          END
      SUBROUTINE AVM(Q,NYRS,AV)
C     NDAY=NUMBER OF DAYS IN A GIVEN MONTH
C     NYRS=NUMBER OF YEARS OF DATA TO BE READ IN
C     ICOUNT COUNTS 12 MONTHS IN A YEAR
      DIMENSION AV(12,50),Q(12,32,50)
     1            ,ND(12)
      ND(1)=31
      ND(2)=28
      ND(3)=31
      ND(4)=30
      ND(5)=31
      ND(6)=30
      ND(7)=31
      ND(8)=31
      ND(9)=30
      ND(10)=31
      ND(11)=30
      ND(12)=31
      DO 1 IYEAR=1,NYRS
      ICOUNT=0
      DO 1 MONTH=1,12
      NDAY=ND(MONTH)
      ICOUNT =ICOUNT+1
      SUM=0
   20 DO 13 IDAY=1,NDAY
   13 SUM=SUM+Q(MONTH,IDAY,IYEAR)
      AV(ICOUNT,IYEAR)=SUM/NDAY
    1 CONTINUE
      RETURN
          END
      SUBROUTINE OUT2
      DIMENSION NL(8)
      DATA NL/0,1,2,3,4,5,6,7/
      WRITE(6,6004)NL
 6004 FORMAT('1',30X,'CORRELATION COEFFICIENTS OF TRANSFORMED HIS
                                                             TORICAL
     1 FLOWS'///1X,'SITE',3X,'MONTH',8(7X,'LAG',I2),3X,'SITE'/)
      RETURN
      END
      SUBROUTINE OUT3(T11,T12,T22,I,II,J)
      DIMENSION T11(4),T12(7),T22(4)
      WRITE(6,6005)I,J,T22,II
      WRITE(6,6005)I,J,T11,II
      WRITE(6,6006)I,J,T12,II
 6005 FORMAT(I4,I7,2X,4G12.5,48X,I6)
 6006 FORMAT(I4,I7,14X,7G12.5,I6)
      RETURN
      END
      SUBROUTINE OUT6(NSITES,GAV,GSD,GSKEW,GCURT)
      DIMENSION GAV(4,8,12),GSD(4,8,12),
     1          GSKEW(4,8,12),GCURT(4,8,12),NL(8),NM(7),T1(4),T2(
                                                            7)
      DATA NM/1,2,3,4,3,2,1/
      DATA NL/0,1,2,3,4,5,6,7/
      WRITE(6,6014)
 6014 FORMAT('1',27X,'STATISTICS OF GENERATED FLOWS')
      WRITE(6,6015)
 6015 FORMAT(///10X,'SITE',3X,'MONTH',3X,'WEEK',6X,'MEAN',5X,
     1          'STD DEV',4X,'SKEWNESS',4X,'KURTOSIS'/)
      DO 10 I=1,NSITES
      DO 10 J=1,12
      DO 10 L=1,4
   10 WRITE(6,6016)I,J,L,GAV(L,I,J),GSD(L,I,J),GSKEW(L,I,J),
     1                 GCURT(L,I,J)
 6016 FORMAT(I13,I7,I8,4F12.5)
      RETURN
      END
      SUBROUTINE OUT1(NSITES,NTRAN,XQAV,XQSD,XSKEW,XCURT,QAV,QSD,
     1                       SKEW,CURT)
      DIMENSION ATRAN(3),XQAV(4,12,8),XQSD(4,12,8),XSKEW(4,12,8),
     1           XCURT(4,12,8),QAV(4,12,8),QSD(4,12,8),SKEW(4,12,
                                                             8),
     2          CURT(4,12,8)
      DATA ATRAN/4H  NO,4H LOG,4HSQRT/
      WRITE(6,6000)
 6000 FORMAT('1',26X,'STATISTICS OF HISTORICAL FLOWS')
      WRITE(6,6001)
 6001 FORMAT(///10X,'SITES',3X,'MONTH',3X,'WEEK',6X,'MEAN',5X,'ST
                                                               D DEV',
     1          4X,'SKEWNESS',4X,'KURTOSIS'/)
      DO 10 I=1,NSITES
      DO 10 J=1,12
      DO 10 L=1,4
   10 WRITE(6,6002)I,J,L,XQAV(L,J,I),XQSD(L,J,I),XSKEW(L,J,I),
     1              XCURT(L,J,I)
      WRITE(6,6003) ATRAN(NTRAN)
 6002 FORMAT(I13,I7,I8,4G12.5)
 6003 FORMAT('1',33X,A4,' TRANSFORMATION'//
     1         22X,'STATISTICS OF TRANSFORMED HISTORICAL FLOWS')
      WRITE(6,6001)
      DO 15 I=1,NSITES
      DO 15 J=1,12
      DO 15 L=1,4
   15 WRITE(6,6002)I,J,L,QAV(L,J,I),QSD(L,J,I),SKEW(L,J,I),CURT(L
                                                               ,J,I)
      RETURN
      END
/*
//GO.FT01F001 DD UNIT=181,VOLUME=SER=XXX,LABEL=(,BLP),DISP=(,PASS
                                                               )
//GO.FT04F001 DD UNIT=180,LABEL=(,BLP),DISP=NEW,
                                                                     X
//               VOLUME=SER=YYY,
                                                                     X
//               DCB=(RECFM=U,BLKSIZE=800,DEN=1,TRTCH=ET)
/*EOF
```

A4.2 Normal

A4.2.1 Purpose

The NORMAL program is designed to test the frequency distribution of the historical gage data to determine if the data are normally distributed in the statistical sense. The program then makes six transformations to change the scale of these data and it tests the resulting frequency distributions to determine if the transformed variable has a normal, or Gaussian distribution. This program is intended to be used in preparation to generate synthetic gage data by FLASH and to simulate stream flows and water quality by WASP. The result of NORMAL should be used to determine the transformation to be selected for subroutine TRANS in the program FLASH.

Many natural random functions have values which are symmetrically distributed about their mean value. Hydrologic functions are natural and random but in almost every case, their frequency distributions are not normal, but, rather, are skewed right. This type of distribution has a preponderance of values less than the mean, but the larger values extend well to the right of the mean. Because many statistical procedures are available for analysis of normal data, advantage is gained if these skewed hydrologic data are converted to a normal distribution. This is possible by making a non-linear transformation of the data. Most hydrologic data can be normalized by considering the logs of the ordinate values, flows or rainfalls, instead of the direct values.

The objective is to maintain the statistical appearance of the historical data in generating the synthetic data. Because the data usually have a skewed distribution, it will be necessary to use a model which preserves the mean, standard deviation and skewness of the historical data, as well as the serial correlation. If the historical data were normal distributed, it would be possible to eliminate the skewness parameter from consideration because the mean and standard deviation completely describe the statistical properties of a normally distributed population. Thus, using data made normal by transformation eliminates the need of a skewness term in the generating equation.

More importantly, in considering the multivariate process there is no general multivariate gamma distribution available (the skewed data approximate a gamma distribution). On the other hand, information readily available about multivariate distributions is based upon the assumption of multivariate normality. This is due to the fact that marginal and conditional distributions derived from multivariate distributions also are normal, as are linear combinations of normal variates. These properties are utilized in the multiple-lag model which is the basis of the flow generator. It is necessary to transform the historical data to render their distribution normal to reduce the error that would otherwise result.

NORMAL reads edited and filled historical data and, then, in a "do loop" taking each station in order, transforms the data, computes the mean, standard deviation and skewness of the transformed data, prepares a histogram to show the distribution of the transformed data, computes the cumulative frequency of the transformed data and compares the cumulative frequency, cell by cell, with the cumulative frequency of a normal distribution. The comparison is made by determining the difference between the cumulative frequency of the transformed historical data and the cumulative frequency of a normal distribution having the same cell bounds.

The test for normality is made using these differences in cumulative frequency values and the Kolmogorov-Smirnov test for goodness of fit. The Kolmogorov-Smirnov test is a non-parametric or distribution free test similar to, but reportedly more restrictive than, the Chi-Square test. The difference in cumulative frequency between the transformed historical data and data obtained from the normal distribution must be less than the tabulated "critical" value. The "critical" values have been tabulated by Massey (9). For a sample size greater than 35 and a level of significance of $\alpha = 0.05$, the "critical" value is given by $1.36/N^{1/2}$ where N is the sample size. If any cell difference exceeds the "critical" value, the hypothesis that the distribution of the historical data is normal is rejected.

As an example, suppose the maximum cell difference of 0.0487 is obtained for a sample of 480. The critical value is $1.36/480^{1/2} =$ 0.062. The hypothesis that the distribution tested is normal can be accepted since $0.0487 < 0.062$. This indicates that in 5 percent of the samples of size 480, the maximum relative deviation between the sample cumulative distribution and the normal cumulative distribution will be at least 0.062.

Note that if the difference in any cell is greater than the critical value, the hypothesis is rejected and the data must be considered non-normal.

The transformations used in the program are of two general forms:

$$(1) \quad q = Q^b$$

$$(2) \quad q = \log (Q+\alpha)$$

where q is the transformed value, Q is the untransformed value, b and α are constants. In the program coding (see A4.2.7), values for b are: +0.25, +0.50 and +0.75; and values for α are 0 (log-normal transform), $+0.25\ \mu$ and $0.50\ \mu$, where μ is the mean value of Q. Note that b=1.00 results in no transformation. If one wishes to make a transformation of a different form, it is an easy matter to change the program coding.

The program is designed to provide the differences in cumulative frequency for a complete set of station data and for a week by week consideration of the data; that is, all data points for week L of the year are considered separately. This allows testing of the complete set of each weekly set of data. If one wishes to bypass the weekly computations and consider only the complete data set, statement number 19 of NORMAL - MAIN must be "GO TO 51".

The program computes the cumulative frequency, for each cell limit, for each station and transformation and computes the difference between that cumulative frequency and the normal cumulative frequency. The maximum of these differences is then checked against the critical value. If the maximum difference is less than the critical value, "good fit" is written, and if it is greater than the critical value, "not a good fit" is written. The maximum difference and the critical value also are written.

In general, one or more of the transformations included in the program will convert the historical data into a normal distribution. If more than one transformation is successful, the assumption is made that the one giving the least maximum cell difference should be selected. On the other hand, data for which a normalizing transform has not been found should not be used to generate gage flow data in FLASH.

A4.2.2 Program Components

The program NORMAL is made up of the subroutines listed below with their lengths in bytes.

NORMAL MAIN	23,428
DIFCHK	796
TRFM	658
HISTGM	1,210

Function length - 20,208
Total Program length - 46,672

A4.2.2.1 Subroutine NORMAL - MAIN

Subroutine NORMAL - MAIN is the controlling subroutine which reads in all data, computes the statistical data, calls the supporting subroutines and writes output array headings.

A4.2.2.2 Subroutine DIFCHK

This subroutine computes the cumulative distribution of the transformed data, performs the comparison with the normal cumulative distribution and writes out array containing both cumulative distribution and the absolute value of their difference.

A4.2.2.3 Subroutine TRFM

TRFM sets up the transformation constants, determines the transformation called for and transforms the data.

A4.2.2.4 Subroutine HISTGM

Subroutine HISTGM arranges the transformed data in order of increasing magnitude, computes the mean, standard deviation and skewness of the transformed data, normalizes the transformed data and classifies the data into 17 cells, according to magnitude. These data are the cell counts which make up the histogram and are the basis for computation of the cumulative distribution of the transformed data.

A4.2.3 Program Input

All program input enters through NORMAL MAIN.

Card # 1 (I2) NNSTA = the number of gaging stations for which data are to be read.

Card # 2 (10I8)NSTA(I) = the identifying number of the gaging stations, maximum of ten.

Card # 3 (16I5) IYR1 (I) = the beginning year of data for station I.
IYR2 (I) = the ending year of data for station I.

Data Cards (I8, I4, I2, 4F8.2)

NSTA = identifying station number, field is I8.
N2 = the year identifying the data on the card, field is I4.
N3 = the month identifying the data on the card, field is I2.
WK(K,L,J) = four weekly average flows for the month, year and station identified therewith, 4-F8.2 fields.

A4.2.4 Program Output

Program output is written by NORMAL-MAIN and DIFCHK. The output is in four parts described as follows:

(1) The number of stations and the identifying station number with starting and ending years of data are written out once at the beginning of the output package.

(2) For each station, the mean, standard deviation and skewness for the untransformed data are written. When weekly data are called for, this output is in an array of 48 rows and 4 columns. The rows are one for each "week" of the year. The column headings are week, mean, standard deviation, and skewness. When the complete set of data are considered at one time, the output is printed in three lines, one each for the mean, standard deviation and skewness.

(3) For each station and transformation, the following information is written. First, the identifying transformation number is written. This is followed by a horizontal array in which the first three numbers are the mean, standard deviation and skewness of the transformed data, followed by 17 numbers representing the number of the data items which fall into the 17 cells of the histogram.

(4) For each station and transformation, an array is written to output the cumulative frequency distribution for the data, the cumulative frequency distribution for the normal curve and the absolute value of the difference between the two. The array is 17 rows by four columns, one row for each cell of the distribution

and one column each for the station number, the cumulative frequency for the data, the cumulative frequency for the normal curve and the difference. The maximum difference and critical value of the test statistics are written and if the maximum difference is less than critical, "good fit" is written; otherwise, "not a good fit" is written.

A4.2.5 Dictionary of Variables

Following is a list of the variables used in NORMAL and a brief definition of each:

AVER	Average.
BOUND (I)	Cell limit (for normalized data), Ith cell.
C (I)	Coefficient, Ith transform.
COMPAR	Cumulative frequency.
D1	Coefficient, standard deviation formula.
D2	Coefficient, skewness formula.
D3	Coefficient, skewness formula.
DAVE	Mean, double precision.
DEV	Deviation, equals data value minus mean.
DIF	Difference, cum. dist. of data and normal.
DSD	Standard deviation, double precision.
DSKEW	Skewness, double precision.
FIRST	Mean of data, first moments.
FIT	Test statistic, Kolmogorov-Smirnov critical value.
FN	Number of data points.
IFREQ (I)	Number of data points in cell I.
INT (I)	Same as IFREQ (I).
IT	Number identifying transform.
IYR1 (I)	Beginning year of data, station I.
IYR2 (I)	Ending year of data, station I.
KOUNT (I)	Cell count for Ith cell.
N2	Year.
N3	Week.
NNSTA	Number of gage stations.
NSTA (I)	Identifying number, gage station I.
NYR	Number of years of data.
SD	Standard deviation.
SECOND	Standard deviation of data, second moment.
SKEW	Skewness, relative number.
SUMFRQ	Sum of IFREQ (I), cumulative frequency of data.
TEMP	Dummy variable, flow.
THIRD	Skewness of data, third moment.

X (IYR)	Dummy variable, flow data.
X (J)	Flow data.
XBAR	Mean x
XMDIF	Maximum difference.
XNORM (I)	Cumulative frequency, normal distribution, Ith cell.

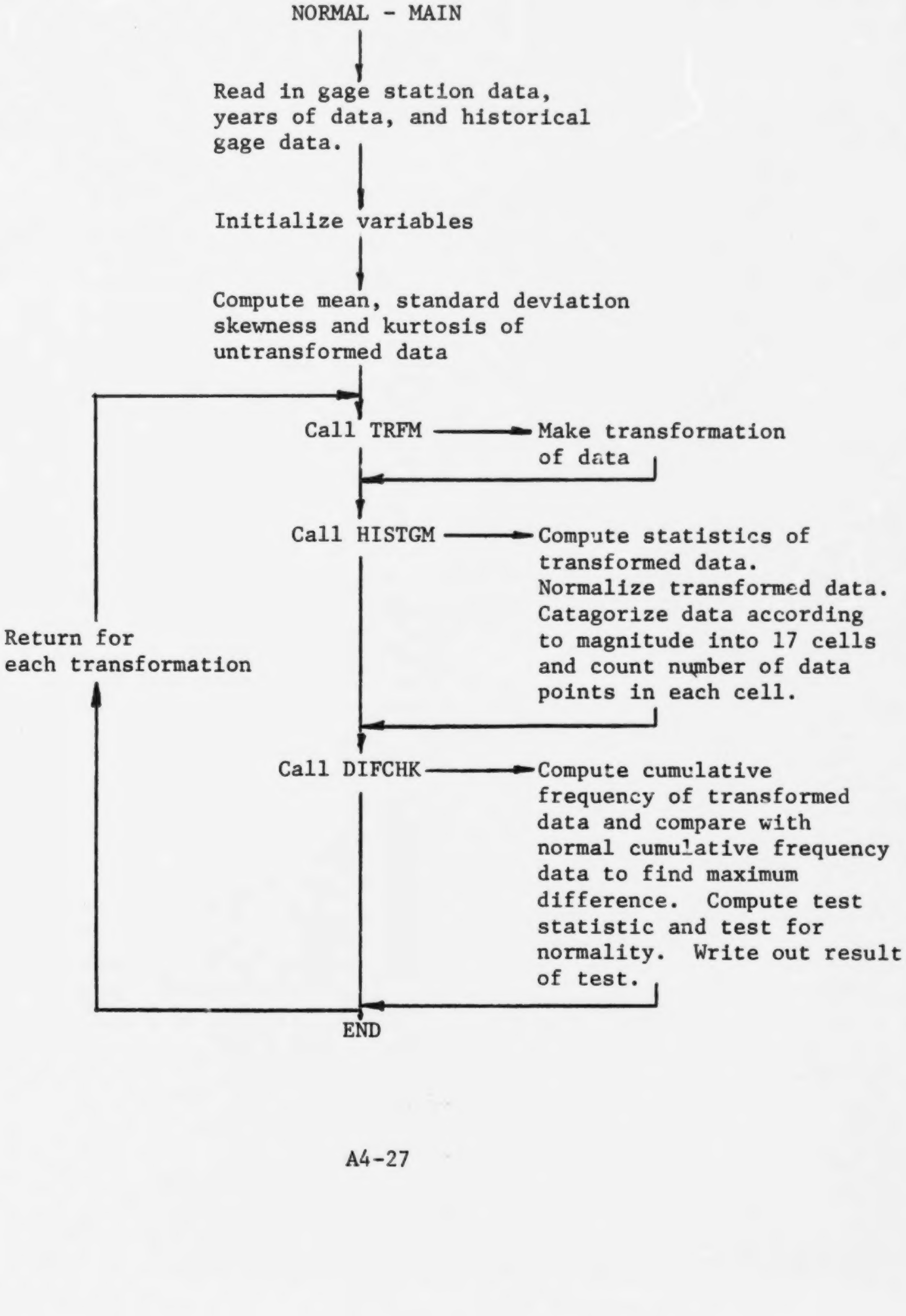
A4.2.6 Program Logic

Figure A4-3 is a diagram of program logic for NORMAL.

A4.2.7 Program Coding

The program coding for NORMAL follows.

FIGURE A4-3
PROGRAM LOGIC - NORMAL



NORMAL - MAIN

Read in gage station data, years of data, and historical gage data.

Initialize variables

Compute mean, standard deviation skewness and kurtosis of untransformed data

Call TRFM

Make transformation of data

Call HISTGM

Compute statistics of transformed data. Normalize transformed data. Catagorize data according to magnitude into 17 cells and count number of data points in each cell.

Return for each transformation

Call DIFCHK

Compute cumulative frequency of transformed data and compare with normal cumulative frequency data to find maximum difference. Compute test statistic and test for normality. Write out result of test.

END

```
//NORMAL   JOB (1143,47,020,10,0000),'ALEMAN                    ',CLASS
                                                                  =S
// EXEC F4GCXS       FORT G COMPILE (NODECK), EXECUTE, CLASS S
//FORT.SYSIN DD *
      DIMENSION WK(50,4,12),X(2400),TITLE(18),IFREQ(17),XNORM(17)
                                                                  ,
     1NSTA(15),IYR1(15),IYR2(15)
      READ(5,500)NNSTA,TITLE
  500 FORMAT(I2/18A4)
      READ(5,502)(NSTA(I),I=1,NNSTA)
  502 FORMAT(10I8)
      READ(5,503)(IYR1(I),IYR2(I),I=1,NNSTA)
  503 FORMAT(16I5)
      WRITE(6,504)NNSTA,(I,NSTA(I),IYR1(I),IYR2(I),I=1,NNSTA)
  504 FORMAT('1',///T20,'DATA DISTRIBUTION CHECK'/I8,' STATIONS',
                                                             //,8(T5
     A,'STATION('
     1,I2,')=',I8,' STARTING YEAR=',I4,'    FINAL YEAR=',I4/))
      READ(5,501)(XNORM(I),I=1,17)
  501 FORMAT(8F10.4)
      DO 900  I=1,NNSTA
      NYR=IYR2(I)-IYR1(I)+1
      DO 12 K=1,NYR
      DO 12 J=1,12
   12 READ(5,505)NSTA(I),N2,N3,(WK(K,L,J),L=1,4)
  505 FORMAT(I8,I4,I2,4F8.2)
      NWEEKS=48*NYR
      GO TO 51
99999 WRITE(6,600) TITLE
  600 FORMAT(1H1,///1H0,10X,18A4/11X,'WEEK',8X,'MEAN',2X,'STANDAR
                                                             D DEVIA
     1TION',4X,'SKEWNESS')
 1111 DO 25 JMO=1,12
      DO 25 LWK=1,4
      AVER=0
      SD=0
      SKEW=0
      DO 15 IYR=1,NYR
   15 AVER=AVER+WK(IYR,LWK,JMO)
      AVER=AVER/NYR
      DO 20 IYR=1,NYR
      SD=SD+(WK(IYR,LWK,JMO)-AVER)**2
   20 SKEW=SKEW+(WK(IYR,LWK,JMO)-AVER)**3
      SD=SQRT(SD/(NYR))
      SKEW=SKEW/(NYR)
      SKEW=SKEW/(SD**3)
   25 WRITE(6,601)JMO,LWK,AVER,SD,SKEW
  601 FORMAT(2X,I4,7X,I2,F12.4,8X,F12.4,F12.4)
      DO 50 IT=1,7
      DO 50 JMO=1,12
      DO 50 LWK=1,4
      DO 40 IYR=1,NYR
   40 X(IYR)=WK(IYR,LWK,JMO)
      CALL TRFM(X,IT,NYR,AVER,SD)
      CALL HISTGM(X,NYR,FIRST,SECOND,THIRD,IFREQ)
      WRITE(6,200)
  200 FORMAT(1H0,60X,'FREQUENCY DISTRIBUTION'/1H0,10X,'WEEK',8X,'
                                                                   MEAN',
     1  2X,'STD. DEVN.',4X,
     2  'SKEWNESS',3X,'1',3X,'2',3X,'3',3X,'4',3X,'5',3X,
     2  '6',3X,'7',3X,'8',3X,'9',2X,'10',2X,'11',2X,'12',2X,'13',
                                                                   2X,
     4  '14',2X,'15',2X,'16',2X,'17')
      WRITE(6,210)JMO,LWK,FIRST,SECOND,THIRD,IFREQ
  210 FORMAT(1H ,4X,I4,2X,I4,3F12.2,17(1X,I3))
      WRITE(6,249)JMO,LWK
  249 FORMAT(1H0,'MONTH=',I4,'  WEEK=',I4)
   50 CALL DIFCHK(XNORM,NYR,IFREQ,IT,ISTAT)
   51 AVER=0
      SD=0
      SKEW=0
      DO 125 IYR=1,NYR
      DO 125 JMO=1,12
      DO 125 LWK=1,4
  125 AVER=AVER+WK(IYR,LWK,JMO)
      AVER=AVER/NWEEKS
      DO 120 IYR=1,NYR
      DO 120 JMO=1,12
      DO 120 LWK=1,4
      DEV=WK(IYR,LWK,JMO)-AVER
      SD=SD+(DEV)**2
  120 SKEW=SKEW+(DEV)**3
      SD=SQRT(SD/NWEEKS)
      SKEW=SKEW/NWEEKS
      SKEW=SKEW/(SD**3)
      WRITE(6,620)NSTA(I),AVER,NSTA(I),SD,NSTA(I),SKEW
  620 FORMAT(1H0,'MEAN OF DATA FOR STATION',I9,' IS',F14.4/
     1        1H0,'STANDARD DEVIATION OF DATA FOR STATION',I9,' IS
                                                                 ',F14.4
     2       /1H0,'SKEWNESS OF DATA FOR STATION',I9,' IS',F14.4)
      DO 890 IT=1,3
      WRITE(6,205)
  205 FORMAT(1H0,60X,'FREQUENCY DISTRIBUTION'/1H0,8X,'MEAN',
     1  2X,'STD. DEVN.',4X,
     2  'SKEWNESS',4X,'1',4X,'2',4X,'3',4X,'4',4X,'5',4X,
     2  '6',4X,'7',4X,'8',4X,'9',3X,'10',3X,'11',3X,'12',3X,'13',
                                                                   3X,
     4  '14',3X,'15',3X,'16',3X,'17')
      J=1
      DO 901 IYR=1,NYR
      DO 901 JMO=1,12
      DO 901 LWK=1,4
      X(J)=WK(IYR,LWK,JMO)
  901 J=J+1
      CALL TRFM(X,IT,NWEEKS,AVER,SD)
      CALL HISTGM(X,NWEEKS,FIRST,SECOND,THIRD,IFREQ)
      WRITE(6,211)FIRST,SECOND,THIRD,IFREQ
  211 FORMAT(1H ,3F12.2,17(1X,I4))
      CALL DIFCHK(XNORM,NWEEKS,IFREQ,IT,I)
  890 CONTINUE
  900 CONTINUE
      STOP
      END
      SUBROUTINE DIFCHK(XNORM,N,IFREQ,IT,ISTAT)
      DIMENSION IFREQ(17),XNORM(17)
      XMDIF=0.0
      SUMFRQ=0
      XN=N
      WRITE(6,600)IT
  600 FORMAT(1H0,'TRANSFORMATION',I4)
      WRITE(6,603)
  603 FORMAT(1H0,'STATION',4X,'CUMULATIVE DIST. FOR DATA',4X,
     1        'CUMULATIVE DIST. FOR NORMAL',4X,'DIFFERENCE')
      DO 100 I=1,17
      SUMFRQ=SUMFRQ+IFREQ(I)
      COMPAR=SUMFRQ/XN
      DIF=COMPAR-XNORM(I)
      DIF=ABS(DIF)
      IF(XMDIF.LE.DIF) XMDIF=DIF
  100 WRITE(6,602)ISTAT,COMPAR,XNORM(I),DIF
  602 FORMAT(1H ,I7,16X,F13.4,19X,F12.4,2X,F12.4)
      FIT=1.36/SQRT(XN)
      WRITE(6,606) XMDIF,FIT
  606 FORMAT(1H,'MAXIMUM DIFFERENCE= ',F12.4,4X,'FIT COEFF.= ',F1
                                                              2.4)
      IF(XMDIF.LE.FIT) GO TO 105
      WRITE(6,604)
  604 FORMAT(1H0,'NOT A GOOD FIT'//)
      GO TO 110
  105 WRITE(6,605)
  605 FORMAT(1H0,'GOOD FIT'//)
  110 RETURN
      END
      SUBROUTINE TRFM(X,I,N,XBAR,SD)
      DIMENSION X(2400),C(7)
      DATA C/0.10,0.05,-.10/
      IF(I-3) 49,50,100
   49 DO 25 J=1,N
   25 X(J)=X(J)**C(I)
      GO TO 100
   50 DO 51 J=1,N
      X(J)=X(J)+C(I)*XBAR
      IF(X(J).LT.0) X(J)=1.
   51 X(J)=ALOG10(X(J))
  100 RETURN
      END
      SUBROUTINE HISTGM(X,N,AVE,SD,SKEW,IFREQ)
      DIMENSION X(1),INT(17),IFREQ(17),BOUND(16)
      DIMENSION KOUNT(20)
      DATA BOUND/-3.75,-3.25,-2.75,-2.25, -1.75,-1.25,-0.75,-0.25,
     1      0.25,0.75,1.25,1.75,2.25,2.75,3.25,3.75/
C     ARRANGE DATA IN ORDER OF INCREASING MAGNITUDE
C     COMPUTE MEAN, STANDARD DEVIATION,AND SKEWNESS
      XN=N
      DOUBLE PRECISION DAVE,DSD,DSKEW,TEMP,DXN,D3,D2,D1
      DXN=N
      D1=1
      D2=2
      D3=3
      DAVE=0.0
      DSD=0.0
      DSKEW=0.0
      DO 60 I=1,N
      TEMP=X(I)
      DAVE=DAVE+TEMP
      DSD=DSD+TEMP*TEMP
   60 DSKEW=DSKEW+TEMP*TEMP*TEMP
      DAVE=DAVE/DXN
      AVE=DAVE
      SD=(DSD-DXN*DAVE*DAVE)/(DXN-D1)
      SD=SQRT(SD)
      SKEW=(DSKEW-D3*DAVE*DSD+D2*DXN*DAVE*DAVE*DAVE)/DXN
      SKEW=SKEW/SD**3
C     NORMALIZE DATA
      DO 65 I=1,N
   65 X(I)=(X(I)-AVE)/SD
      DO 79 I=1,17
   79 KOUNT(I)=0
      DO 90 I=1,N
      IF(X(I).LE.BOUND(1)) GO TO 86
      IF(X(I).GT.BOUND(16)) GO TO 87
      DO 85 J=2,16
      IF(X(I).LE.BOUND(J)) GO TO 82
      GO TO 85
   82 K=J
      GO TO 89
   85 CONTINUE
   86 K=1
      GO TO 89
   87 K=17
   89 KOUNT(K)=KOUNT(K)+1
   90 CONTINUE
      DO 95 I=1,17
      INT(I)=KCUNT(I)
   95 IFREQ(I)=INT(I)
      RETURN
      END
/*EOF
```

A4.3 TFLOW, Gage Data Transformation Program

A4.3.1 Purpose

The TFLOW program converts the synthetic gage data generated in FLASH to unregulated stream flows at all reach points in the watershed. The program receives data indicating the reach configuration of the watershed, develops and indexes a computation sequence to fit the watershed configuration and computes the elements of a transformation matrix which transforms gage station data throughout the watershed into stream flow data at all reach points in the watershed.

TFLOW serves two purposes. Primarily, it is an important link in the simulation of streamflow data and secondarily, it can be used as an aid in determining which gaging stations in the watershed will be best to use as basis gages in the simulation. A basis gage is one the historical data of which are used to develop the parameters for generating synthetic gage data. The secondary use allows the selection of the best set of gaging stations to be used when more than ten gaging stations are available in a watershed or when, in the interest of saving machine time or machine storage, fewer gaging stations can be used. Essentially, this secondary purpose entails the selection of certain gages as basis gages, using their data to compute the corresponding gage data at other gages, designated "estimate" gages. Then the correlation coefficients between the computed and actual data at the "estimate" gages are computed. A high correlation coefficient at a given "estimate gage" indicates that, for the set of basis gages used, it is possible with small loss in overall accuracy to eliminate that particular "estimate" gage in the final selection of basis gages.

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regulation, quality, etc.) are to be made. It is not necessary that reaches be numbered in any particular order, so long as each reach has its individual number, because the subroutine sorts out the reaches in the proper order for computation and assigns each reach an internal index which governs the order of computation. However, for other reasons, a reach numbering scheme is recommended in Section 6.3.1. This internal index, JR (I), is set up for each reach so that no reach in the sequence is upstream from one having a lower index. Computation begins at the upstream reach and progresses downstream.

A4.3.2.4 Subroutine TRAN

Subroutine TRAN makes the necessary computations to develop the transformation matrix to convert gage data into flow data at any reach point in the watershed. The subroutine is supplied, through TGEN, with the reach location and upstream drainage area for each gage and the upstream drainage area for each reach point. The elements of the transformation matrix are computed as the weights, $w_{i,j}$, which are coefficients in the equation:

$$Q_i = \sum_{=1}^{NG} w_{i,j} x_j \;.\;.\;.\;.\;. \qquad \text{Eq. A4.2}$$

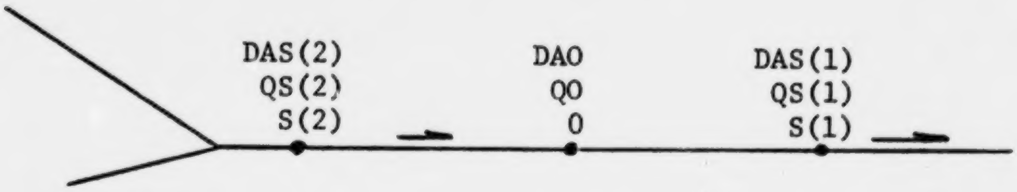
where, Q_i = the computed flow at reach point, i, i=1. . .NR
$w_{i,j}$ = the weighting coefficient for the ith reach and the jth gage.
x_j = the jth gage flow.
NG = the number of gages being used.
NR = the number of reaches in the system.

Thus, the flow at any reach point is a linear combination of the appropriate weights and gage flows.

The weights are determined as a proportion of areas upstream from the gages and reach points in accordance with the procedures described below for various combinations of relative gage-reach locations.

In the development below, the reach at which streamflow is to be computed is called the object site and the gaging stations, where flows are known, are called source sites. QO is the unknown object flow, DAO is the area upstream of the object site, QS(I) is the flow at the source site I and DAS (I) is the area upstream from the source site I. The source-object configurations are divided into five categegories described as follows.

Case I. The object site is located between two source sites, with no branches having source sites entering between the two given source sites.



The streamflow per unit area at the object site, QO/DAO, is computed as the weighted sum of the streamflows per unit area at the source sites, the weights being determined by linear interpolation between flows per unit area for the source sites.

$$\frac{QO}{DAO} = \alpha_1 \frac{QS(1)}{DAS(1)} + \alpha_2 \frac{QS(2)}{DAS(2)} \;\ldots\ldots\; \text{[Eq. A4.3]}$$

where:

$$\alpha_1 = \frac{DAO - DAS2}{DAS(1) - DAS(2)} \;\ldots\ldots\ldots\; \text{[Eq. A4.4]}$$

$$\alpha_2 = \frac{DAS(1) - DAO}{DAS(1) - DAS(2)} \;\ldots\ldots\ldots\; \text{[Eq. A4.5]}$$

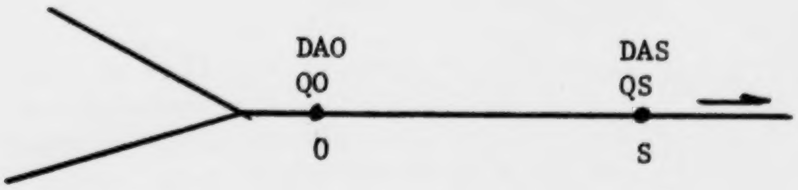
Then,

$$WT(.,1) = \frac{QO}{QS\ (1)} = \alpha_1 \frac{DAO}{DAS(1)} = \frac{DAO}{DAS(1)} \frac{DAO-DAS(2)}{DAS(1)-DAS(2)} \text{[Eq. A4.6]}$$

$$WT(.,2) = \frac{QO}{QS(2)} = \alpha_2 \frac{DAO}{DAS(2)} = \frac{DAO}{DAS(2)} \frac{DAS(1) - DAO}{DAS(1) - DAS(2)} \;.\; \text{[Eq. A4.7]}$$

All other weighting coefficients relating this object site to other source sites are zero.

Case II. The object site is upstream of the source site and there are no other source sites upstream of the source site.



The streamflow per unit area at the object site, QO/DAO, is computed as the weighted sum of the streamflows per unit area at the source sites, the weights being determined by linear interpolation between flows per unit area for the source sites.

The object site may be located on either the main stem of the river or on a branch tributary. The flow rate at the object site is in direct proportion to the flow rate at the source site. That is:

$$\frac{QO}{DAO} = \frac{QS}{DAS} \quad \text{. . . . . [Eq. A4.8]}$$

or:

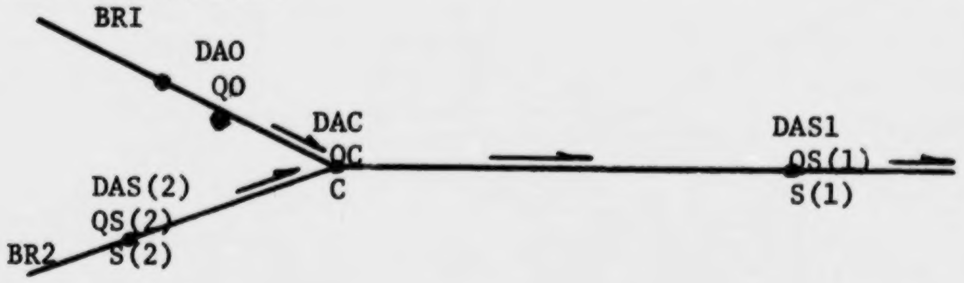
$$\frac{QO}{QS} = \frac{DAO}{DAS} \quad \text{. . . . . [Eq. A4.9]}$$

and:

$$WT(.,1) = \frac{DAO}{DAS} \quad \text{. . . . . [Eq. A4.10]}$$

The weighting coefficients for all other source sites located downstream of the source site used here are zero.

Case III. The object site is located upstream of source site (1) on BRI and there is another source site (2) located on another upstream branch, BR2.



Either BR1 or BR2 may be the main stem.

In this case, it is necessary to compute the flow at the confluence of BR1 and BR2, which will be called QC, and the upstream area which will be called DAC. The flow rate at the confluence QC/DAC is computed by applying Case I, so that:

$$QC = QS(1) \times (WT(c,1)) + QS(2) \times (WT(c,2)) \quad . \; . \; [\text{Eq. A4.11}]$$

By case II,

$$\frac{QO}{DAO} = \frac{QC}{DAC} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. A4.12}]$$

or:

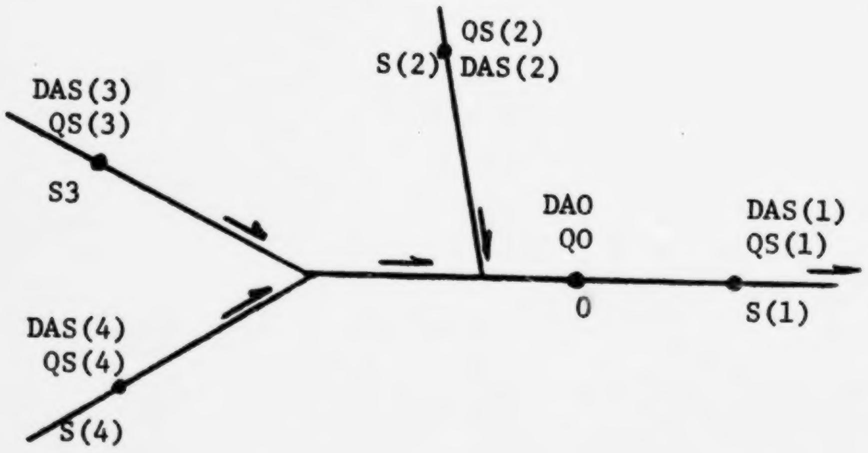
$$QC = \frac{(QO)\ (DAC)}{DAO} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. A4.13}]$$

and by substitution and clearing:

$$QO = \frac{DAO}{DAC}\ (QS(1)\ WT(c,1) + QS(2)\ WT(c,2)) \quad [\text{Eq. A4.14}]$$

The weighting coefficients for source sites upstream of source site (2) and downstream from source site (1) are all zero.

Case IV. The object site is located upstream of a source site (1) and downstream of more than one source site, each of which is located on a tributary to the object site location as shown.



The first step is to calculate the total gaged flow, QG, and total gaged upstream areas, DAG, for the source gages upstream of the object site:

$$QG = QS(2) + QS(3) + QS(4) \quad \ldots\ldots \quad \text{[Eq. A4.15]}$$

$$DAG = DAS(2) + DAS(3) + DAS(4) \quad \ldots\ldots \quad \text{[Eq. A4.16]}$$

The summations in these equations extend to other comparable source sites if they exist. The flow at the object site is computed as the weighted average of the sum of the upstream source site flows and the downstream source site flow. This case is then converted to Case 1, and:

$$\frac{QO}{DAO} = \alpha_1 \frac{QS(1)}{DAS(1)} + \alpha_2 \frac{QG}{DAG} \quad \ldots\ldots \quad \text{[Eq. A4.17]}$$

and:

$$\alpha_1 = \frac{DAO - DAG}{DAS(1) - DAG} \quad \ldots\ldots \quad \text{[Eq. A4.18]}$$

$$\alpha_2 = \frac{DAS(1) - DAO}{DAS(1) - DAG} \quad \ldots\ldots \quad \text{[Eq. A4.19]}$$

then:

$$QO = \frac{DAO}{DAS(1)} \left(\frac{DAO-DAG}{DAS(1)-DAG}\right) QS(1) + \frac{DAO}{DAG} \left(\frac{DAS(1) - DAO}{DAS(1) - DAG}\right) QG$$

$$\text{[Eq. A4.20]}$$

from which:

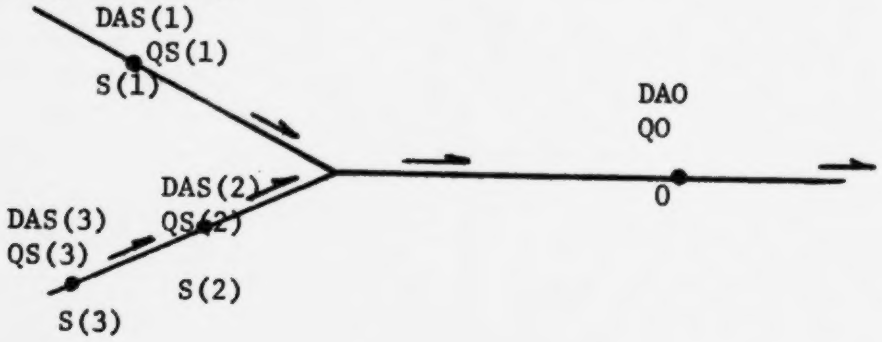
$$WT(.,j) = \frac{QO}{QS(1)} = \frac{DAO}{DAS(1)} \left(\frac{DAO - DAG}{DAS(1) - DAG}\right) \quad \ldots \quad \text{[Eq. A4.21]}$$

and:

$$WT(.,j) = \frac{QO}{QG} = \frac{DAO}{DAG} \left(\frac{DAS(1) - DAO}{DAG(1) - DAG}\right) \quad \ldots\ldots \quad \text{[Eq. A4.22]}$$

The weighting coefficients of any source sites downstream of source site 1 and upstream of source sites 2, 3 and 4 on the same tributaries are zero.

Case V. The object site is located downstream of all source sites. The general case would be where the object site is a downstream point having a number of gages upstream, each of which is on a tributary as shown.



As in Case IV, compute QG and DAG, the sums of flows and areas for those gages which are upstream of the object site, using only the gage on any single tributary which is nearest the object site. In the example shown:

$$QG = QS(1) + QS(2) \quad . \; . \; . \; . \; . \; . \; . \quad [Eq.\ A4.23]$$

$$DAG = DAS(1) + DAS(2) \quad . \; . \; . \; . \; . \; . \quad [Eq.\ A4.24]$$

The problem then reverts to that of Case II, excepting the object site is downstream. Equating the flow rates per unit area at the object and source sites gives:

$$\frac{QO}{DAO} = \frac{QG}{DAG} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [Eq.\ A4.25]$$

and:

$$WT(.,j) = \frac{QO}{QG} = \frac{DAO}{DAG} \quad . \; . \; . \; . \; . \; . \quad [Eq.\ A4.26]$$

In all cases, do not use source sites which are upstream of source sites adjacent to the object site.

Any combination of stream gages and an object site can be classified under one of the five cases described above. It is important that the sequence of reaches begin with the downstream reach in the watershed and proceed upstream. It then can be assumed that, as the computation progresses upstream, all downstream flows are known. This is necessary because the subroutine uses the computed flow in the next downstream reach as a source site for the object reach. The source sites upstream of the object site are considered explicit

A4.3.2.5 UPGAGE

Subroutine UPGAGE is called by subroutine TRAN when, in the progress of computing the transform elements, it is necessary to search out and identify upstream gage sites. Subroutine UPGAGE searches the reaches upstream from the object site for gage locations and, in any upstream branch, the search is discontinued when the first gage is found in the current branch being searched. The gage number is returned to subroutine TRAN which associates its location and upstream drainage area from data read in by subroutine TGEN and computes the desired weighting coefficient.

A4.3.2.6 IREACH

Function IREACH is used by subroutines TGEN, TRAN and UPGAGE. In the search for upstream reaches, function IREACH is used to determine the upstream reach index number.

A4.3.3 Program Input

The input to TFLOW, when it is used in selecting basis gages, is entered in TFLOW-MAIN and subroutines WEEKLY and TGEN. The input is described as follows:

(1) For TFLOW-MAIN

Card # 1 (2I5) NYR = Number of years of data to be read.
NGT = Number of gages for which data are to be read.
Card # 2 (10I8) IGT = The gage numbers of the gages which are to be read I = 1 . . . NGT with a maximum of eight gages.
Card # 3 (I5) NFE = The number of flow estimates to be made; i.e., the number of estimate gages. NFE=NGT - number of basis gages.
Card # 4 (I8,I5) LG(I) = gage numbers for the estimate gages I=1, . . ., NFE.
LR(I)= reach numbers corresponding to the location of gage numbers LG(I). I=1, . . ., NFE.

(2) For subroutine WEEKLY - The data to be read are edited and checked historical gage data on magnetic tape or punched cards. It is necessary that the length of record for each gage be the same; i.e., NYR years. Data must be "weekly" average flows, Q(I,K,L,J) where I = 1, NGT for a maximum of eight; K = 1,... NYR for a maximum of 50 years; L = 1, 2, 3, 4, the weekly index; and J=1...12, the monthly index.

(3) For subroutine TGEN

Card # 1 (2I5) NR = number of reaches (maximum of 50).
NG = number of gages in the watershed (maximum of 10).

Card # 2 (4I5, 6F5.0) There are NR cards with the following on each card, each card representing one reach:

NOR(I)= external number of reach (I). I is the assigned reach number, the external index number and the order the data are read into the the computer.

NUR(I,J)= external number of the Jth reach upstream from NOR(I).

J = 1, 2 or 3 depending upon the number of branches at reach NOR (I). J cannot exceed 3.

DAU(I,J)= the drainage area upstream from reach NOR (I) in the direction of branch J, in square miles.

FL(I)= the length of the reach NOR(I), in feet.

SLOPE(I) = the slope of the hydraulic gradient in reach NOR (I).

ROUGH(I) = the value of Manning's "n" for reach NOR(I).

NOR(I) takes one I5 field, NUR (I,J) takes one to three I5 fields, depending upon watershed configuration, DAU(I,J) uses one to three F5.0 fields, while FL(I), SLOPE(I) and ROUGH(I) each use an F5.0 field. If the velocity of flow in each reach is to be computed by empirical formula, such as described in Section 6.2, it will not be necessary to provide SLOPE(I) and ROUGH(I) data.

Card # 3 (2I5, F5.0) There are NG cards, each containing the following data for the gages:

NGAGE(I) = the external number for gage (I).

NGR(I) = the external reach number in which NGAGE(I) is located.

DAG(I) = the drainage area upstream from NGAGE(I), in square miles.

A4.3.4 Program Output

The output from TFLOW is in the form of four arrays. The first array has the following column headings:

(1) External Reach Number
(2) Upstream Reaches 1
(3) Upstream Reaches 2
(4) Upstream Reaches 3
(5) Upstream Drainage Areas 1
(6) Upstream Drainage Areas 2
(7) Upstream Drainage Areas 3
(8) Downstream Reach Internal Index
(9) Downstream Reach External Number
(10) Total Upstream Area
(11) External Index
(12) Reach Computation Sequence External Number

There will be NR rows in this array, one for each reach.

The external reach number is the reach number assigned by the operator to the various reaches of the watershed. Input data are entered in this order but output data are printed in the order of the internal index. The watershed reach numbers should be assigned in accordance with the scheme recommended in Section 6.2. Although the only restriction in numbering the reaches are that no two reaches can have the same number and the maximum number of reaches is 50, experience has shown that numbering as suggested above will result in an indexing scheme that is easy to use.

The upstream reaches 1, 2 and 3 refer to the external reach numbers upstream from the current reach (I). For instance, if the current reach is 15 and there is a main stream reach, 16, upstream, then "Upstream Reaches 1" is 16. If there is also a branch (say reach 27) at the upstream end of reach 15, then "Upstream Reaches 2" is 27. If there is a second branch upstream of reach 15, then "Upstream Reaches 3" will be the number of that second branch. Similarly, upstream drainage areas 1, 2 and 3 refer to the total area, in square miles, above the current reach, 1 corresponding to the upstream main stem, 2 corresponding to the first upstream branch and 3 corresponding to the second upstream branch.

Downstream reach, internal and external, are the internal and external reach numbers downstream from the current reach. These downstream reach indices are determined by the program. Total upstream area is the total area upstream of the current reach. This column is also determined by the program. The external index is printed again for ease in reference to the current reach. The reach computational sequence is the order, in external reach numbers, of the sequence of computation used in TFLOW and in the simulation programs which follow.

The second array has four columns with headings as follows:
(1) Basic Gage Number
(2) Reach Containing Gage
(3) Area Upstream of Gage
(4) Internal Index

There will be one row for each basis gage. The first column lists the gage numbers. The second column lists the external reach number which locates the gage, while the third column lists the drainage area upstream of the gage. The fourth column is an internal index of gages.

The third array is the output of subroutine TRAN. The elements of the array are the weights used in equation [A4.2] to compute streamflows at reach points. The array will have a number of columns equal to the number of basis gages; each column heading is one of the basis gage numbers. The rows correspond to reaches. The flow at the upstream end of reach i is the sum of the products of ith row of weights multiplied by the corresponding gage flows. The array lists the rows according to the external index.

The fourth array contains the results of the comparison of flows generated for estimate gage (I) and the flows of that same gage (I) considered as a basis gage. The output lists the external basis gage numbers and for each estimate gage, a row of data containing:
(1) The estimate gage number
(2) Estimated mean flow
(3) Observed mean flow
(4) Estimated standard deviation
(5) Observed standard deviation
(6) Correlation coefficient
(7) Weight coefficients, one for each basis gage
The key in selection of the basis gages is the value of the correlation coefficient. A high value (maximum value possible is 1.00) indicates that the basis gages can be used to generate the flows at the estimate gage site with acceptable accuracy and

thus eliminate the need to use that gage as a basis gage in the simulation of flows. The elimination of one basis gage in the simulation decreases the order of the correlation matrix by four and results in saving in computation time throughout the simulation.

A4.3.5 Dictionary of Variables

Following is a list of the variables used in TFLOW and a brief definition of each:

DAG (J)	Drainage area upstream of gage J.
DAU (I,J)	Drainage area upstream of reach I in direction of branch J.
FN	Number of weeks of data.
GDA	Sum of drainage areas.
IGT (I)	Identifying number of gage I.
IYR1	Beginning year of data.
JGU	Gage number of upstream gage.
KGB (I)	Internal data set index for basis gage I.
KGC (I)	Internal data set index for object gage I at reach LR(I).
KR (I)	Internal reach index for object gage I.
LG (I)	Gage number of object gage I.
NFE	Number of object gages.
NG	Number of gages.
NGAGA (I)	Identifying number of gage I in basis.
NGR (JJ)	Number of reach containing gage JJ.
NGT	Total number of gages.
NGU	Number of gages upstream.
NOR (I)	Number of upstream reach I.
NR	Number of reaches.
NUR (I,J)	Number of reach upstream of reach I in direction of branch J.
NYR	Number of years of data.
QEST	Estimated flows.
QWEEK (I, J, K)	Weekly average flow for week K, year J and gage I.
TDA (I)	Total drainage area upstream of reach I.
WT (I, J)	Weight coefficient for reach I, gage J.
X (I)	Sum of flow estimates for station I.
X2 (I)	Sum of squares of flow estimates for station I.
XY (I)	Sum of cross products, flow data x estimated data.
Y (I)	Sum of flow data for station I.
Y2(I)	Sum of squares of flow for station I.
ZY	Dummy variable.

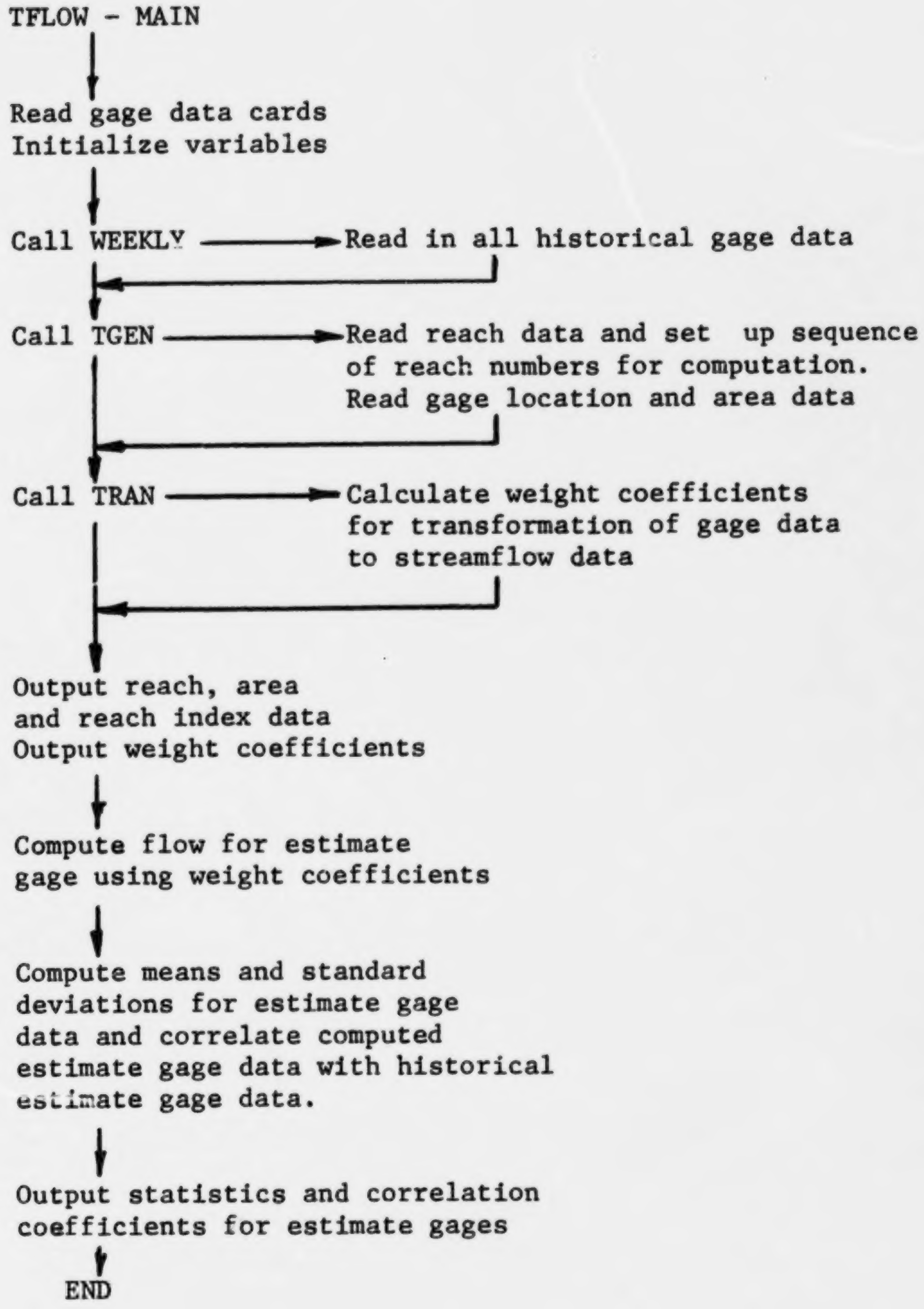
A4.3.6 Program Logic

Figure A4-4 is a diagram of program logic for TFLOW.

A4.3.7 Program Coding

The program coding for TFLOW follows.

FIGURE A4-4
PROGRAM LOGIC TFLOW


TFLOW - MAIN
Read gage data cards Initialize variables
Call WEEKLY
Read in all historical gage data
Call TGEN
Read reach data and set up sequence of reach numbers for computation. Read gage location and area data
Call TRAN
Calculate weight coefficients for transformation of gage data to streamflow data
Output reach, area and reach index data Output weight coefficients
Compute flow for estimate gage using weight coefficients
Compute means and standard deviations for estimate gage data and correlate computed estimate gage data with historical estimate gage data.
Output statistics and correlation coefficients for estimate gages
END

```
//TFLOW    JOB (1143,47,005,06,1000),'ALEMAN                 ',CLASS
                                                              =M
// EXEC F4GCXM      FORT G COMPILE (NODECK), EXECUTE, CLASS M
//FORT.SYSIN DD *
C  NGT=TOTAL NUMBER OF GAGES
C  NG=NUMBER OF GAGES IN BASIS
C  NFE=NUMBER OF FLOW ESTIMATES
C  (LG(I),I=1,NFE)=GAGE NUMBERS FOR GAGES TO CHECK FLOW ESTIMATES
C  (LR(I),I=1,NFE)=REACH NUMBERS OF REACHES AT WHICH FLOW ESTIMAT
                                                              ES ARE
C  TO BE MADE
C  (IGT(I),I=1,NGT)=GAGE NUMBER FOR DATA SET I
C  QWEEK(I,J,K)=WEEKLY FLOW FOR WEEK K DURING YEAR J FOR GAGE I
C  QEST= WEEKLY FLOW ESTIMATE
C  KR(I)=INTERNAL REACH INDEX FOR ESTIMATE I
C  KGB(I)=INTERNAL DATA SET INDEX FOR BASIS GAGE I ,I=1,NG
C  KGC(I)=INTERNAL DATA SET INDEX FOR CHECKING ESTIMATE I AT REAC
                                                              H LR(I)
C  NOR(I)=REACH NUMBER FOR REACH I
C  NGAGE(I)=NUMBER OF GAGE I IN BASIS
C  STREAMFLOW  EXTRAP/INTER  ROUTINE (SEIR)
      DIMENSION IGT(10),QWEEK(10,50,12,4),Q(50,48),
     1          LG(10),LR(10),DALG(10),KGB(10),KR(10),
     1          KGC(10),X(10),X2(10),Y(10),Y2(10),XY(10),W(10)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1     TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW3/JR(50),WT(50,10)
C  READ STREAMFLOW DATA
      READ(5,5001) NYR,NGT
 5001 FORMAT(2I5)
      READ (5,5002) (IGT(I),I=1,NGT)
 5002 FORMAT(10I8)
      DO 98 I=1,NGT
      DO 98 K=1,NYR
      DO 98 J=1,12
      DO 98 L=1,4
   98 QWEEK(I,K,J,L)=0.0
      CALL WEEKLY(NYR,NGT,QWEEK)
C  COMPUTE TRANFORMATION MATRIX
   30 CALL TGEN
      CALL TRAN
      WRITE(6,6103) (NGAGE(I),I=1,NG)
      WRITE(6,6105)
      DO 31 I=1,NR
   31 WRITE(6,6104) NOR(I),(WT(I,J),J=1,NG)
 6103 FORMAT('1',9X,'ENTERNAL'/
     112X,'REACH',9X,'COEFFICIENT OF FLOW AT GAGE'/
     111X,'NUMBER',10I10)
 6105 FORMAT(1X)
 6104 FORMAT(I15,6X,10F10.3)
C  READ FLOW ESTIMATE INFORMATION
```

```
      READ(5,5003) NFE
 5003 FORMAT(I5)
      READ(5,5004) (LG(I),LR(I),I=1,NFE)
 5004 FORMAT(5(I8,I5))
C     SET-UP ARRAY OF INTERNAL REACH
C     INDICIES CORRESPONDING
C     TO THE FLOW ESTIMATES
      DO 15 I=1,NFE
      M=0
   11 M=M+1
      IF(LR(I).EQ.NOR(M))GO TO 15
      IF(M.LT.NR) GO TO 11
      WRITE(6,6000) LR(I)
 6000 FORMAT(' REACH NUMBER MISSING',I8)
      CALL EXIT
   15 KR(I)=M
C  SET-UP INTERNAL INDICIES FOR GAGES TO
C  CHECK FLOW ESTIMATES
      DO 25 I=1,NFE
      M=0
   21 M=M+1
      IF(LG(I).EQ.IGT(M)) GO TO 25
      IF(M.LT.NGT)GO TO 21
      WRITE(6,6001) LG(I)
 6001 FORMAT(' GAGE NUMBER MISSING',I8)
      CALL EXIT
   25 KGC(I)=M
C  SET-UP INTERNAL INDICIES FOR BASIS GAGES
      DO 35 I=1,NG
      M=0
   32 M=M+1
      IF(NGAGE(I).EQ.IGT(M)) GO TO 35
      IF(M.LT.NGT) GO TO 32
      WRITE(6,6001) NGAGE(I)
      CALL EXIT
   35 KGB(I)=M
C  INTIALIZE STATISTICS
      WRITE(6,6200) (I,I=1,NG)
 6200 FORMAT(///12X,'EXTERNAL',3X,'INTERNAL',3X,'VERIFICATION',3X
                                                                    ,
     1'INTERNAL'/3X,'FLOW',7X,'REACH',6X,'REACH',8X,'GAGE',9X,'GA
                                                                  GE',
     14X,'BASIC GAGE NUMBERS'/1X,'ESTIMATE',4X,'NUMBER',6X,'INDEX
                                                                  ',7X,
     1'NUMBER',7X,'INDEX',10I7)
      WRITE(6,6201)
 6201 FORMAT(1X)
      DO 36 I=1,NFE
   36 WRITE(6,6202)I,NOR(KR(I)),KR(I),IGT(KGC(I)),KGC(I),(IGT(KGB
                                                                  (J)),
     1J=1,NG)
```

```
6202 FORMAT(I6,I11,I11,I13,I12,2X,10I7)
     DO 50 I=1,NFE
     X(I)=0.
     Y(I)=0.
     X2(I)=0.
     Y2(I)=0.
  50 XY(I)=0.
C  COMPUTE MEANS, STDS, AND CORR. COEF.
     DO 100 IYR=1,NYR
     DO 100 J=1,12
     DO 100 L=1,4
     DO 100 I=1,NFE
     QEST=0.
     ZY=QWEEK(KGC(I),IYR,J,L)
     Y(I)=Y(I)+ZY
     Y2(I)=Y2(I)+ZY**2
     DO 110 K=1,NG
 110 QEST=QEST+WT(KR(I),K)*QWEEK(KGB(K),IYR,J,L)
     XY(I)=XY(I)+ZY*QEST
     X(I)=X(I)+QEST
 100 X2(I)=X2(I)+QEST**2
     FN=NYR*48
     DO 150 I=1,NFE
     X(I)=X(I)/FN
     X2(I)=SQRT(X2(I)/FN-X(I)**2)
     Y(I)=Y(I)/FN
     Y2(I)=SQRT(Y2(I)/FN-Y(I)**2)
     XY(I)=(XY(I)/FN-X(I)*Y(I))/(X2(I)*Y2(I))
     WRITE(6,6010) (NGAGE(J),J=1,NG)
6010 FORMAT(1H1,35X,'EVALUATION OF STREAMFLOW INTERPOLATION/
    1EXTRAPOLATION ROUTINE'///30X,'BASIS GAGES=',10I7)
     WRITE(6,6011) (NGAGE(J),J=1,NG)
6011 FORMAT(////16X,'EST',7X,'OBS',7X,'EST',7X,'OBS',15X,'COEFFI
                                                         CIENTS
    1FOR GAGE'/7X,'GAGE',4X,'MEAN',6X,'MEAN',7X,'STD',7X,'STD',5
                                                         X,
    1'CORR',2X,10I7)
 150 WRITE(6,6002)IGT(KGC(I)),X(I),Y(I),X2(I),Y2(I),XY(I),(WT(KR
                                                         (I),J),
    1J=1,NG)
6002 FORMAT(3X,I7,4F10.2,F9.4,10F7.3)
     WRITE(6,6003)
6003 FORMAT(1X)
     END
     SUBROUTINE WEEKLY (NYR,NGT,Q)
     DIMENSION Q(10,50,12,4),IGT(10)
     DO 10 I=1,NGT
     DO 10 K=1,NYR
     DO 10 J=1,12
  10 READ(5,200) IGT(I),IYR1,J,(Q(I,K,J,L),L=1,4)
 200 FORMAT(I8,I4,I2,4F8.2)
```

```
      RETURN
      END
      SUBROUTINE TRAN
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1       TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION JGU(10),NIU(50),NID(50)
C    *************************
C    COMPUTE FLOW IN LAST REACH DOWNSTREAM
C    *************************
      I = JR(NR)
C    IS THERE A GAGE IN THIS REACH
      J = 0
      DO 5 JJ=1,NG
      IF (NGR(JJ) - NOR(I)) 5,6,5
    6 J = JJ
    5 CONTINUE
      IF (J) 10,10,15
C    THERE IS A GAGE - CASE 1
   15 DO 16 JJ=1,NG
   16 WT(I,JJ) = 0.
      WT(I,J) = TDA(I)/DAG(J)
      GO TO 100
C    THERE IS NO GAGE - CASE 2
   10 CALL UPGAGE (I,NGU,JGU)
      IF (NGU) 21,21,22
   21 WRITE (6,6000)
 6000 FORMAT (10X,'NO GAGES')
      STOP
   22 DO 23 JJ=1,NG
   23 WT(I,JJ) = 0.
      GDA = 0.
      DO 25 JJ=1,NGU
   25 GDA = GDA+DAG(JGU(JJ))
      DO 24 JJ=1,NGU
      J = JGU(JJ)
   24 WT(I,J) = TDA(I)/GDA
C    *********************************
C    CONTINUE UPSTREAM
C    *********************************
  100 IU = I
      IB = 1
      NID(IB) = IU
C    IS THERE A REACH UPSTREAM
  105 NU = NUR(IU,1)
      IF (NU) 110,110,115
C    THERE IS NO REACH UPSTREAM
  110 IB = IB - 1
C    HAVE ALL REACHES BEEN COMPLETED
      IF (IB) 200,200,120
```

```
C  TRANSFORM IS COMPLETE
  200 RETURN
C  CONTINUE CALCULATIONS
  120 IU = NIU(IB)
      ID = NID(IB)
      GO TO 150
C  THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
  115 IF (NUR(IU,2)) 125,125,130
C  THERE IS NO BRANCH
  125 NU = NUR(IU,1)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS A BRANCH - ARE THERE TWO
  130 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      IF (NUR(IU,3)) 135,135,140
C  THERE IS ONLY ONE BRANCH
  135 NU = NUR(IU,2)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS ANOTHER BRANCH
  140 NU = NUR(IU,2)
      IB = IB + 1
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      NU = NUR(IU,3)
      ID = IU
      IU = IREACH(NU)
C  IS THERE A GAGE IN THIS REACH
  150 J = 0
      DO 151 JJ=1,NG
      IF (NGR(JJ)-NCR(IU)) 151,152,151
  152 J = JJ
  151 CONTINUE
      IF (J) 160,160,165
C  THERE IS A GAGE IN THIS REACH
  165 DO 166 JJ=1,NG
  166 WT(IU,JJ) = 0.
      WT(IU,J) = TDA(IU)/DAG(J)
      GO TO 105
C  THERE IS NO GAGE IN THIS REACH
  160 CALL UPGAGE (IU,NGU,JGU)
      IF (NGU) 180,180,161
  161 GDA = 0.
      DO 170 JJ=1,NGU
      J = JGU(JJ)
  170 GDA = GDA + DAG(J)
```

```
      DO 171 JJ=1,NG
  171 WT(IU,JJ) = 0.
      A1 = (TDA(ID) - TDA(IU))/(TDA(ID) - GDA)
      DO 172 JJ=1,NGU
      J = JGU(JJ)
  172 WT(IU,J) = A1*TDA(IU)/GDA
      A1 = (TDA(IU) - GDA)/(TDA(ID) - GDA)/TDA(ID)
      DO 173 J=1,NG
  173 WT(IU,J) = WT(IU,J) + A1*WT(ID,J)*TDA(IU)
      GO TO 105
C  THERE IS NO GAGE UPSTREAM - CASE 2
  180 A1 = TDA(IU)/TDA(ID)
      DO 181 J=1,NG
  181 WT(IU,J) = A1*WT(ID,J)
      GO TO 105
      END
      SUBROUTINE TGEN
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1      TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION IR(50)
      DIMENSION NNOR(50)
      READ (5,5001) NR,NG
 5001 FORMAT (2I5)
      DO 1 I=1,NR
      NNOR(I)=0
    1 IDR(I) = 0
      DO 5 I=1,NR
    5 READ (5,5002) NOR(I),(NUR(I,J),J=1,3),
     1      (DAU(I,J),J=1,3),FL(I),SLOPE(I),ROUGH(I)
 5002 FORMAT (4I5,6F5.0)
      DO 6 I=1,NG
    6 READ (5,5003) NGAGE(I),NGR(I),DAG(I)
 5003 FORMAT(I8,I5,F5.0)
C  DETERMINE SEQUENCE OF REACH NUMBERS
      DO 15 I=1,NR
   15 IR(I) = 0
      DO 20 N=1,NR
      I = 1
   23 IF (IR(I)) 21,21,22
   22 I = I + 1
      IF(I.GT.NR) GO TO 20
      GO TO 23
   21 K = 0
      DO 25 J=1,3
      IF (NUR(I,J)) 25,25,26
   26 NUP = NUR(I,J)
      L = IREACH(NUP)
      IF (IR(L)) 27,27,25
   27 K = 1
   25 CONTINUE
      IF (K) 30,30,22
   30 JR(N) = I
      IR(I) = 1
      DO 36 K=1,3
      IF (NUR(I,K)) 36,36,37
   37 NC = NUR(I,K)
      ID = IREACH(NC)
      IDR(ID) = I
   36 CONTINUE
   20 CONTINUE
      DO 45 I=1,NR
      TCA(I) = 0.
      DO 45J=1,3
   45 TCA(I) = TCA(I) + DAU(I,J)
C  *****************
      WRITE(6,6000)
 6000 FORMAT('1EXTERNAL',49X,'DOWNSTREAM REACH',6X,'TOTAL',15X,
     1'REACH COMPUTATION SEQUENCE'/
     13X,'REACH',4X,'UPSTREAM REACHES',3X,'UPSTREAM DRAINAGE AREA
                                                           S',
     1       3X,'INTERNAL  EXTERNAL',3X,'UPSTREAM',3X,'INTERNAL',
                                                           7X,
     1'INTERNAL  EXTERNAL'/2X,'NUMBER',7X,'1',4X,'2',4X,'3'8X,'1'
                                                           ,7X,
     1'2',7X,'3',6X,'INDEX',7X,'NUMBER',5X,'AREA',7X,'INDEX',8X,'
                                                           INDEX',
     17X,'NUMBER'/)
      DO 800 I=1,NR
      IF(IDR(I).NE.0)NNOR(IDR(I))=NOR(IDR(I))
  800 CONTINUE
      DO 46 I=1,NR
   46 WRITE(6,6001)NOR(I),(NUR(I,J),J=1,3),(DAU(I,J),J=1,3),IDR(I
                                                           ),
     1NNOR(IDR(I)),TCA(I),I,JR(I),NOR(JR(I))
 6001 FORMAT(I6,I10,I5,I5,F11.1,F8.1,F8.1,I8,I11,F12.1,I10,6X,I8,
                                                           I11)
      WRITE(6,6002)
 6002 FORMAT(///43X,'BASIC',7X,'REACH',7X,'AREA'/
     143X,'GAGE',5X,'CONTAINING',3X,'UPSTREAM',3X,'INTERNAL'/
     142X,'NUMBER',7X,'GAGE',7X,'OF GAGE',5X,'INDEX'/)
      DO 47 I=1,NG
   47 WRITE(6,6003) NGAGE(I),NGR(I),DAG(I),I
 6003 FORMAT(38X,I8,I12,F13.1,I10)
C  **************
      RETURN
      END
      SUBROUTINE UPGAGE (I,NGU,JGU)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1       TCA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION JGU(10),NIU(50)
C   DETERMINE THE NUMBER AND IDENTITY OF GAGES UPSTREAM
C   OF THIS REACH. DISCOUNT FURTHER SEARCH WHEN A GAGE
C   IS ENCOUNTERED. I IS CURRENT REACH, NGU IS NUMBER OF
C   GAGES UPSTREAM AND JGU ARE INDICIES OF THESE GAGES
      NGU = 0
      IB = 1
      IU = I
C   IS THERE A REACH UPSTREAM
    5 NU = NUR(IU,1)
      IF (NU) 10,10,15
C   THERE IS NO REACH UPSTREAM
   10 IB = IB - 1
C   IS THE SEARCH COMPLETE
      IF (IB) 100,100,20
C   SEARCH COMPLETE
  100 RETURN
C   CONTINUE SEARCH
   20 IU = NIU(IB)
      GO TO 24
C   THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
   15 IF (NUR(IU,2)) 25,25,30
C   THERE IS NO BRANCH
   25 IU = IREACH(NU)
C   IS THERE A GAGE
   24 J = 0
      DO 26 JJ=1,NG
      IF (NGR(JJ) - NOR(IU)) 26,27,26
   27 J = JJ
   26 CONTINUE
      IF (J) 35,35,40
C   THERE IS NO GAGE IN THIS REACH
   35 GO TO 5
C   THERE IS A GAGE IN THIS REACH
   40 NGU = NGU + 1
      JGU(NGU) = J
C   CONTINUE SEARCHING
      GO TO 10
C   THERE IS A BRANCH
   30 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB - 1) = IREACH(NU)
C   IS THERE A SECOND BRANCH
      IF (NUR(IU,3)) 45,45,50
C   THERE IS NO SECOND BRANCH
   45 NU = NUR(IU,2)
      GO TO 25
C   THERE IS A SECOND BRANCH
   50 IB = IB + 1
      NU = NUR(IU,2)
      NIU(IB-1) = IREACH(NU)
      NU = NUR(IU,3)
      GO TO 25
      END
      FUNCTION IREACH (NU)
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1        TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      II = 1
    3 IF (NCR(II) - NU) 1,2,1
    1 II = II + 1
      IF (II.EQ.NR) GO TO 2
      GO TO 3
    2 IREACH = II
      RETURN
      END
```

A4.4 FLASH - Synthetic Gage Data Generator Program

A4.4.1 Purpose

The FLASH program is designed to generate synthetic gage data that are statistically indistinguishable from the available historical data. The program generates average weekly gage flow for one to ten gage sites. A detailed development of the methodology is contained in Section 6.4.

The FLASH program generates a trace of data for each gage site selected. It is preferable that gage sites selected correspond to the sites for which historical data are available for input. However, it is possible to utilize parameters developed by FLASH from historical data to generate data for another site, provided proper modifications are made. When parameters developed for a given site are used for another site, lesser confidence can be placed in the result.

The synthetic data are developed according to the model:

$$Q_{t,i} = A_i(\tau) + \frac{1}{4n} \sum_{i=1}^{n} \sum_{j=1}^{4} B_{\tau-j,i}\, Q_{t-j,i} + R_t\, \sigma_{\tau,i} C_{\tau-j,i}$$

[Eq. A4.27]

where: $Q_{t,i}$ = the generated flow for the current week, t, for site i.

$A_i(\tau)$ = a deterministic component developed fron analysis of historical data for that week, τ, of the year corresponding to the current week, t, site i.

$B_{\tau-j,i}$ = a correlation coefficient developed from historical data which relates the current flow to the previous four generated flows for all stations.

$Q_{t-j,i}$ = the generated flows for the four previous weeks for all n sites, j = 1,2,3,4.

n = number of sites; program maximum is ten sites.

$C_{\tau-j,i}$ = a parameter related to the variance of the generated flows such that a sequence of generated flows has the same variance as the historical data.

$\sigma_{\tau,i}$ = the standard deviation of weekly historical data, week τ, station i.
R_t = a standard normal random deviate; different for each t.
τ = a week of the year corresponding to the week, t: $\tau = 1,\ldots 48$.

Thus, the generated flow data-point is made up of a deterministic component plus a factor which correlates the current flow to previously generated flows for the site being computed as well as for all other sites, plus a random component which preserves the variance of the historical data. The deterministic component is the "least squares" best fit mean value of the historical flow for the week, τ, of the year corresponding to the data point being generated. The correlation component preserves the dependence between flows spanning a short time period, one month in this case, and flows from gage sites close enough spatially to be influenced by the same hydrometeorological conditions. The A, B and C parameter values are developed in FLASH from edited historical data supplied as input. The development of these parameters is described in detail in Section 6.

The data generated are to be used as a replacement for observed gage data to extend the amount of data available for analysis and simulation. The gage data generated are subsequently transformed into streamflow values at other points in the river system, appropriately routed through controlling structures, to simulate the stream flow throughout the system. This transformation is carried out in the program TFLOW, a description of which is contained in A4.3.

A4.4.2 Program Components
The program components of FLASH and their lengths are:

FLASH MAIN	244,080	ITRAN	896
GEN	1,324	RAN	460
COREL	3,120	RRN	396
TRANS	2,520	STA2	1506
FCOEF	1,140	STA3	904
STA1	1,458	STA4	1152
OUT3	612	OUT1	1596
EIGEN	22,652	OUT2	424
WFLOW	888	OUT4	864
OUT5	738	OUT6	1076
OUTP	1,048	MEAN	938
INP	1,036	S	9612
FUNCTIONS	20,792		

Total program length = 321,248 bytes

A4.4.2.1 FLASH MAIN

This program reads in the program controlling information and serves as the coordinating program to call the numerous subroutines which perform the operations, compute statistics and write out information.

The program controlling information is supplied on one card. The details are contained in A4.4.3, Program Input, and A4.4.4, Program Output, which follow. The controlling information indicates: (1) whether the flow generator parameters are to be developed from historical data supplied with the current "run", or previously developed parameters are to be used, (2) the number of years of data that are to be generated, (3) the number of sites for which data are to be generated, (4) if the historical data to be used are to be transformed for use or not, and (5) other miscellaneous instructions for handling the transfer and output of data.

Although the program is a complete package which utilizes a set of edited and filled historical data to generate any number of years of synthetic data, it also is set up to generate synthetic flows from parameters previously developed. If more than one generating "run" is to be made from a given set of historical data, the parameters developed in the first "run" on that set of data can be used for all subsequent "runs". This eliminates the need for developing the same parameters for each "run" and results in a saving of machine time. This can be used only when successive "runs" are to be based on a given set of historical data and the numbers and locations of the gages for which data are to be generated are not changed.

When the flow generating parameters are to be developed from historical data, FLASH MAIN calls subroutine WFLOW which reads and stores all the historical data that are used as a basis of the generating process. Then subroutine TRANS is called, which in turn calls subroutine MEAN, FCOEF and OUT1 before returning to FLASH MAIN. Subroutine TRANS calls MEAN to compute the mean, standard deviation, skewness and kurtosis of the untransformed historical data. This done, TRANS performs the required transformation of the data. The data may be (1) used without transformation, (2) given a square root transformation or (3) given a logarithmic transformation. If, through the use of NORMAL, it appears that a transform other than these two is better, the program should be changed to incorporate the best transformation. These transformations are used to make the historical data normally distributed and to render the means

$\sigma_{\tau,i}$ = the standard deviation of weekly historical data, week τ, station i.
R_t = a standard normal random deviate; different for each t.
τ = a week of the year corresponding to the week, t: $\tau = 1,\ldots 48$.

Thus, the generated flow data-point is made up of a deterministic component plus a factor which correlates the current flow to previously generated flows for the site being computed as well as for all other sites, plus a random component which preserves the variance of the historical data. The deterministic component is the "least squares" best fit mean value of the historical flow for the week, τ, of the year corresponding to the data point being generated. The correlation component preserves the dependence between flows spanning a short time period, one month in this case, and flows from gage sites close enough spatially to be influenced by the same hydrometerological conditions. The A, B and C parameter values are developed in FLASH from edited historical data supplied as input. The development of these parameters is described in detail in Section 6.

The data generated are to be used as a replacement for observed gage data to extend the amount of data available for analysis and simulation. The gage data generated are subsequently transformed into streamflow values at other points in the river system, appropriately routed through controlling structures, to simulate the stream flow throughout the system. This transformation is carried out in the program TFLOW, a description of which is contained in A4.3.

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The program controlling information is supplied on one card. The details are contained in A4.4.3, Program Input, and A4.4.4, Program Output, which follow. The controlling information indicates: (1) whether the flow generator parameters are to be developed from historical data supplied with the current "run", or previously developed parameters are to be used, (2) the number of years of data that are to be generated, (3) the number of sites for which data are to be generated, (4) if the historical data to be used are to be transformed for use or not, and (5) other miscellaneous instructions for handling the transfer and output of data.

Although the program is a complete package which utilizes a set of edited and filled historical data to generate any number of years of synthetic data, it also is set up to generate synthetic flows from parameters previously developed. If more than one generating "run" is to be made from a given set of historical data, the parameters developed in the first "run" on that set of data can be used for all subsequent "runs". This eliminates the need for developing the same parameters for each "run" and results in a saving of machine time. This can be used only when successive "runs" are to be based on a given set of historical data and the numbers and locations of the gages for which data are to be generated are not changed.

When the flow generating parameters are to be developed from historical data, FLASH MAIN calls subroutine WFLOW which reads and stores all the historical data that are used as a basis of the generating process. Then subroutine TRANS is called, which in turn calls subroutine MEAN, FCOEF and OUT1 before returning to FLASH MAIN. Subroutine TRANS calls MEAN to compute the mean, standard deviation, skewness and kurtosis of the untransformed historical data. This done, TRANS performs the required transformation of the data. The data may be (1) used without transformation, (2) given a square root transformation or (3) given a logarithmic transformation. If, through the use of NORMAL, it appears that a transform other than these two is better, the program should be changed to incorporate the best transformation. These transformations are used to make the historical data normally distributed and to render the means and variances more independent. After the data are transformed, MEAN is again called to determine the mean, standard deviation, skewness and kurtosis of the transformed data. These statistical parameters are printed out for reference and for later comparison with the same parameters computed from the generated data.

The subroutine FCOEF utilizes the transformed historical data to develop the parameters and constants needed to compute the deterministic component of the generated data. This subroutine is described in more detail below.

Continuing, FLASH MAIN calls COREL, S and EIGEN which are used to develop the correlation matrices and perform the necessary operations on them to compute the B and C parameters of equation [A4.27]. Subroutine OUT 5 prints out these parameters and subroutine OUTP is called if it is desired to record these parameters on magnetic tape for future use.

If the synthetic flow generator is to use previously computed parameters B and C, subroutine INP is called to read in these parameters. Subroutine STA 1 utilizes the statistics. Subroutine RAN generates the standard normal random numbers, subroutine S is called again to provide starting values for $Q_{t-j,i}$ and subroutine GEN is called to compute the $Q_{t,i}$ values of the generating flow trace.

All operations subsequent to subroutine TRANS have been performed on transformed data. Now, subroutine ITRAN is called to perform the inverse transformations to convert back to the original units. Following this, subroutines STA 4 and OUT 6 are called to compute the statistics on the generated data and to print them.

A4.4.2.2 WFLOW

The subroutine WFLOW reads in the edited and checked historical data for selected gages and for all of the historical data which are to be used as a statistical basis for generating the synthetic data. The program is set up to utilize historical data for up to ten gages or stations and for up to 50 years of data for each station. It is necessary that the same number of years of historical data be used for each of the stations and the beginning and ending years for the historical data must be the same for all gages used.

WFLOW reads either a magnetic data tape or cards, depending upon the control information initially placed into FLASH MAIN as variable IHIST. If IHIST equals 0, the subroutine reads from tape and if IHIST equals 1, cards are to be read. The formats are detailed in A4.4.3. All data are read into storage because subsequent processes require several returns to these data during the course of computing the generator parameters and making the necessary statistical checks.

A4.4.2.3 TRANS
The subroutine TRANS is a multiple purpose subroutine in that it in turn calls subroutines MEAN, FCOEF and OUT 1 before returning to FLASH MAIN. TRANS calls MEAN to compute the first four statistical moments of the historical data prior to the transformation of the data. Following this, TRANS performs the desired transformation of the historical data depending upon the control information initially established in FLASH MAIN for the variable ITRAN. If ITRAN equals 1, no transformation is made; if ITRAN equals 2, the log transformation is made and if ITRAN equals 3, the square root transformation is made.

Following the transformation, subroutine MEAN again is called to compute the first four statistical moments of the transformed data.

TRANS then calls subroutine FCOEF which computes the Fourier coefficients which, along with the data mean, are the constants in the periodic formula which is a "least squares best fit" approximation of the historical data. This periodic formula provides the deterministic components, $A(\tau)$, in equation [A4.27]. More detail about FCOEF is contained below.

Subroutine FCOEF returns the computed Fourier coefficients, as well as the computed deterministic components $A\ (\tau)$ for each gaging site, back to TRANS.

Next, subroutine TRANS determines the difference between the computed deterministic component, $A(\tau)$, and the corresponding historical data points for "week" τ and, to normalize, divides this difference by the standard deviation of the historical flows for that week. These normalized deviations, one for each data point in the set of historical data, then are returned to FLASH MAIN for transfer to subroutine COREL, which is described in detail below.

Finally, subroutine TRAN calls OUT 1, which prints out the first four statistical moments for the transformed and untransformed historical data.

A4.4.2.4 MEAN

Subroutine MEAN computes the first, second, third and fourth statistical moments of the data supplied to it. The moments are first computed about origin zero according to the formulas:

$$\upsilon_1 = \frac{\Sigma X(I)}{N} \quad . . . . . \quad \text{[Eq. A4.28]}$$

$$\upsilon_2 = \frac{\Sigma X(I)^2}{N} \quad . . . . . \quad \text{[Eq. A4.29]}$$

$$\upsilon_3 = \frac{\Sigma X(I)^3}{N} \quad . . . . . . \text{[Eq. A4.30]}$$

$$\upsilon_4 = \frac{\Sigma X(I)^4}{N} \quad . . . . . . \text{[Eq. A4.31]}$$

where X(I) are the data values, N is the number of data values summed and υ_i are the moments.

The usual form for the second, third and fourth moments is for these moments to be taken about the data mean as the origin. In this case:

$$\mu_1 = \frac{\Sigma(X(I) - \upsilon_1)}{N} = 0 \quad . . \quad \text{[Eq. A4.32]}$$

$$\mu_2 = \frac{\Sigma(X(I) - \upsilon_1)^2}{N} \quad . . . . \quad \text{[Eq. A4.33]}$$

$$\mu_3 = \frac{\Sigma(X(I) - \upsilon_1)^3}{N} \quad . . . . \quad \text{[Eq. A4.34]}$$

$$\mu_4 = \frac{\Sigma(X(I) - \upsilon_1)^4}{N} \quad . . . . \quad \text{[Eq. A4.35]}$$

where μ_i = the ith moment about the mean, υ_1 = the data mean, X(I) and N are as above.

The subroutine directly computes the values for μ_2, μ_3 and μ_4 from the values of ν_i by the formulas:

$$\mu_2 = \nu_2 - \nu_1^2 \quad \ldots\ldots \text{[Eq. A4.36]}$$

$$\mu_3 = \nu_3 - 3\nu_1 \nu_2 + 2\nu_1^3 \quad . \quad \text{[Eq. A4.37]}$$

$$\mu_4 = \nu_4 - 4 \nu_1 \nu_3 + 6 \nu_1^2 \nu_2 - 3 \nu_1^4 \quad \text{[Eq. A4.38]}$$

The second moment about the mean is the variance, which is a measure of the "spread" of the data points about the mean. The square root of the second moment about the mean is defined as the standard deviation, which is also a measure of the "spread" in the units of the data.

The third moment about the mean, called "skewness", is a measure of distortion from symmetry. Normally (Gaussian) distributed data are symmetrical about their mean. A right skewness indicates that more of the data items are less than the mean while left skewness indicates that the majority of data items is greater than the mean. The fourth moment about the mean is called kurtosis. It is a measure of the magnitude of the peak of the distribution in relation to the peak of normally distributed data. Both skewness and kurtosis are expressed as numerical parameters, the magnitude of which is indicative of the shape of the distribution.

The parameter for skewness is given by:

$$\alpha_3 = \frac{\mu_3}{\mu_2{}^{3/2}} = \frac{\mu_3}{\sigma^3} \quad \ldots . \text{[Eq. A4.39]}$$

If $\alpha_3 = 0$, the data are normally distributed. The parameter for kurtosis is given by:

$$\beta_2 = \frac{\mu_4}{\sigma_2{}^2} = \frac{\mu_4}{\sigma^4} \quad \ldots\ldots . \text{[Eq. A4.40]}$$

Values of β_2 equal to 3 indicate normally distributed data, values greater than 3 indicate data having a more peaked distribution than normal data, while values of β_2 less than 3 indicate a distribution having a maximum less than the normal.

The program prints out the computed moments of the untransformed historical data, the transformed historical data and the generated synthetic data. The program uses the mean (1st moment) and variance (2nd moment about the mean) in the computation of those parameters which are used in generating the synthetic data. The third and fourth moments are useful only in the comparison of the statistical properties of the historical, transformed historical and generated data.

A4.4.2.5 Subroutine FCOEF

It is well known that weather varies in a cyclic fashion with a primary period of one year. Runoff and gaged data also follow a periodic pattern with an annual frequency. Subroutine FCOEF uses the historical data, transformed or not, as the case may be, to develop a periodic formula which represents the "best fit" of the data. The formula provides the deterministic component, $A(\tau)$, in equation [A4.27], which serves as a basis for generating synthetic data.

The form of the periodic function is:

$$\begin{aligned} A(\tau) = f(\tau) = A_o + A_1(\tau)\cos\theta + A_2(\tau)\cos 2\theta + \ldots \\ A_n(\tau)\cos n\theta + B_1(\tau)\sin\theta + B_2(\tau)\sin 2\theta \\ + - - - + B_n(\tau)\sin n\theta, \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{[Eq. A4.41]} \end{aligned}$$

where τ is the weekly index; that is, $\tau = 1$ is the 1st "week" of the year, $\tau = 2$ is the 2nd "week" of the year, $\tau = 1,2,\ldots,48$. θ is the angular value of τ; that is, for an annual period (360°), $\theta = (\frac{360}{48})\tau = 7.5\tau$ is the angular measure of one "week". For example for $\tau = 20$, the twentieth "week" of the year, $\theta = 7.5 \times 20 = 150°$. Equation [A4.41] is an infinite series and typically, the more terms used to describe the function, that is, the higher the value of n, the more accurate is the approximation to the true function. In this subroutine, provision is made to compute up to six harmonics (up to n=6) which provides an adequate approximation.

The function $f(\tau)$ is not known at the outset and subroutine FCOEF develops the function from the historical data. If y are the data points, then $y(\tau)$ are the data points for week, τ, of the year. Let N be the number of years of data available. The coefficients A_i and B_i in equation [A4.41] are given by:

$$A_o = \frac{1}{48N} \sum_{n=1}^{N} \sum_{i=1}^{48} y_{n,i} \quad . \; . \; . \; . \; . \quad \text{[Eq. A4.42]}$$

or A_o = the arithmetic mean of all data points.

$$A_k(\tau) = \frac{2}{N} \sum_{n=1}^{N} y_n(\tau) \cos k\,(7.5\tau) \quad . \; . \; . \; . \quad \text{[Eq. A4.43]}$$

$$B_k(\tau) = \frac{2}{N} \sum_{n=1}^{N} y_n(\tau) \sin k\,(7.5\tau) \quad . \; . \; . \; . \quad \text{[Eq. A4.44]}$$

where: $y_n(\tau)$ are N data points, all data points for the "week" τ in the record of N years. k is the harmonic number.

Thus, subroutine FCOEF computes the constants $A_o, \ldots A_k$ and $B_1, \ldots B_k$, which are needed to compute $A(\tau)$, the estimated or expected value of the flow. Following this, the subroutine then computes $A(\tau)$, the estimated flow for each of the 48 "weeks." This is done for each gage site.

In subroutine FCOEF, the historical flow data are converted to normalized deviations from the estimated flow data using the relation:

$$x = (Q - A(\tau))/\sigma_Q \quad . \; . \; . \; . \quad \text{[Eq. A4.45]}$$

These normalized deviations are used in all subroutines until they are transformed back in subroutine ITRAN which is near the end of the program.

A4.4.2.6 OUT 1

Subroutine OUT 1 prints the statistics of the historical flows, the first four statistical moments of the untransformed data and, if required, of the transformed data. Values are provided for each of the 48 "weeks" of the year and for each gaging site.

A4.4.2.7 COREL

Subroutine COREL utilizes the normalized deviations of the historical flows (or transformed flows) from the corresponding estimated flows, $A(\tau)$, to compute the various correlation coefficients needed ultimately to develop the B and C coefficients in equation [A4.27]. This subroutine is called from FLASH MAIN.

In the course of its operation, it calls subroutines S and OUT 3.

Correlation is a measure of the degree to which variables vary together or a measure of the intensity of association (1). The simple linear correlation coefficient is generally defined as:

$$r = \frac{\Sigma(X - \bar{x})(Y - \bar{y})/(n - 1)}{\sqrt{(X - \bar{x})^2/n - 1}\sqrt{Y - \bar{y})^2/n - 1}} \text{ . [Eq. A4.46]}$$

or, since the values (X - x) amd (Y - y) have already been computed by TRANS, the equivalent form is:

$$r = \frac{\Sigma xy}{\sqrt{x^2y^2}} \text{ . . . . [Eq. A4.47]}$$

where x and y are deviations from the mean. The correlation coefficient is a pure number, independent of units, which equals its maximum value of unity, when x and y vary identically, and its minimum value of negative one and when the two variables always vary in an opposite manner.

The rationale for the procedure used to develop the required correlation coefficients and correlation matrices was established in Section 6.5. The computational methods needed to obtain the necessary correlation coefficients and to set them up in matrix form are described below. Briefly, the purpose is to establish the correlation between historical flow values at the various gaging stations in the river basin and the correlation between historical flow values for "week" L and "weeks" L-1, L-2,..., L-7, for all gaging stations. Thus there are: (1) autocorrelations between flows at the same station for different time frames (a temporal correlation); (2) correlations between flows at different stations in the watershed for the same time frame (a spatial correlation); and (3) correlations between flows at different stations and different time frames (a temporal and spatial correlation). The problem is, therefore, multivariate.

The temporal correlations are made over the period from the current time frame under consideration backwards in time for seven time frames for a total eight correlations, one with no time lag and seven with time lags of from 1 to 7 time frames. This is done so that the flows being generated will exhibit correlation with the previously generated flows, in agreement with the assumption that natural river flows autocorrelate with the flows observed in the previous four week period. Recall from Section 6.4 that to correlate temporally for one month in multivariate analysis, it was necessary to obtain the submatrix S_{22}, which was made up of correlations in the previous month, and submatrix S_{12} which was made up of correlations between the two months. The spatial correlations are made because, in a given basin, the hydrometerological conditions which cause the flow at one station also cause a corresponding flow at a nearby station. By considering both spatial and temporal correlation multiple traces of streamflow data, one for each station, that will be not unlike those observed in a record of flows for a river basin, are provided.

The correlations are computed for all historical flow records that are being used as a basis for the generated flows. The correlation coefficients are computed and then are placed in a matrix array upon which mathematical operations are performed to produce the desired coefficients. In actuality, four matrices are set up, each of which is a sub-matrix of the correlation matrix:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

The procedure is illustrated using a three-station example.

The array S_{11} is shown in Figure A4-5. For the column and row headings, note the two digits separated by a hyphen. The first digit is the station number and the second digit is the number of time frames lagging the current time frame. For instance, 1-0 indicates station 1 at the current time frame while 1-2 indicates station 1 at the time frame two weeks lagging. The a_{ij} are correlation coefficients which correspond to their appropriate station-lag relationships. For instance, a_{17} is the coefficient which indicates the correlation between station 1 - current time frame and station 2 - lag of 2 weeks. The value of a_{17} will be the same as a_{71}. Accordingly, the value of any a_{ii} will be unity.

A modification of the computational procedure is made to produce less variability in correlation coefficients representing the same time difference. The values of a_{12}, a_{23}, and a_{34} are computed, averaged and then the average value is used in each of these positions. The coefficient a_{12} represents a one-lag time difference as does a_{23} and a_{34}, so the average one-lag coefficient is used. Similarly, a_{21}, and a_{43} are averaged from these one-lag coefficients. Further, a_{13} and a_{24} are averages of the two-lag coefficients, as are a_{31} and a_{42}.

The same procedure is used in computing the elements representing correlations between two stations. Thus, a_{15}, a_{26}, a_{37} and a_{48} are all identical, representing the average of the different station-no lag correlations. Correspondingly, a_{51}, a_{62}, a_{73} and a_{84} have the same value, which is the same as for a_{15}, and so on. The computer programming computes the coefficients and sets up the array as shown.

The array S_{22} is shown in Figure A4-6. This array is set up in the same manner as array S_{11}, excepting that lag-four through lag-seven coefficients are computed. Thus, array S_{22} contains correlation coefficients for the four through seven "weeks" following the current time frame. For example, the element b_{23} represents the correlation between station 1 - lag 6 and station 1 - lag 5. Again, b_{12}, b_{23} and b_{34} represent the average of the one-lag coefficient for these four through seven "weeks" before the current time frame.

The array S_{12} is shown in Figure A4-7. This array contains the correlations between the lag-zero through lag-three values and the lag-four through lag-seven values. The array is developed in the same manner as for S_{11} and S_{22}. Array S_{21} is merely the transpose of array S_{12}.

Subroutine COREL calls subroutine S which makes the transposition. Subroutine OUT3 is called, if desired for reference, to write out the correlation coefficients that have been computed for the current time frame. This subroutine can be removed, if desired, without loss of program continuity.

A4.4.2.8 S

Subroutine S is a utility program developed for the program library of the Department of Environmental Engineering, University

Figure A4-5
Array S_{11} Correlation Sub-Matrix
(Three station example)

$$\begin{array}{llcccccccccccc}
 & & & \text{Station 1} & & & & \text{Station 2} & & & & \text{Station 3} & & \\
 & & \text{1-0} & \text{1-1} & \text{1-2} & \text{1-3} & \text{2-0} & \text{2-1} & \text{2-2} & \text{2-3} & \text{3-0} & \text{3-1} & \text{3-2} & \text{3-3} \\
 & \text{1-0} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{110} & a_{111} & a_{112} \\
\text{Station 1} & \text{1-1} & a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & \cdots & \vdots & \vdots & \vdots & \vdots \\
 & \text{1-2} & a_{31} & a_{32} & a_{33} & a_{34} & \cdots & \vdots & a_{37} & \cdots & & & & \\
 & \text{1-3} & a_{41} & a_{42} & a_{43} & a_{44} & \cdots & & \vdots & a_{48} & \cdots & & & \\
 & & & & & & & & & \vdots & & & & \\
 & \text{2-0} & a_{51} & a_{52} & \cdots & & & & & & & & & \\
 & \text{2-1} & a_{61} & a_{62} & \cdots & & & & & & & & & \\
\text{Station 2} & \text{2-2} & a_{71} & a_{72} & a_{73} & \cdots & & & & & & & & \\
 & \text{2-3} & a_{31} & \cdot & \cdot & a_{84} & \cdots & & & & & & & \\
 & & \vdots & \vdots & \vdots & \vdots & & & & & & & & \\
 & \text{3-0} & a_{91} \cdots & & & & & & & & & & & \\
 & \text{3-1} & a_{101} \cdots & & & & & & & & & & & \\
\text{Station 3} & \text{3-2} & a_{111} \cdots & & & & & & & & & & & \\
 & \text{3-3} & a_{121} \cdots & & & & & & & & & & & a_{1212}
\end{array}$$

Figure A4-6
Array S_{22} Correlation Sub-Matrix
(Three station example)

$$
\begin{array}{ll|cccc|cccc|cccc}
 & & \text{Station 1} & & & & \text{Station 2} & & & & \text{Station 3} & & & \\
 & & 1\text{-}4 & 1\text{-}5 & 1\text{-}6 & 1\text{-}7 & 2\text{-}4 & 2\text{-}5 & 2\text{-}6 & 2\text{-}7 & 3\text{-}4 & 3\text{-}5 & 3\text{-}6 & 3\text{-}7 \\
\hline
 & 1\text{-}4 & b_{11} & b_{12} & b_{13} & b_{14} & \cdots & & & & & & \cdots & b_{112} \\
 & 1\text{-}5 & b_{21} & b_{22} & b_{23} & b_{24} & \cdots & & & & & & & \vdots \\
\text{Station 1} & 1\text{-}6 & b_{31} & b_{32} & b_{33} & b_{34} & \cdots & & & & & & & \\
 & 1\text{-}7 & b_{41} & b_{42} & b_{43} & b_{44} & \cdots & & & & & & & \\
 & & \vdots & \vdots & \vdots & \vdots & & & & & & & & \\
 & 2\text{-}4 & & & & & & & & & & & & \\
 & 2\text{-}5 & & & & & & & & & & & & \\
\text{Station 2} & 2\text{-}6 & & & & & & & & & & & & \\
 & 2\text{-}7 & & & & & & & & & & & & \\
 & 3\text{-}4 & & & & & & & & & & & & \\
 & 3\text{-}5 & & & & & & & & & & & & \\
\text{Station 3} & 3\text{-}6 & & & & & & & & & & & & \\
 & 3\text{-}7 & b_{121} & \cdots & & & & & & & & & \cdots & b_{1212}
\end{array}
$$

Figure A4-7
Array S_{12} Correlation Sub-Matrix
(Three station example)

$$
\begin{array}{llcccccccccccc}
 & & \multicolumn{4}{c}{\text{Station 1}} & \multicolumn{4}{c}{\text{Station 2}} & \multicolumn{4}{c}{\text{Station 3}} \\
 & & 1\text{-}4 & 1\text{-}5 & 1\text{-}6 & 1\text{-}7 & 2\text{-}4 & 2\text{-}5 & 2\text{-}6 & 2\text{-}7 & 3\text{-}4 & 3\text{-}5 & 3\text{-}6 & 3\text{-}7 \\
 & 1\text{-}0 & c_{11} & c_{12} & c_{13} & c_{14} & \cdots & & & & & & \cdots & c_{112} \\
 & 1\text{-}1 & c_{21} & c_{22} & c_{23} & c_{24} & \cdots & & & & & & & \vdots \\
\text{Station 1} & 1\text{-}2 & c_{31} & c_{32} & c_{33} & c_{34} & \cdots & & & & & & & \\
 & 1\text{-}3 & c_{41} & c_{42} & c_{43} & c_{44} & \cdots & & & & & & & \\
 & 2\text{-}0 & \vdots & \vdots & \vdots & \vdots & & & & & & & & \\
 & 2\text{-}1 & & & & & & & & & & & & \\
\text{Station 2} & 2\text{-}2 & & & & & & & & & & & & \\
 & 2\text{-}3 & & & & & & & & & & & & \\
 & 3\text{-}0 & & & & & & & & & & & & \\
 & 3\text{-}1 & & & & & & & & & & & & \\
\text{Station 3} & 3\text{-}2 & & & & & & & & & & & & \\
 & 3\text{-}3 & c_{121} & \cdots & & & & & & & & & \cdots & c_{1212}
\end{array}
$$

of Florida. This program is included in its entirety in the coding which follows. The subroutine S performs common matrix operations, matrix inversion and a determinant evaluation as tabulated below.

Operation Name	Operation Number	Operation
ADD	1	$A = B+C$
SUBTRACT	2	$A = B-C$
MULTIPLY	3	$A = BxC$
INVERT	4	$A = B^{-1}$
DETERMINANT	5	$\|A\| = N$
TRANSPOSE	6	$A = A^T$
EQUAL	7	$A = B$
CLEAR	8	A,B,C = 0 (clear matrix space)
INPUT	9	A (read in matrix A)
OUTPUT	10	A (write matrix A)
SCALAR MULT	11	$A = \alpha B$

The operations in the S subroutine used in the flow generating program, FLASH MAIN, and its subroutines are: 2, substract; 3, multiply; 4, invert and 6, transpose. The subroutine calls S, operation 6 to make the transposition of matrix S_{12} to S_{21} and does so without destroying S_{12}. FLASH MAIN calls S, operation 4 to invert matrix S_{22} and in so doing replaces S_{22} with S_{22}^{-1}. Then S is called to multiply S_{12} and S_{22}^{-1} and places the product in array B(1,1,J). Following this, a matrix multiplication of S_{21} and the array B(1,1,J) is made and the result is placed in S_{22}'. The last matrix manipulation subtracts S_{22}' from S_{11} and places the result in C(1,1,J). The reason for making these matrix operations is described in Section 6.5.

The matrix operation of subroutine S, excepting for operation 4, invert, and operation 5, determinant evaluation, are simple and straightforward. Matrix inversion and determinant evaluation involve considerably more computation, especially when the order is greater than 4 or 5.

The problem of matrix inversion is one of finding a square matrix $\underline{A}^{-1}$ such that:

$$\underline{A}^{-1}\,\underline{A} = \underline{A}\,\underline{A}^{-1} = \underline{I} \quad . \; . \; . \; . \; . \; . \; . \; [\text{Eq. A4.48}]$$

where $\underline{A}$ is a square matrix of order n:

$$\underline{A} = (a_{ij}) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \ldots\ldots \text{[Eq. A4.49]}$$

and $\underline{I}$ is the identity matrix. Finding the inverse, $\underline{A}^{-1}$, is the same as solving the system of linear equations:

$$\begin{matrix} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}\,x_n = f_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}\,x_n = f_2 \\ \cdot \qquad\qquad\qquad\qquad \cdot \\ \cdot \qquad\qquad\qquad\qquad \cdot \\ \cdot \qquad\qquad\qquad\qquad \cdot \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}\,x_n = f_n \end{matrix} \quad \ldots\ldots \text{[Eq. A4.50]}$$

The matrix has one, and only one, inverse and the system has one, and only one, solution if, and only if, $\underline{A}$ is non-singular; that is, if the determinant of $\underline{A}$ is non-zero.

The system of linear equations can be written in the form:

$$\underline{A}\,\underline{x} = f \quad \ldots\ldots\ldots\ldots\ldots \text{[Eq. A4.51]}$$

and then:

$$\underline{x} = \underline{A}^{-1}\,\underline{f} \quad \ldots\ldots\ldots\ldots\ldots \text{[Eq. A4.52]}$$

If:

$$A^{-1} = (c_{ij}) \quad \ldots\ldots\ldots\ldots \text{[Eq. A4.53]}$$

then, by components:

$$x_i = \sum_{j=1}^{n} c_{ij}\,f_j \quad \ldots\ldots\ldots\ldots \text{[Eq. A7.54]}$$

$i = 1, 2, \ldots n$. This relationship allows computation of the values of c_{ij} after the above system of equations is solved for x_i. Thus, matrix inversion is equivalent to solving a system of linear equations.

The best known method for solving this problem is that of Gauss, often called "Gaussian Elimination." The method uses one equation to eliminate one variable in all other equations and successively uses the remaining equations to eliminate the remaining variables, until the last equation has only one variable remaining. The result is a upper triangular matrix, $\underline{A}^{(n)}$:

$$\underline{A}^{(n)} = \begin{matrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \cdots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & \cdots & a_{2n}^{(2)} \\ 0 & 0 & a_{33}^{(3)} & \cdots & a_{3n}^{(3)} \\ \vdots & & & & \vdots \\ 0 & \cdots & & & a_{nn}^{(n)} \end{matrix} \quad . \text{ [Eq. A4.55]}$$

After finding $a_{nn}^{(n)}$, the value of x_n can be found by:

$$a_{nn}^{(n)} x_n = f_n \text{ . . . . [Eq. A4.56]}$$

Then, by successively computing the x_i and substituting, all values of x can be found and the system is solved. Having the values of x_i, the inverse, $\underline{A}^{-1}$, can be found component by component.

The classical method for finding the inverse is through the use of the relation:

$$\underline{A}^{-1} = \frac{1}{(\det A)} \underline{A}^{+} \text{ . . . . [Eq. A4.56a]}$$

where $\underline{A}^{+}$ is the adjoint of $\underline{A}$, defined as the transpose of the matrix of cofactors of $\underline{A}$. If $\underline{A}$ is of order 2, computation of the inverse is trivial, and as the order increases to more than 4, the computation becomes laborious. For this reason, the systematic method of Gauss, with later variations, is usually selected for machine computation.

One of the variations of the Gauss method, the so-called Crout reduction method, has an advantage that, although the number of computation operations is the same, there is a reduction in the number of intermediate values that must be stored. For this reason, the method of Crout, modified by making row interchanges for increased accuracy, is used in subroutine S for matrix inversion and determinant evaluation (2).

Determinant evaluation is performed in a part of the operation required to invert. Referring to the triangular matrix obtained by Gaussian elimination, the value of the determinant of A, (det A), is obtained by:

$$(\det A) = a_{11}^{(1)} a_{22}^{(2)} a_{33}^{(3)} \cdots a_{nn}^{(n)} . \quad \text{[Eq. A4.56b]}$$

i.e., the product of the main diagonal elements of the triangularized matrix.

A4.4.2.9 EIGEN
Subroutine EIGEN computes the eigenvalues and eigenvectors of the correlation matrix developed by subroutine COREL and the successive matrix operations of subroutine S. The reason for making these computations is developed in Section 6.5.

The fact that the correlation matrix is symmetric with real elements considerably simplifies the computation of eigenvalues and eigenvectors. A symmetric matrix has the properties that: (1) the eigenvalues are real, (2) the eigenvectors corresponding to different eigenvalues are orthogonal, and (3) the correlation matrix can be diagonalized by a similarity transformation. The transforming matrix has as its columns the orthonormal set of eigenvectors for the correlation matrix and, furthermore, the resulting diagonal matrix has as its diagonal elements the eigenvalues of the correlation matrix. The symmetric matrix is a special type of Hermitian matrix, one having no complex components, and the computational advantages which are manifest in Hermitian matrices apply also to the correlation matrix.

The classical method for determining eigenvalues for a real symmetric matrix is that of Jacobi (1846) in which the transformation is made by a sequence of two-dimensional rotations (3). The process is iterative because an element which is reduced to zero by rotation in one plane is made nonzero by another rotation in a different plane. Theoretically, an infinite number of rotations would be required but in practice the process is stopped when the value of the off-diagonal elements is zero (to working accuracy).

The method for determining the eigenvalues used in EIGEN is that of Givens (1954) which is similar to the Jacobi method except that a less complete reduction is attempted. In Given's method, a real symmetric tri-diagonal matrix is produced by orthogonal similarity transformations (3). The method is not iterative and thus a considerable savings in computation results. Given's method also is based upon plane rotations. The first row off-diagonal elements are reduced to zero. These zero elements are not affected by the next set of rotations, which generate zero in off-diagonal elements of the second row. This method is continued successively for all rows until the tri-diagonal matrix is formed.

There are approximately $4n^3/3$ multiplications to be made using Given's method compared with $2n^3$ multiplications needed for one iteration in the complete reduction method of Jacobi. where n is the order (3). Although the tridiagonal matrix which results in Given's method usually is not close, numerically, to the completely reduced matrix, the eigenvalues and eigenvectors resulting from both methods are very nearly the same, within working accuracy.

The remainder of the problem is to solve the symmetric tridiagonal matrix for the eigenvalues. The method used is again that of Givens (3). There are (2n-1) independent elements in the tridiagonal matrix and the solution for its eigenvalues is much simpler than the original matrix.

The tridiagonal matrix generally has non-zero superdiagonal elements. Although roots of multiplicity k will give (k-1) zero superdiagonal elements, the occurrence of zero superdiagonal elements does not necessarily imply that there are any multiple roots. The matrix will be of the form:

$$C = \begin{bmatrix} \alpha_1 & \beta_2 & 0 & \cdots & & & 0 \\ \beta_2 & \alpha_2 & \beta_3 & \cdots & & & 0 \\ 0 & \beta_3 & \alpha_3 & \beta_4 & \cdots & & 0 \\ \cdot & \cdot & \beta_4 & \alpha_4 & \cdots & & 0 \\ \vdots & & & & & & \vdots \\ & & & & & & \beta_n \\ 0 & \cdots & & & 0 & \beta_n & \alpha_n \end{bmatrix} \quad \ldots\ldots \quad \text{[Eq. A4.57]}$$

If the leading principal minor of order r of ($\underline{C} - \lambda\underline{I}$) is Pr ($\lambda$), and if P_o (λ) is defined to be 1, then:

$$P_i (\lambda) = \alpha_i - \lambda \;.\;.\;.\;.\;.\;.\;.\;.\;. \text{[Eq. A4.57 a]}$$

and for successive minors of C:

$$P_i (\lambda) = (\alpha_i - \lambda)\, P_{i-1} (\lambda) - \beta_i^{\,2}\, P_{i-2} (\lambda) \;.\;. \text{[Eq. A4.58]}$$

$$i = 2,\ldots,n.$$

The zeros of P_r (λ) are the eigenvalues.

If the P_i (λ) are evaluated for a given value $\lambda = \mu$ then, by the Sturm sequence property, the number of agreements in sign of consecutive numbers of this sequence is the number of eigenvalues which are greater than μ in values (4). This property is used to locate the individual eigenvalues by magnitude. If for two values of μ, a_o and b_o, $b_o > a_o$, then:

$$S(a_o) \geq K \;.\;.\;.\;.\;.\;.\;.\;.\;. \text{[Eq. A4.59]}$$

and:

$$S(b_o) < K \;.\;.\;.\;.\;.\;.\;.\;.\;. \text{[Eq. A4.60]}$$

then, λ_k lies between a_o and b_o. By successively reducing the interval, $b_o - a_o$, the eigenvalues can be separated. After separation, further reduction increases the precision of the value. Suitable starting values for a_o and b_o are $\pm \,||C||_{\infty}$, the maximum value of the norm of matrix $\underline{C}$.

After an eigenvalue is isolated by reducing the interval (usually done by successive bisections), it is possible to switch to an iterative technique for faster convergence to the value having the desired working accuracy. EIGEN computes the n eigenvalues by this method.

A4.4.2.10 OUT 4

Subroutine OUT 4 writes out the parameter estimates of weekly flows which have been computed by subroutine FCOEF. These are values of A_i (τ) in equation [A4.27] developed from analysis of the historical data. Subroutine OUT 4 also writes the monthly standard deviations for each month of the year for each station. These data also are obtained through analysis of the historical data and are used also in equation [A4.27].

A4.4.2.11 OUT 5

Subroutine OUT 5 writes out the parameters B and C developed from the correlation matrix of the historical data. These are the B and C parameters of equation [A4.27]. The matrix write-out operation in subroutine S is used to write out these parameters.

A4.4.2.12 OUTP

Subroutine OUTP records the parameters B and C on magnetic tape for future use. If one wishes to make several synthetic data generating runs based upon a given set of historical data from a given set of gaging stations, the output of these parameters on tape and their subsequent use in subroutine INP allows the bypassing of all of the FLASH program previous to this point. If any change is made in the historical data used, however, a new set of parameters is needed.

A4.4.2.13 INP

Subroutine INP provides the necessary statements to handle and read the parameter data supplied on magnetic tape.

A4.4.2.14 STA 1

Subroutine STA 1 is called to initialize all the variables used in the computation of the statistics of the generated flow data.

A4.4.2.15 GEN

Subroutine GEN computes the probabilistic components of the genereted flow; that is, that portion of equation[A4.27]which is added to the deterministic component A_i (τ) to obtain the generated flow data.

A4.4.2.16 ITRAN

Subroutine ITRAN performs two transformations, or more accurately described, re-transformations. In subroutine FCOEF, the data were converted to normalized deviations and in subroutine TRAN the data were given a normalizing transformation, depending upon the information fed by one of the control cards. Subroutine ITRAN returns the normalized deviations to transformed flow data by the equation:

$$Q = x\ (\sigma_Q) + QEST \quad . \ . \ . \ . \quad [\text{Eq. A4.61}]$$

where Q is the transformed generated flow, x is the normalized deviation, σ_Q is the standard deviation of the transformed flows and QEST is the mean of the transformed flows. Following this, the inverse transform, corresponding to the transform used in subroutine TRAN, is used to return the flow data back to the original form. Subroutine ITRAN also calls subroutine STA3.

A4.4.2.17 STA 3

Subroutine STA 3 makes the summations necessary to compute the first, second, third and fourth statistical moments about zero. These sums are transferred to subroutine STA 4.

A4.4.2.18 STA 4

Subroutine STA 4 receives the summation needed to compute the mean, standard deviation, skewness and kurtosis from subroutine STA 3 and computes these statistical parameters for the generated flows.

Finally, FLASH-MAIN can, depending upon the control statements provided, write out the generated flow data or it can place the generated data on magnetic tape for subsequent use. It would be an easy matter to change the output to punched cards if cards are more suitable than the magnetic tape. Also, it is possible to include other main programs; i.e., TFLOW and WASP in an overall "do loop" to generate gage data, compute unregulated stream flows, compute regulated stream flows and simulate water quality for all for a given time frame before proceeding to the next time frame. The total program length of these combined programs is in excess of 426,000 bytes.

A4.4.3 Program Input

The program input for FLASH is as follows:

(1) For FLASH-MAIN:

Card # 1 (9I5) NYR = Number of years of historical weekly data to be used. When parameters are used, NYR=0.

NYRG = Number of years of data to be generated.

NSITES= Number of gage sites from which historical data are to be used and for which data are to be generated; the number of basis gages.

NTRAN = The transformation option code = 1, for no transformation of historical data.
= 2 for log transformation.
= 3 for square-root transform.

IRAN = Initial random number; 5 digits.

ISAVE = 1, if generated data are to be written on magnetic tape.

= 0, otherwise.
IPARAM = 1, if generator parameters are to be written on magnetic tape, for future use as input.
= 0, otherwise.
NPRINT = The number of years of generated data to be printed as program output.
IHIST = 1, if historical data are to be read from punched data cards.
= 0, if historical data are to be read from an edited and filled data tape.

(2) For subroutine WFLOW: Historical data are read. For input on magnetic tape, data should be placed on the tape according to the format: N1, N2, N3, Q(K,L,J,I), L = 1,4, J = 1,12, K = 1, NYR

where:
N1 = The gaging station number, I.
N2 = The year, K.
N3 = The month, J.
Q(K,L,J,I)= Four weekly flows.

For input on punched cards, the format should be Q(K,L,J,I) L = 1,4,4F8.2 to be read in nested "do loops" for J months, K years and I stations.

(3) For subroutine INP: When the parameters needed for generating synthetic data have been developed previously and are stored on tape for repeated use, the parameters are read in by subroutine INP. See the output from subroutine OUTP. The tape will contain:

(a) QEST (L,J,I), L = 1,4, J = 1,12, I = 1, NSITES.
(b) QVAR (J,I), J = 1,12 I = 1, NSITES.
(c) B(J,K,I), J = 1,N, K = 1,N, I = 1,12.
(d) C (J, K,I) J = 1,N, K = 1,N I = 1,12.

The format and data are established by OUTP, provided the control card for FLASH-MAIN (card #1) contains IPARAM = 1.

A4.4.4 Program Output

The program output for FLASH follows.

(1) From subrotuine OUT1. The output from subroutine OUT1 is the result of the computation, by subroutine TRAN, of the statistical parameters for the historical gage flow data. Two sets are printed out; the first set is for the edited and filled data without transformation and the second set is for the edited and filled data, but computed using the transformed data.

The output contains the mean, standard deviation, skewness and kurtosis computed for each gaging site for each week of the year. The sums used in computing the statistical moments are taken for week i over the number of years of historical data used; i.e., if 30 years of historical data are used, the mean is computed by taking one-thirtieth of the sum of the 30 data values for the ith week.

These two sets of statistical parameters allow the comparison of the parameters of the transformed and untransformed data and, further, allow comparing these parameters with those of the generated data which are printed out by subroutine OUT6. The output amounts to 48N lines for each set, where N is the number of gage sites.
(2) From subroutine OUT3. The output from subroutine OUT3 is the set of correlation coefficients computed by subroutine COREL. These coefficients are the elements that make up the arrays S_{11}, S_{12} and S_{22} described in section A4.4.2.7.

Referring to Figure A4-7A, the correlations in any row are between the site listed on the left and the site listed on the right. If the site numbers are the same, the correlation is serial in time, as shown in the first three lines. The lag 0 serial correlation is 1.000, as shown. The value 0.0352, indicated by (1) in the figure, is the average correlation in month 1 between the flow at site 1 in current frame and the flow at site 1 which occurred three time frames previously. Similarly, the value - 0.0527, indicated by (2), is the average correlation in the previous month, (actually the 12th month in the case shown), between the flow at site 1 and the flow at site 1 which occurred three weeks previously. That is, row two shows the correlation coefficients corresponding to those of row one, excepting they are for the month just passed.

The third row shows average correlation coefficients between the flow in the current time frame and the flows which occurred one to seven weeks previously.

The data shown in rows 4 through 6 in Figure A4-7A are average correlation coefficients between flows at site 1 and site 2 for the current month, the previous month and for the seven lag periods, respectively. In this case, and elsewhere, where the site numbers are different, the coefficients are a combination of serial (time) correlation and spatial correlation.

Figure A4-7A
Sample of Output
Subroutine OUT3

```
SITE   MONTH   LAG 0    LAG 1   LAG 2   LAG 3      LAG 4    LAG 5    LAG 6    LAG 7     SITE

 1       1    1,000 0   .4379   .0872   .0352(1)                                          1

 1       1    1,000 0   .3036   .0318  -.0527(2)                                          1

 1       1              .3489   .0357   .3007    -.0114   -.1633   -.1432   -.5543       1

 1       1    .9100     .4093   .0588  -.1918                                             2

 1       1    .9141     .3896   .0491   .1043                                             2

 1       1              .0720   .0680  -.0949    -.1141   -.0844   -.0778   -.5656       2

                                 etc
```

This output amounts to $36N^2$ lines, where N is the number of gaging sites used.

(3) From subroutine OUT4. The output from subroutine OUT4 is the result of the computations made in subroutine FCOEF. The output consists of two sets of data; first, the estimated average weekly flow for each site for each week of the year and secondly, the weekly standard deviation for each site for each month. These data were developed by fitting a Fourier series of six harmonics to the historical data and represent the "best fit" for each week based upon the N years of historical data used. The estimated average weekly flow corresponds to the $A_i(\tau)$ and the weekly standard deviation corresponds to the σ_i in equation [A4.27].

The number of lines of output for subroutine OUT4 is 24N, where N is the number of sites used.

(4) From subroutine OUT5. The output from subroutine OUT5 also is two sets of data. The first set consists of the values of the parameter B and the second set lists the values of parameter C. Both are used in equation [A7.27]. These parameters are the coefficients which result from the multivariate correlation techniques described in Section 6.4.

The output is in the form of a 4N x 4N matrix (N is the number of sites being used) for each of the two parameters. Because only seven columns can be spaced on a printout sheet, it is necessary to print the matrix in a folded format with the number of folds equal to the next larger integral value of $p = 4N/7$ if 4N is not exactly divisible by seven.

The total lines of output for each of the two sets is given by $48Np'$ where p' is the integer next greater than p.

(5) From FLASH-MAIN. FLASH-MAIN prints out the generated data or places the generated data on magnetic tape, depending upon the value given ISAVE on data card number 1. If ISAVE = 1 the generated data will be placed on tape, if ISAVE = 0 the output will be printed. It would be an easy modification to have the output in the form of punched cards.

The format of the data on tapes will have the generated flows for the first week for all sites spread across the tape; the second week data will appear in the next row and so on through the months and years. The format of the printed data is such that the columns correspond to the months of the year, with the first four rows in the first column giving the generated flows for site 1 for the first month, the first four rows in the second month, and so on. The data for site 2 follow in the 5th through 8th rows, for site 3 in the 9th through 12th rows, and so on. The year number is printed and the data for each year are separated.

(6) From subroutine OUT6. Subroutine OUT6 prints out the data computed by subroutine STA4; namely, the statistics of generated flow data. These data correspond to and are in the same format as the output from subroutine OUT1 for untransformed historical data, so that comparison of the statistical parameters is facilitated. OUT6 prints the statistical parameters, mean, standard deviation, skewness and kurtosis for the generated data for each site for each month of the year with sums taken over the number of years of data generated. The number of lines of output is 48N, where N is the number of sites.

A4.4.5 Dictionary of Variables.

Following is a list of the variables used in FLASH and a brief definition of each.

AO	Fourier constant, flow estimate equation.
ASI(J)	Fourier constant, flow estimate equation, COEFF. Jth harmonic.
AV	Mean.
B	Generating parameter - regression coefficient.
BCO(J)	Fourier constant, flow estimate equation, COEFF. Jth harmonic.
C	Generating parameter - variance coefficient.
CURT	Kurtosis, gage flows, transformed data.
DEV	Deviation, equals Q-QEST.
GAV	Mean, generated flows.
GCURT	Kurtosis, generated flows.
GQ(J,L.I)	Generated flow, Jth month, Lth week, Ith station, deviation.
GSD	Standard deviation, generated flows.
GSKEW	Skewness, generated flows.
IHIST	Control variable - historical data source.
IPARAM	Control variable - parameters on tape.
NPRINT	Control variable - years of output.
IRAN	Initial random number.
ISAVE	Control variable - records B & C parameters.
N1	Year number, data.
N2	Month number, data.
N3	Week number, data.
NFREQ	Number of harmonics, estimating equation.
NSITES	Number of basis gages to be used.
NSP	Number of data points.
NTRAN	Control variable - selects transform.
NYR	Number of years of historical gage data to be read.
NYRG	Number of years of synthetic gage data to be generated.
Q(K,L,J,I)	Gage flow, Kth year, week, L, month J, station I, transformed.
QAV	Mean gage flow, transformed data.

QEST	Expected value of flow, = A(τ).
QPR(K)	Flow for Kth previous time frame.
QSD	Standard deviation, gage flows, transformed data.
QVAR	Variance of computed flow estimates.
R(I)	Random number, Ith number generated.
S11(L,I,LL,II)	Correlation matrix, current month, weeks L & LL, sites I & II.
S12(L,I,LL,II)	Correlation matrix, lag month, week L this mo. week LL last.
S22(L,I,LL,II)	Correlation matrix, previous month.
S3	Skewness.
S4	Kurtosis.
SKEW	Skewness, gage flows, transformed data.
SL12(M)	Summing variable.
SLL12(M)	Summing variable.
SUML1(M)	Summing variable.
SUMLL1(M)	Summing variable.
SUML2(M)	Summing variable.
SUMLL2(M)	Summing variable.
T11(M)	Element of correlation matrix, lag 0 through 3, week M.
T12(M)	Element of correlation matrix, cross.
T22(M)	Element of correlation matrix, lag 4 through 7, week M.
XCURT	Kurtosis, gage flows, untransformed data.
XQAV	Mean gage flow, untransformed data.
XQSD	Standard deviation of gage flow, untransformed data.
XSKEW	Skewness, gage flows, untransformed data.

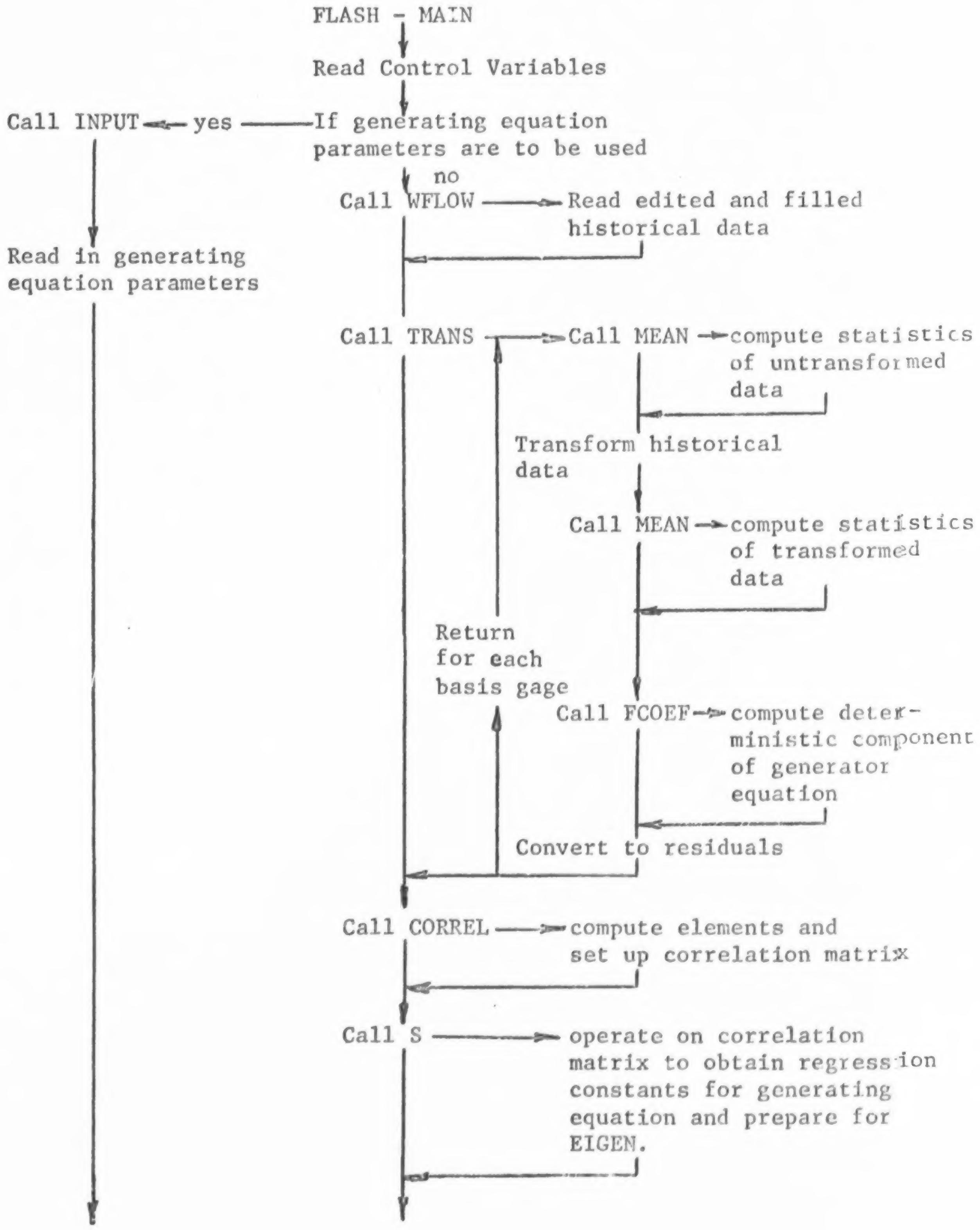
A4.4.6 Program Logic.

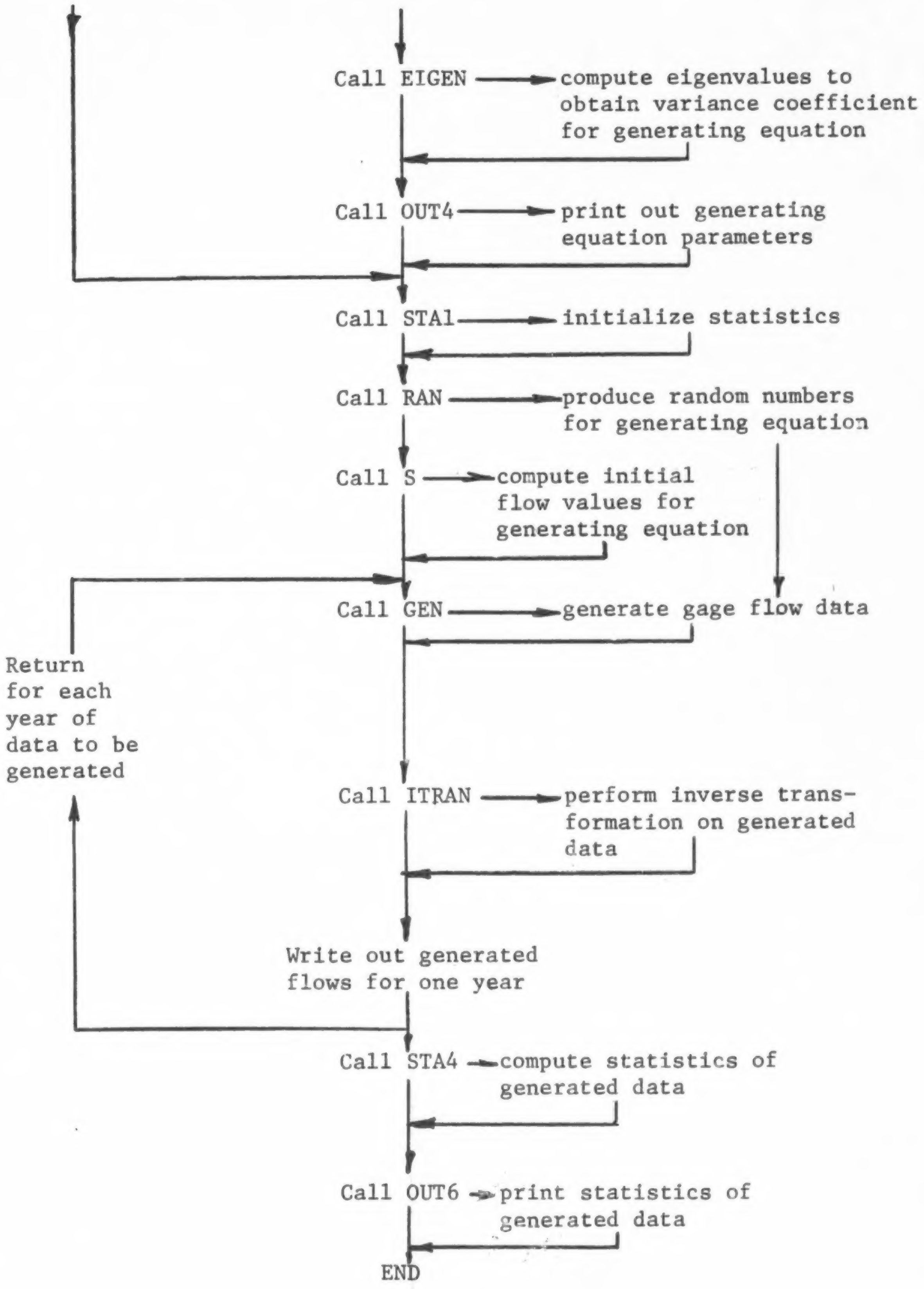
Figure A4-8 is a diagram of program logic for FLASH.

A4.4.7 Program Coding.

The program coding for FLASH follows.

FIGURE A4-8
PROGRAM LOGIC - FLASH


FLASH - MAIN
Read Control Variables
Call INPUT
yes
If generating equation parameters are to be used
no
Call WFLOW
Read edited and filled historical data
Read in generating equation parameters
Call TRANS
Call MEAN
compute statistics of untransformed data
Transform historical data
Call MEAN
compute statistics of transformed data
Return for each basis gage
Call FCOEF
compute deterministic component of generator equation
Convert to residuals
Call CORREL
compute elements and set up correlation matrix
Call S
operate on correlation matrix to obtain regression constants for generating equation and prepare for EIGEN.


Call EIGEN
compute eigenvalues to obtain variance coefficient for generating equation
Call OUT4
print out generating equation parameters
Call STA1
initialize statistics
Call RAN
produce random numbers for generating equation
Call S
compute initial flow values for generating equation
Call GEN
generate gage flow data
Return for each year of data to be generated
Call ITRAN
perform inverse transformation on generated data
Write out generated flows for one year
Call STA4
compute statistics of generated data
Call OUT6
print statistics of generated data
END

```
//FLASH    JOB (1143,47,020,18,0000),'ALEMAN                  ',CLASS
                                                              =L
// EXEC FORTRAN
//SOURCE   DD *
C   FLORIDA SYNTHETIC HYDROLOGY MODEL(FLASH)
      DIMENSION Q(50,4,12,8),XQAV(4,12,8),
     1          XQSD(4,12,8),XSKEW(4,12,8),XCURT(4,12,8),QAV(4,12
                                                              ,8),
     2          QSD(4,12,8),SKEW(4,12,8),CURT(4,12,8),AO(8),ASI(6
                                                              ,8),
     3          BCO(6,8),QEST(4,12,8),        QVAR(12,8),S11(32,3
                                                              2),
     4         S12(32,32),S21(32,32),S22(32,32),B(32,32,12),
     5         C(32,32,12),EVAL(32),QPR(32),GQ(4,8,12),GAV(4,8,12
                                                              ),
     6          GSD(4,8,12),GSKEW(4,8,12),GCURT(4,8,12)
  200 READ(5,5000)NYR,NYRG,NSITES,NTRAN,IRAN,ISAVE,IPARAM,NPRINT,
                                                              IHIST
 5000 FORMAT(9I5)
      IF(NYR.EQ.0)GO TO 50
      CALL WFLOW(NYR,NSITES,Q,IHIST)
      N=4*NSITES
      CALL TRANS(NYR,NSITES,NTRAN,Q,XQAV,XQSD,XSKEW,XCURT,QAV,QSD
                                                              ,SKEW,
     1           CURT,AO,ASI,BCO,QEST,      QVAR)
      CALL OUT2
      DO 10 J=1,12
      CALL COREL(NYR,NSITES,Q,QVAR,S11,S12,S21,S22,J)
      CALL S(32,4,S22,S22,0,N,N,0,TAG)
      CALL S(32,3,B(1,1,J),S12,S22,N,N,N,0)
      CALL S(32,3,S22,B(1,1,J),S21,N,N,N,0)
      CALL S(32,2,C(1,1,J),S11,S22,N,N,N,0)
      CALL EIGEN(32,C(1,1,J),EVAL,N,N)
      DO 10 I=1,N
      XL=0.
      IF(EVAL(I).LE.0.)GO TO 11
      XL=SQRT(EVAL(I))
   11 DO 10 II=1,N
   10 C(II,I,J)=C(II,I,J)*XL
      CALL OUT4(NSITES,QEST,QVAR)
      CALL OUT5(NSITES,B,C)
      IF(IPARAM.GT.0)CALL OUTP(NSITES,QEST,QVAR,B,C)
      GO TO 100
   50 CALL INP(NSITES,QEST,QVAR,B,C)
  100 IF(NYRG.LE.0)RETURN
      IF(ISAVE.GT.0)REWIND 9
      CALL STA1(NSITES,GAV,GSD,GSKEW,GCURT)
      N=4*NSITES
      CALL RAN (IRAN,N,S11)
      CALL S(32,3,QPR,C(1,1,12),S11,N,N,1,0)
      CALL S(32,10,QPR,C,C,N,1,6,0)
      DO 101 K=1,NYRG
      CALL GEN(IRAN,NSITES,QPR,B,C,GQ,NPRINT)
      CALL ITRAN (NSITES,NTRAN,GQ,QVAR,QEST,GAV,GSD,GSKEW,GCURT)
      IF(NPRINT.LE.0)GO TO 160
      WRITE (6,6001) K,(((GQ(J,L,I),I=1,12),J=1,4),L=1,NSITES)
 6001 FORMAT (//1X,'GENERATED FLOWS FOR YEAR',I3/(1X,12G10.3))
  160 IF(ISAVE.LE.0)GO TO 101
      WRITE(9)(((GQ(L,J,I),J=1,NSITES),L=1,4),I=1,12)
  101 CONTINUE
      CALL STA4(NSITES,GAV,GSD,GSKEW,GCURT,NYRG)
      CALL OUT6(NSITES,GAV,GSD,GSKEW,GCURT)
      IF (ISAVE.LE.0) CALL EXIT
      END FILE 9
      REWIND 9
      CALL EXIT
      END
      SUBROUTINE GEN(IRAN,NSITES,QPR,B.C,GQ,NPRINT)
      DIMENSION QPR(32),GQ(32,12),B(32,32,12),C(32,32,12),R(32),
     1GQ1(32,12)
      N=4*NSITES
      XN=N
      CALL S(32,10,QPR,C,C,N,1,6,0)
      DO 10 J=1,12
      CALL RAN(IRAN,N,R)
      DO 21 K=1,N
      GQ(K,J)=0.
      GQ1(K,J)=0.0
      DO 20 M=1,N
      GQ(K,J)=GQ(K,J)+C(K,M,J)*R(M)
   20 GQ1(K,J)=GQ1(K,J)+B(K,M,J)*QPR(M)
   21 GQ(K,J)=GQ(K,J)+GQ1(K,J)/XN
      DO 10 K=1,N
   10 QPR(K)=GQ(K,J)
      RETURN
      END
      SUBROUTINE COREL(NYR,NSITES,Q,QVAR,S11,S12,S21,S22,J)
      DIMENSION Q(50,4,12,8),S11(4,8,4,8),S12(4,8,4,8),S21(4,8,4,
                                                           8),
     1           S22(4,8,4,8),T11(4),T22(4),T12(7),QVAR(12,8),NM(7
                                                           )
     1          ,SL12(7),SLL12(7),SUML1(4),SUMLL1(4),SUML2(4),SUML
                                                           L2(4)
      DATA NM/1,2,3,4,3,2,1/
      N=4*NSITES
      JJ=J-1
      IF(J.EQ.1)JJ=12
      DO 10 I=1,NSITES
      DO 10 II=1,NSITES
      DO 5 M=1,7
      SL12(M)=0
      SLL12(M)=0
 5 T12(M)=0.
   DO 6 M=1,4
   T11(M)=0.
   SUML1(M)=0
   SUMLL1(M)=0
   SUML2(M)=0
   SUMLL2(M)=0
 6 T22(M)=0.
   DO 20 L=1,4
   DO 20 LL=1,L
   M=L-LL+1
   DO 20 K=1,NYR
   T11(M)=T11(M)+Q(K,L,J,I)*Q(K,LL,J,II)
   SUML1(M)=SUML1(M)+Q(K,L,J,I)**2
   SUMLL1(M)=SUMLL1(M)+Q(K,LL,J,II)**2
   KK=K
   IF(JJ.LT.12)GO TO 21
   KK=K-1
   IF(KK.LT.1)KK=NYR
21 T22(M)=T22(M)+Q(KK,L,JJ,I)*Q(KK,LL,JJ,II)
   SUML2(M)=SUML2(M)+Q(KK,L,JJ,I)**2
20 SUMLL2(M)=SUMLL2(M)+Q(KK,LL,JJ,II)**2
   DO 25 L=1,4
   DO 25 LL=1,4
   M=L-LL+4
   DO 25 K=1,NYR
   KK=K
   IF(JJ.LT.12)GO TO 24
   KK=K-1
   IF(KK.LT.1)KK=NYR
24 T12(M)=T12(M)+Q(K,L,J,I)*Q(KK,LL,JJ,II)
   SL12(M)=SL12(M)+Q(K,L,J,I)**2
25 SLL12(M)=SLL12(M)+Q(KK,LL,JJ,II)**2
   DO 30 M=1,4
   T11(M)=T11(M)/SQRT(SUML1(M)*SUMLL1(M))
30 T22(M)=T22(M)/SQRT(SUML2(M)*SUMLL2(M))
   DO 35 M=1,7
35 T12(M)=T12(M)/SQRT(SL12(M)*SLL12(M))
   CALL CUT3(T11,T12,I,II,J)
   DO 40 L=1,4
   DO 40 LL=1,L
   M=L-LL+1
   S11(L,I,LL,II)=T11(M)
   S11(LL,I,L,II)=T11(M)
   S22(L,I,LL,II)=T22(M)
40 S22(LL,I,L,II)=T22(M)
   DO 10 L=1,4
   DO 10 LL=1,4
   M=L-LL+4
10 S12(L,I,LL,II)=T12(M)
   CALL S(32,6,S21,S12,C,N,N,0,0)
   RETURN
   END
   SUBROUTINE TRANS(NYR,NSITES,NTRAN,Q,XQAV,XQSD,XSKEW,XCURT,
  1                    QAV,QSD,SKEW,CURT,AO,ASI,BCO,QEST       ,
                                                           QVAR)
   DIMENSION Q(50,4,12,8),XQAV(4,12,8),XQSD(4,12,8),XSKEW(4,12
                                                            ,8),
  1          AO(8),ASI(6,8),BCO(6,8),QVAR(12,8),XCURT(4,12,8),
  2          QAV(4,12,8),QSD(4,12,8),SKEW(4,12,8),CURT(4,12,8)
                                                         ,

  3          QEST(4,12,8)
    DO 10 I=1,NSITES
    DO 5 J=1,12
    DO 5 L=1,4
    CALL MEAN(Q(1,L,J,I),NYR,XQAV(L,J,I),XQSD(L,J,I),XSKEW(L,J,
                                                          I),

  1          XCURT(L,J,I))
    IF(NTRAN-2)4,20,30
 20 DO 25 K=1,NYR
 25 Q(K,L,J,I)=ALOG(Q(K,L,J,I))
    GO TO 4
 30 DO 35 K=1,NYR
 35 Q(K,L,J,I)=SQRT(Q(K,L,J,I))
  4 CALL MEAN(Q(1,L,J,I),NYR,QAV(L,J,I),QSD(L,J,I),SKEW(L,J,I),
  1          CURT(L,J,I))
  5 CONTINUE
    CALL FCOEF(QAV(1,1,I),48,6,AO(I),ASI(1,I),BCO(1,I),QEST(1,1
                                                          ,I))

    DO 40 J=1,12
    DEV=0.
    DO 45 L=1,4
    DO 45 K=1,NYR
    X=Q(K,L,J,I)-QEST(L,J,I)
    DEV=DEV+X**2
 45 Q(K,L,J,I)=X
    FN=4*NYR
    QVAR(J,I)=SQRT(DEV/FN)
    DO 40 L=1,4
    DO 40 K=1,NYR
 40 Q(K,L,J,I)=Q(K,L,J,I)/QVAR(J,I)
 10 CONTINUE
    CALL OUT1(NSITES,NTRAN,XQAV,XQSD,XSKEW,XCURT,QAV,QSD,SKEW,C
                                                          URT)

    RETURN
    END
    SUBROUTINE WFLOW (NYR,NSITES,Q,IHIST)
    DIMENSION Q(50,4,12,8)
    IF(IHIST)4,4,14
  4 IO=12
    GO TO 15
 14 IO=5
```

```
  15 DO 20 I=1,NSITES
     DO 20 K=1,NYR
     DO 20 J=1,12
  20 READ(IC,5000)(Q(K,L,J,I),L=1,4)
5000 FORMAT (14X,4F8.2)
     RETURN
     END
     SUBROUTINE ITRAN(NSITES,NTRAN,GQ,QVAR,QEST,GAV,GSD,GSKEW,GC
                                                              URT)
     DIMENSION GQ(4,8,12),QVAR(12,8),QEST(4,12,8),GAV(4,8,12),
    1          GSD(4,8,12),GSKEW(4,8,12),GCURT(4,8,12)
     DO 10 I=1,NSITES
     DO 10 J=1,12
     DO 10 L=1,4
     GQ(L,I,J)=GQ(L,I,J)*QVAR(J,I)+QEST(L,J,I)
     IF(NTRAN-2)10,20,30
  20 GQ(L,I,J)=EXP(GQ(L,I,J))
     GO TO 10
  30 GQ(L,I,J)=GQ(L,I,J)**2
  10 CONTINUE
     CALL STA3(NSITES,GQ,GAV,GSD,GSKEW,GCURT)
     RETURN
     END
     SUBROUTINE RAN(IX,N,R)
     DIMENSION R(1)
     DO 10 I=1,N
     R(I)=0.
     DO 20 J=1,12
     R(I)=R(I)+RRN(IX)
  20 CONTINUE
  10 R(I)=R(I)-6.
     RETURN
     END
     FUNCTION RRN(IX)
     IX=IX*65539
     IF(IX)5,6,6
   5 IX=IX+2147483647+1
   6 RRN=IX
     RRN=RRN*.4656613E-9
     RETURN
     END
     SUBROUTINE STA1(NSITES,GAV,GSD,GSKEW,GCURT)
     DIMENSION GAV(4,8,12),GSD(4,8,12),GSKEW(4,8,12),GCURT(4,8,1
                                                              2)
     DO 5 J=1,12
     DO 5 L=1,4
     DO 5 I=1,NSITES
     GAV(L,I,J)=0.
     GSD(L,I,J)=0.
     GSKEW(L,I,J)=0.
   5 GCURT(L,I,J)=0.
```

```
      RETURN
      END
      SUBROUTINE STA3 (NSITES,GQ,GAV,GSD,GSKEW,GCURT)
      DIMENSION GAV(4,8,12),GSD(4,8,12),GSKEW(4,8,12),GCURT(4,8,1
                                                                2)
      DIMENSION GQ(4,8,12)
      DO 20 J=1,12
      DO 20 I=1,NSITES
      DO 20 L=1,4
      X=GQ(L,I,J)
      GAV(L,I,J)=GAV(L,I,J)+X
      X=X*GQ(L,I,J)
      GSD(L,I,J)=GSD(L,I,J)+X
      X=X*GQ(L,I,J)
      GSKEW(L,I,J)=GSKEW(L,I,J)+X
      X=X*GQ(L,I,J)
   20 GCURT(L,I,J)=GCURT(L,I,J)+X
      RETURN
      END
      SUBROUTINE STA4(NSITES,GAV,GSD,GSKEW,GCURT,N)
      DIMENSION GAV(4,8,12),GSD(4,8,12),GSKEW(4,8,12),GCURT(4,8,1
                                                                2)
      DO 30 J=1,12
      DO 30 I=1,NSITES
      DO 30 L=1,4
      GAV(L,I,J)=GAV(L,I,J)/N
      GSD(L,I,J)=GSD(L,I,J)/N
      GSKEW(L,I,J)=GSKEW(L,I,J)/N
      GCURT(L,I,J)=GCURT(L,I,J)/N
      GCURT(L,I,J)=GCURT(L,I,J)-4*GSKEW(L,I,J)*GAV(L,I,J)+
     1              6*GSD(L,I,J)*GAV(L,I,J)**2-3*GAV(L,I,J)**4
      GSKEW(L,I,J)=GSKEW(L,I,J)-3*GSD(L,I,J)*GAV(L,I,J)+
     1             2*GAV(L,I,J)**3
      GSD(L,I,J) = SQRT(ABS(GSD(L,I,J)-GAV(L,I,J)**2))
      GSKEW(L,I,J)=GSKEW(L,I,J)/GSD(L,I,J)**3
      GCURT(L,I,J)=GCURT(L,I,J)/GSD(L,I,J)**4
   30 CONTINUE
      RETURN
      END
      SUBROUTINE OUTP(NSITES,QEST,QVAR,B,C)
      DIMENSION QEST(4,12,8),QVAR(12,8),B(32,32,12),C(32,32,12)
      N=4*NSITES
      NTAPE=10
      REWIND NTAPE
      WRITE(NTAPE)(((QEST(L,J,I),L=1,4),J=1,12),I=1,NSITES)
      WRITE (NTAPE)((QVAR(J,I),J=1,12),I=1,NSITES)
      WRITE(NTAPE)(((B(J,K,I),J=1,N),K=1,N),I=1,12)
      WRITE(NTAPE)(((C(J,K,I),J=1,N),K=1,N),I=1,12)
      END FILE NTAPE
      REWIND NTAPE
      RETURN
      END
      SUBROUTINE INP(NSITES,QEST,QVAR,B,C)
      DIMENSION QEST(4,12,8),QVAR(12,8),B(32,32,12),C(32,32,12)
      N=4*NSITES
      NTAPE=10
      REWIND NTAPE
      READ(NTAPE)(((QEST(L,J,I),L=1,4),J=1,12),I=1,NSITES)
      READ(NTAPE)((QVAR(J,I),J=1,12),I=1,NSITES)
      READ(NTAPE)(((B(J,K,I),J=1,N),K=1,N),I=1,12)
      READ(NTAPE)(((C(J,K,I),J=1,N),K=1,N),I=1,12)
      REWIND NTAPE
      RETURN
      END
      SUBROUTINE MEAN(X,N,AV,SD,S3,S4)
      DIMENSION X(1)
      AV=0.
      SD=0.
      S3=0.
      S4=0.
      DO 10 I=1,N
      AV=AV+X(I)
      SD=SD+X(I)**2
      S3=S3+X(I)**3
   10 S4=S4+X(I)**4
      AV=AV/N
      SD=SD/N
      S3=S3/N
      S4=S4/N
      S4=S4-4*S3*AV+6*SD*AV**2-3*AV**4
      S3 = S3 - 3*SD*AV + 2*AV**3
      SD=SQRT(((SD-AV**2)*N)/(N-1))
      S3=S3/SD**3
      S4=S4/SD**4
      RETURN
      END
        SUBROUTINE EIGEN(IDIM,A,EVAL,N,M)
C
C     EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC MATRIX
C
      DIMENSION A(IDIM,IDIM),B(60,60),EVAL(IDIM),S(60),C(60),
     1          D(60),IND(60),U(60)
       DOUBLE PRECISION ANORM,ANORM2,TAU,P,DIAG(60),VALU(60),VALL
                                                             (60),
     1  T1,T2,T,SUPERD(60),Q(60),DSI,DCO,BETA
C
C     CALCULATE NORM OF MATRIX
C
       MAXIT= 50*N*N
       IT=0
    3 ANORM2 = 0.
    4 DO 6 I=1,N
    5 DO 6 J=1,N
    6 ANORM2 = ANORM2 + A(I,J)**2
    7 ANORM =DSQRT (ANORM2)
C
C     GENERATE IDENTITY MATRIX
C
    9 IF (M) 10,45,10
   10 DO 40 I=1,N
   12 DO 40 J=1,N
   20 IF (I-J) 35,25,35
   25 B(I,J) = 1.
   30 GO TO 40
   35 B(I,J) = 0.
   40 CONTINUE
C
C     PERFORM ROTATIONS TO REDUCE MATRIX TO JACOBI FORM
C
   45 IEXIT = 1
   50 NN = N-2
   52 IF (NN) 890,170,55
   55 DO 160 I=1,NN
   60 II = I + 2
   65 DO 160 J=II,N
   70 T1 = A(I,I+1)
   75 T2 = A(I,J)
        IF(T2) 1910,160,1910
 1910   T=DSQRT(T1**2 +T2**2)
        CO=T1/T
         SI=T2/T
   90 DO 105 K=1,N
   95 T2 = CO *A(K,I+1)+SI *A(K,J)
  100 A(K,J) = CO *A(K,J)-SI *A(K,I+1)
  105 A(K,I+1) = T2
  110 DO 125 K=1,N
  115 T2 = CO *A(I+1,K) + SI *A(J,K)
  120 A(J,K) = CO *A(J,K) - SI *A(I+1,K)
  125 A(I+1,K) = T2
  128 IF (M) 130,160,130
  130 DO 150 K=1,N
  135 T2 = CO *B(K,I+1) + SI *B(K,J)
  140 B(K,J) = CO *B(K,J) - SI *B(K,I+1)
  150 B(K,I+1) = T2
  160 CONTINUE
C
C     MOVE JACOBI ELEMENTS AND INITIALIZE EIGENVALUE BOUNDS
C
  170 DO 200 I=1,N
  180 DIAG(I) = A(I,I)
  190 VALU(I) = ANORM
  200 VALL(I) = -ANORM
  210 DO 230 I=2,N
  220 SUPERD(I-1) = A(I-1,I)
  230 Q(I-1) = (SUPERD(I-1))**2
C
C     DETERMINE SIGNS OF PRINCIPAL MINORS
C
  235 TAU = 0.
  240 I = 1
  260 MATCH = 0
        IT=IT+1
  270 T2 = 0.
  275 T1 = 1.
  277 DO 450 J=1,N
  280 P = DIAG(J) - TAU
  290 IF (T2) 300,330,300
  300 IF (T1) 310,370,310
  310 T = P*T1 - Q(J-1)*T2
  320 GO TO 410
  330 IF (T1) 335,350,350
  335 T1 = -1.
  340 T = -P
  345 GO TO 410
  350 T1 = 1.
  355 T = P
  360 GO TO 410
  370 IF (Q(J-1)) 380,350,380
  380 IF (T2) 400,390,390
  390 T = -1.
  395 GO TO 410
  400 T = 1.
C
C    COUNT AGREEMENTS IN SIGN
C
  410 IF (T1) 425,420,420
  420 IF (T) 440,430,430
  425 IF (T) 430,440,440
  430 MATCH = MATCH + 1
  440 T2 = T1
  450 T1 = T
C
C     ESTABLISH TIGHTER BOUNDS ON EIGENVALUES
C
  460 DO 530 K=1,N
  465 IF (K - MATCH) 470,470,520
  470 IF (TAU - VALL(K)) 530,530,480
  480 VALL(K) = TAU
  490 GO TO 530
  520 IF (TAU - VALU(K)) 525,530,530
  525 VALU(K) = TAU
  530 CONTINUE
  540 IF (VALU(I) - VALL(I) - 5.0D-8) 570,570,550
  550 IF (VALU(I)) 560,580,560
  560 IF(DABS (VALL(I)/VALL(I) - 1.) - 5.0D-8) 570,570,580
  570 I = I + 1
        IT=0
  575 IF (I - N) 540,540,590
  580 TAU = (VALL(I) + VALU(I))/2.
        IF(IT-MAXIT) 260,260,581
  581  WRITE(6,6001)IT,I,VALL(I),VALU(I)
 6001  FORMAT(15H MAXIT EXCEEDED,2I10,2E20.8)
       GO TO 570
C
C     JACOBI EIGENVECTORS BY ROTATIONAL TRIANGULARIZATION
C
  590 IF (M) 593,890,593
  593 IEXIT = 2
  595 DO 610 I=1,N
  600 DO 610 J=1,N
  610 A(I,J) = 0.
  615 DO 850 I=1,N
  620 IF (I-1) 625,625,621
  621 IF (VALU(I-1) - VALU(I) - 5.0D-7) 730,730,622
  622 IF (VALU(I-1)) 623,625,623
  623 IF(DABS (VALU(I)/VALU(I-1) - 1.) - 5.0D-7) 730,730,625
  625 DCO=1.
  628 DSI=0.
  630 DO 700 J=1,N
  635 IF (J-1) 680,680,640
  640 T=DSQRT(T1**2+T2**2)
       DCO=T1/T
       DSI=T2/T
  650 S(J-1) =DSI
  660 C(J-1) =DCO
  670 D(J-1)= T1*DCO          +T2*DSI
  680 T1 = (DIAG(J) - VALU(I))*DCO - BETA*DSI
  690 T2 = SUPERD(J)
  700 BETA = SUPERD(J)*DCO
  710 D(N) = T1
  720 DO 725 J=1,N
  725 IND(J) = 0
  730 SMALLD = ANORM
  735 DO 780 J=1,N
  740 IF (IND(J) - 1) 750,780,780
  750 IF (ABS (SMALLD) - ABS (D(J))) 780,780,760
  760 SMALLD = D(J)
  770 NN = J
  780 CONTINUE
  790 IND(NN) = 1
  800 PRODS = 1.
  805 IF (NN-1) 810,850,810
  810 DO 840 K=2,NN
  820 II = NN + 1 - K
  830 A(II+1,I) = C(II)*PRODS
  840 PRODS = - PRODS*S(II)
  850 A(1,I) = PRODS
C
C     FORM MATRIX PRODUCT OF ROTATION MATRIX WITH JACOBI VECTOR M
                                                              ATRIX
C
  855 DO 885 J=1,N
  860 DO 865 K=1,N
  865 U(K) = A(K,J)
  870 DO 885 I=1,N
  875 A(I,J) = 0.
  880 DO 885 K=1,N
  885 A(I,J) = B(I,K)*U(K) + A(I,J)
  890 GO TO 941
  941     DO 945 I=1,N
  945  EVAL(I)=  VALU(I)
        RETURN
       END
      SUBROUTINE FCOEF(X,NSP,NFREQ,AO,AS,BC,XEST)
      DIMENSION X(48),XEST(48),AS(6),BC(6)
      T=NSP
      W=2.*3.1416/T
      AO = 0.
      DO 5 I=1,NSP
    5 AO = AO + X(I)
      AO = AO/FLOAT(NSP)
      DO 10 M=1,NFREQ
      AS(M) = 0.
      BC(M) = 0.
      TA=0.
      TB=0.
      DO 15 I=1,NSP
      WT = W*FLOAT(I*M)
      STA=SIN(WT)
      STB=COS(WT)
      AS(M)=AS(M)+X(I)*STA
   15 BC(M)=BC(M)+X(I)*STB
      AS(M)=2.*AS(M)/T
   10 BC(M)=2.*BC(M)/T
      X2 = 0.
      DO 20 I=1,NSP
      X2 = X2 + X(I)**2
      XEST(I) = AO
      DO 20 M=1,NFREQ
      WT = W*FLOAT(I*M)
   20 XEST(I) = XEST(I) + AS(M)*SIN(WT)+BC(M)*COS(WT)
      RETURN
      END
      SUBROUTINE S(KT,NN,A,B,C,IM,JM,KM,DET)
      DIMENSION A(KT,KT),B(KT,KT),C(KT,KT),IN(100),EMP(100)
      IMAX=IM
      JMAX=JM
      KMAX=KM
      GOTO(30,32,34,36,38,40,42,44,46,50,52),NN
   30 DO31I=1,IMAX
      DO31J=1,JMAX
   31 A(I,J)=B(I,J)+C(I,J)
      GO TO 805
   32 DO33I=1,IMAX
      DO33J=1,JMAX
   33 A(I,J)=B(I,J)-C(I,J)
      GO TO 805
   34 DO101I=1,IMAX
      DO35J=1,KMAX
       EMP(J)=0.
      DO35K=1,JMAX
   35  EMP(J)= EMP(J)+B(I,K)*C(K,J)
      DO101K=1,KMAX
  101 A(I,K)= EMP(K)
      GO TO 805
   36 DO37I=1,IMAX
      DO37J=1,IMAX
   37 A(I,J)=B(I,J)
   59 IN(1)=0
      IMAXO=IMAX-1
      TEMP=A(1,1)
      DO70I=2,IMAX
      IF(ABS (TEMP)-ABS (A(I,1)))71,70,70
   71 IN(1)=I
      TEMP=A(I,1)
   70 CONTINUE
      IF(IN(1))73,72,73
   73 IS=IN(1)
      DO74J=1,IMAX
      TEMP=A(1,J)
      A(1,J)=A(IS,J)
   74 A(IS,J)=TEMP
   72 IF(A(1,1))98,99,98
   98 DO75I=2,IMAX
   75 A(I,1)=A(I,1)/A(1,1)
      DO100I=2,IMAX
      IPO=I+1
      IMO=I-1
      DO80L=1,IMO
   80 A(I,I)=A(I,I)-(A(L,I)*A(I,L))
      TEMP=A(I,I)
      IF(I-IMAX)55,83,55
   55 IN(I)=0
      DO81IS=IPO,IMAX
      DO85L=1,IMO
   85 A(IS,I)=A(IS,I)-A(L,I)*A(IS,L)
      IF(ABS (TEMP)-ABS (A(IS,I)))82,81,81
   82 TEMP=A(IS,I)
      IN(I)=IS
   81 CONTINUE
      ISS=IN(I)
      IF(ISS)84,83,84
   84 DO886J=1,IMAX
      TEMP=A(I,J)
      A(I,J)=A(ISS,J)
  886 A(ISS,J)=TEMP
   83 IF(A(I,I))97,99,97
   97 IF(I-IMAX)54,100,54
   54 DO86IS=IPO,IMAX
   86 A(IS,I)=A(IS,I)/A(I,I)
      DO89JS=IPO,IMAX
      DO89L=1,IMO
   89 A(I,JS)=A(I,JS)-(A(L,JS)*A(I,L))
  100 CONTINUE
      DO600JP=1,IMAX
      J=IMAX+1-JP
      A(J,J)=1.0/A(J,J)
      IF(J-1)603,700,603
  603 DO600IP=2,J
      I=J+1-IP
      IPO=I+1
      TEMP=0.0
      DO602L=IPO,J
  602 TEMP=TEMP-A(I,L)*A(L,J)
  600 A(I,J)=TEMP/A(I,I)
  700 DO151J=1,IMAXO
      JPO=J+1
      DO151I=JPO,IMAX
      TEMP=0.0
      IMO=I-1
      DO154L=J,IMO
      IF(L-J)152,153,152
  152 TEMP=TEMP-A(I,L)*A(L,J)
      GO TO154
  153 TEMP=TEMP-A(I,L)
  154 CONTINUE
  151 A(I,J)=TEMP
      DO901I=1,IMAX
      DO900J=1,IMAX
       EMP(J)=0.0
      DO899N=I,IMAX
      IF(N-J)899,897,898
  898  EMP(J)= EMP(J)+A(I,N)*A(N,J)
      GO TO899
  897  EMP(J)= EMP(J)+A(I,N)
  899 CONTINUE
  900 CONTINUE
      DO901J=1,IMAX
```

```
901 A(I,J)= EMP(J)
    DO500I=2,IMAX
    M=IMAX+1-I
    IF(IN(M))502,500,502
502 ISS=IN(M)
    DO503L=1,IMAX
    TEMP=A(L,ISS)
    A(L,ISS)=A(L,M)
503 A(L,M)=TEMP
500 CONTINUE
    DET=0.
    GO TO 805
 99 WRITE             (6,806 )
    DET=1.
806 FORMAT(18H0  SINGULAR MATRIX)
805 RETURN
 38 DO39I=1,IMAX
    DO39J=1,IMAX
 39 A(I,J)=B(I,J)
    N=IMAX
    DET=1.
    I1=1
  1 I3=I1
    SUM=ABS (A(I1,I1))
    DO3I=I1,N
    IF(SUM-ABS (A(I,I1)))2,3,3
  2 I3=I
    SUM=ABS (A(I,I1))
  3 CONTINUE
    IF(I3-I1)4,6,4
  4 DO5J=1,N
    SUM=-A(I1,J)
    A(I1,J)=A(I3,J)
  5 A(I3,J)=SUM
  6 I3=I1+1
    DO7I=I3,N
  7 A(I,I1)=A(I,I1)/A(I1,I1)
    J2=I1-1
    IF(J2)8,11,8
  8 DO9J=I3,N
    DO9I=1,J2
  9 A(I1,J)=A(I1,J)-A(I1,I)*A(I,J)
 11 J2=I1
    I1=I1+1
    DO12I=I1,N
    DO12J=1,J2
 12 A(I,I1)=A(I,I1)-A(I,J)*A(J,I1)
    IF(I1-N)1,14,1
 14 I3=1
    J2=N/2
    IF(2*J2-N)15,16,15
 15 I3=0
    DET=A(N,N)
 16 DO17I=1,J2
    J=N-I+I3
 17 DET=DET*A(I,I)*A(J,J)
    GO TO 805
 40 IF(IMAX-JMAX)41,102,102
 41 IP=IMAX
    GO TO 103
102 IP=JMAX
103 DO106K=1,IP
    DO104I=K,IMAX
104  EMP(I)=B(I,K)
    DO105J=K,JMAX
105 A(J,K)=B(K,J)
    DO106I=K,IMAX
106 A(K,I)= EMP(I)
    GO TO 805
 42 DO43I=1,IMAX
    DO43J=1,JMAX
 43 A(I,J)=B(I,J)
    GO TO 805
 44 DO45I=1,IMAX
    DO45J=1,JMAX
    A(I,J)=0.
    B(I,J)=0.
 45 C(I,J)=0.
    GO TO 805
 46 ID=2
 20  READ (KMAX,47) IN(1),IN(5),EMP(1),IN(2),IN(6),EMP(2),
   1  IN(3),IN(7),EMP(3),IN(4),IN(8),EMP(4)
 47 FORMAT (4(I3,I3,E12.8))
    IF(IN(1))805,805,23
 23 GO TO(19,24),ID
 24 IM=IN(1)
    JM=IN(5)
    ID=1
 19 DO21I=1,4
    I1=IN(I)
    J1=IN(I+4)
    IF(I1)21,21,18
 18 A(I1,J1)= EMP(I)
 21 CONTINUE
    GO TO 20
 50 DO 62 IP=1,JMAX,7
    JPO=IP+6
    IF(JPO-JMAX)61,61,60
 60 JPO=JMAX
 61 WRITE             (KMAX,63)(J,J=IP,JPO)
    DO 62 I=1.IMAX
    WRITE             (KMAX,64)I,(A(I,J),J=IP,JPO)
   62 CONTINUE
      GO TO 805
   63 FORMAT(5HO ROW7(8X,4HCOL.I3,1X))
   64 FORMAT(I4,4X,7E16.8)
   52 DO53I=1,IMAX
      DO53J=1,JMAX
   53 A(I,J)=B(I,J)*DET
      GO TO 805
      END
      SUBROUTINE OUT1(NSITES,NTRAN,XQAV,XQSD,XSKEW,XCURT,QAV,QSD,
     1                     SKEW,CURT)
      DIMENSION ATRAN(3),XQAV(4,12,8),XQSD(4,12,8),XSKEW(4,12,8),
     1          XCURT(4,12,8),QAV(4,12,8),QSD(4,12,8),SKEW(4,12,
                                                              8),
     2         CURT(4,12,8)
      DATA ATRAN/4H  NO,4H LOG,4HSQRT/
      WRITE(6,6000)
 6000 FORMAT('1',26X,'STATISTICS OF HISTORICAL FLOWS')
      WRITE(6,6001)
 6001 FORMAT(///10X,'SITES',3X,'MONTH',3X,'WEEK',6X,'MEAN',6X,'ST
                                                            D DEV',
     1        4X,'SKEWNESS',4X,'KURTOSIS'/)
      DO 10 I=1,NSITES
      DO 10 J=1,12
      DO 10 L=1,4
   10 WRITE(6,6002)I,J,L,XQAV(L,J,I),XQSD(L,J,I),XSKEW(L,J,I),
     1               XCURT(L,J,I)
      WRITE(6,6003) ATRAN(NTRAN)
 6002 FORMAT(I13,I7,I8,4G12.5)
 6003 FORMAT('1',33X,A4,' TRANSFORMATION'//
     1        22X,'STATISTICS OF TRANSFORMED HISTORICAL FLOWS')
      WRITE(6,6001)
      DO 15 I=1,NSITES
      DO 15 J=1,12
      DO 15 L=1,4
   15 WRITE(6,6002)I,J,L,QAV(L,J,I),QSD(L,J,I),SKEW(L,J,I),CURT(L
                                                            ,J,I)
      RETURN
      END
      SUBROUTINE OUT2
      DIMENSION NL(8)
      DATA NL/0,1,2,3,4,5,6,7/
      WRITE(6,6004)NL
 6004 FORMAT('1',30X,'CORRELATION COEFFICIENTS OF TRANSFORMED HIS
                                                           TORICAL
     1 FLOWS'///1X,'SITE',3X,'MONTH',8(7X,'LAG',I2),3X,'SITE'/)
      RETURN
      END
      SUBROUTINE OUT3(T11,T12,I,II,J)
      DIMENSION T11(4),T12(7)
      WRITE(6,6005)I,J,T11,II
      WRITE(6,6006)I,J,T12,II
 6005 FORMAT(I4,I7,2X,4G12.5,48X,I6)
 6006 FORMAT(I4,I7,14X,7G12.5,I6)
      RETURN
      END
      SUBROUTINE OUT4(NSITES,QEST,QVAR)
      DIMENSION QEST(4,12,8),QVAR(12,8)
      WRITE(6,6007)
 6007 FORMAT('1',32X,'PARAMETER ESTIMATES - WEEKLY MEAN'//
     1       17X,'SITE',5X,'MONTH')
      WRITE(6,6008)((I,J,(QEST(L,J,I),L=1,4),J=1,12),I=1,NSITES)
 6008 FORMAT(15X,I5,4X,I5,5X,4G12.5)
      WRITE(6,6009)
 6009 FORMAT('1',21X,'PARAMETER ESTIMATES - WEEKLY STANDARD DEVIA
                                                             TION'
     1       //30X,'SITE',5X,'MONTH')
      WRITE(6,6010)((I,J,QVAR(J,I),J=1,12),I=1,NSITES)
 6010 FORMAT(30X,I3,I9,6X,G12.5)
      RETURN
      END
      SUBROUTINE OUT5(NSITES,B,C)
      DIMENSION B(32,32,12),C(32,32,12)
      N=4*NSITES
      WRITE(6,6011)
 6011 FORMAT('1',40X,'PARAMETER ESTIMATES - B')
      DO 30 J=1,12
      WRITE(6,6012)J
 6012 FORMAT(///48X,'MONTH',I3/)
      CALL S(32,10,B(1,1,J),0,0,N,N,6,0)
   30 CONTINUE
      WRITE(6,6013)
 6013 FORMAT('1',40X,'PARAMETER ESTIMATES - C')
      DO 35 J=1,12
      WRITE(6,6012)J
      CALL S(32,10,C(1,1,J),0,0,N,N,6,0)
   35 CONTINUE
      RETURN
      END
      SUBROUTINE OUT6(NSITES,AUTOC,GAV,GSD,GSKEW,GCURT)
      DIMENSION AUTOC(4,8,4,8,2,12),GAV(4,8,12),GSD(4,8,12),
     1          GSKEW(4,8,12),GCURT(4,8,12),NL(8),NM(7),T1(4),T2(
                                                              7)
      DATA NM/1,2,3,4,3,2,1/
      DATA NL/0,1,2,3,4,5,6,7/
      WRITE(6,6014)
 6014 FORMAT('1',27X,'STATISTICS OF GENERATED FLOWS')
      WRITE(6,6015)
 6015 FORMAT(///10X,'SITE',3X,'MONTH',3X,'WEEK',6X,'MEAN',6X,
     1       'STD DEV',4X,'SKEWNESS',4X,'KURTOSIS'/)
      DO 10 I=1,NSITES
      DO 10 J=1,12
      DO 10 L=1,4
   10 WRITE(6,6016)I,J,L,GAV(L,J,I),GSD(L,J,I),GSKEW(L,J,I),
     1             GCURT(L,J,I)
 6016 FORMAT(I13,I7,I8,4G12.5)
      WRITE(6,6017)NL
 6017 FORMAT('1',29X,'CORRELATION COEFFICIENTS OF GENERATED TRANS
                                                           FORMED
     1 FLOWS'///' SITE',3X,'MONTH',8(7X,'LAG',I2),3X,'SITE'/)
      DO 20 J=1,12
      DO 20 I=1,NSITES
      DO 20 II=1,NSITES
      DO 23 M=1,4
   23 T1(M)=0
      DO 24 M=1,7
   24 T2(M)=0
      DO 25 L=1,4
      DO 25 LL=L,4
      M=LL-L+1
   25 T1(M)=T1(M)+AUTOC(L,I,LL,II,1,J)
      DO 30 L=1,4
      DO 30 LL=1,4
      M=L-LL+5
   30 T2(M)=T2(M)+AUTOC(L,I,LL,II,2,J)
      DO 35 M=1,4
      MM=5-M
   35 T1(M)=T1(M)/MM
      WRITE(6,6018)I,J,T1,II
 6018 FORMAT(I4,I7,2X,4G12.5,48X,I6)
      DO 40 M=1,7
   40 T2(M)=T2(M)/NM(M)
   20 WRITE(6,6019)I,J,T2,II
 6019 FORMAT(I4,I7,14X,7G12.5,I6)
      RETURN
      END
/*
//GO.FT09F001 DD UNIT=TAPE9,VOLUME=SER=XXX,LABEL=(,BLP),DISP=(,PA
                                                            SS)
/*EOF
```

A4.5 WASP - Watershed Simulation Program

A4.5.1 Purpose.

The basic requirement in the employment of simulation for watershed analysis is a methodology or program logic which, when given the operating conditions and parameters, will adequately represent the real-life interactions and print the results in usable form. WASP is a combination of a set of mathematical models which describe the primary interactions of the factors and a program logic which links them all together in a simulation of the river system.

In this study, the river flows and water quality are simulated. Water quality is dependent upon river flows and is, consequently, affected by the results of the flow simulation. Simulated flows are generated by taking gage data generated by FLASH and transforming gage data into unregulated stream flows at reach points in the stream. Then, by application of the operating rules for the devices and procedures which modify the flow in a stream, the regulated flows at each reach point in the stream can be computed. The waste loads are then applied and operated upon by the river system to obtain the conditions of water quality at reach points in the system. The time series traces of regulated flows and water quality parameter values for all points in the watershed are the objectives of the simulation process subsequently described in detail.

The simulation program is controlled by WASP-MAIN which sets up the common blocks, reads control data and calls supporting subroutines. The program passes automatically from subroutine to subroutine to generate simulated unregulated stream flows, regulated stream flows and water quality indicating values for each reach point. This is done for any number of years desired. The program simulates the flow and quality for the whole watershed for one "week," prints out the results, then goes to the next "week" and repeats the process, continuing "week" by "week' until the number of years desired are traversed.

The first operation, to generate unregulated stream flows, makes use of the program TFLOW which is described in A4.3. The only modification in TFLOW needed to adapt it for this use is to delete card #3 and card #4 in the input for TFLOW-MAIN. This deletes the requirement for estimates and consequently no comparisons are computed. The data input to subroutine WEEKLY, for this use of TFLOW, must be the generated gage flows from FLASH, the synthetic gage data generator. The reader is referred to A4.3 for the description of TFLOW and the basis for the computation of the weighting factors which form the elements of the transformation matrix which converts gage data to streamflow data at all reaches.

The values called QNAT, for natural flow, are generated by summing the appropriate weight factors multiplied by their current week gage data:

$$QNAT_i = \sum_{j=1}^{NG} w_{ij}\, x_j \quad . . . . \text{[Eq. A4.62]}$$

where:

$QNAT_i$ = the generated flow at the upper end of reach i.
$w_{i,j}$ = the weight coefficient for the i^{th} reach and j^{th} gage.
x_j = the generated gage flow for j^{th} gage.
NG = number of gages.

The program generates the weighting coefficients only once and they are stored for use again for each week of simulation. The weighting coefficients will remain unchanged as long as the configuration of gages used is unchanged. Additionally, the sequence of computation of the reaches is set up once and used over and over again during the simulation. It takes a realignment or revision of reaches, and their numbering, to change the sequence of computation.

The second operation in the simulation is the conversion of the unregulated flows in each reach to regulated flows. The subroutine REG and its supporting subroutines superimpose the effect of any flow regulating structures or operations on the unregulated flows to produce the desired result - simulated streamflows. REG accounts for the regulation of flow by reservoirs and impoundments, the losses in reservoirs due to evaporation, diversions made from the reservoir or river for water supply and/or irrigation and discharges to the river by waste producers.

The presence of a dam and reservoir in a river system affects the flow downstream, depending upon how the control facilities at the dam are operated. Control structures are usually operated according to a fixed scheme, called "reservoir operating rules." The operating rules usually are fixed for a given reservoir but, because no two are alike, each reservoir has its unique operating rules. In this program, reservoirs have been classified into five different types, according to use, and information needed to simulate operation is input to the program as a series of parameters which are described in detail below. It is not intended that all reservoirs be forced into a five-type classification, according to use, and more or different types can be added, if desired, with only the proper attention to details. The five types set forth here will serve to indicate the manner in which the details are handled.

When regulated river flows are simulated, it is necessary to maintain an inventory of volume of water stored in each reservoir for each time frame encompassed by the simulation period. To do this, it is first necessary to know the relationship between the volume of water in the reservoir and the corresponding depth, and also the relationship between the depth and area of water surface for all volumes from empty to full. These can be developed by least squares fitting of actual reservoir data and expressed in the form:

$$C = a + bD + eD^2 \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.63]}$$

and:

$$A = f + gD + hD^2 \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.64]}$$

where C and A are reservoir capacity and surface area, respectively; D is the depth of water corresponding to C and A; and a, b, e, f, g and h are constants. Because the inventory is kept by volume, the equation relating C and D is more useful in the form:

$$D = \frac{-b + (m+nC)^{1/2}}{2e} \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.65]}$$

because C is the independent variable. In this form:

$$m = b^2 - 4ae \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.66]}$$

$$n = +4e \quad \ldots\ldots\ldots\ldots\ldots \quad \text{[Eq. A4.67]}$$

In maintaining the inventory, the new water volume, C_{t+1}, is obtained by:

$$C_{t+1} = C_t + QIN_{t+1} - EVAP_{t+1} - RREL_{t+1} - DIV_{t+1} \quad \ldots \quad \text{[Eq. A4.68]}$$

where QIN is the flow into the reservoir from upstream, EVAP is the evaporation correction, RREL is the volume released downstream and DIV is the volume diverted elsewhere. QIN is determined by simulation in the upstream reaches for time frame t+1, which is possible because the sequence of computation is upstream to downstream. Evaporation is computed by substituting C_t into equation [A4.65] to get D_t which is, in turn, used to get A_t from equation [A4.64]. The evaporation losses are a function of A_t and are described below. Releases depend upon operating rules which are usually a function of D_{t+1} or of (t+1), while diversions are independent demands which are expressed in time series relations.

Evaporation losses usually can be expressed as a sine function:

$$EVAP = a \sin (L-b) + c \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.69]}$$

where E is in inches per square foot of reservoir surface area per week, a is a constant, L is the current week in the year expressed as a time angle, b is the lag constant and c is the mean annual evaporation. If weekly or monthly average evaporation data are available, a Fourier series fit thereof having a single harmonic will suffice. If pan data are available for several years, it would be more appropriate to include a random term to account for the variance in the data. The formula then would be:

$$E = a \sin (L-b) + c + R\sigma_e \quad \ldots\ldots \quad \text{[Eq. A4.70]}$$

where R is a random number having mean = 0 and variance =1 and σ_e the standard deviation of the evaporation data.

The volume lost to evaporation during the time frame is computed by the relation:

$$QVAP = C\ E\ A \quad \ldots\ldots\ldots\ldots \quad \text{[Eq. A4.71]}$$

where QVAP is in volume units for the time frame, C is a conversion factor to make units compatible, E is the evaporation rate in inches per unit of time and A is in areal units. If QVAP is in 10^6 cubic feet per "week," E is in inches per "week" and A is in 10^6 square feet units, then $C = 1/12$.

Diversions are volumes of water delivered out of the watershed or transported to a downstream reach not adjacent to the control structure. Releases are discharges from the reservoir to the next downstream reach.

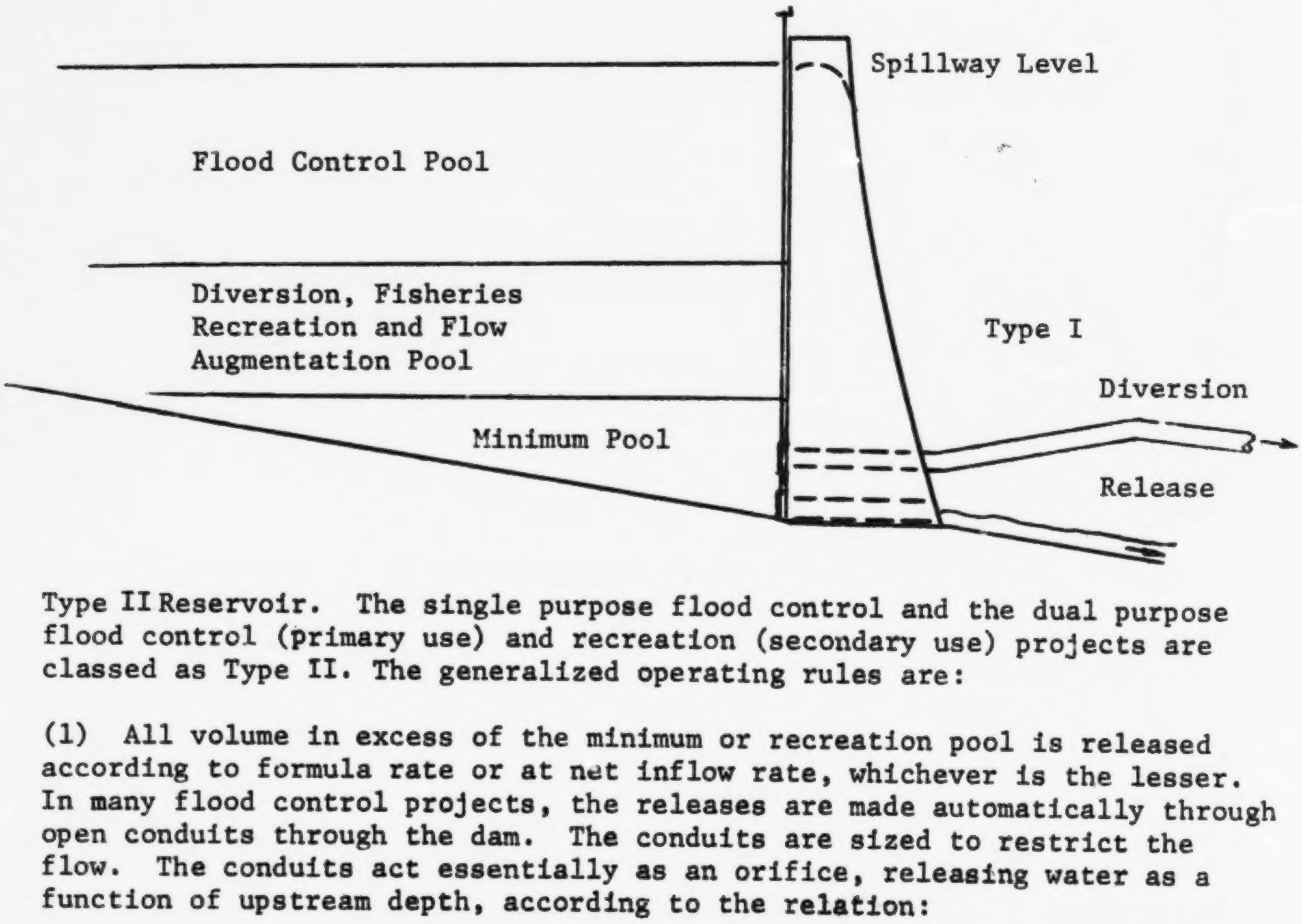
Diversions can be expressed in many different ways; i.e., they may be constant with time or they may vary according to some determinable pattern. If historical data on diverted volumes are available, it is possible to approximate the pattern by harmonic analysis. An auxiliary program is included in Appendix A2 which fits to the data a periodic curve having up to six harmonics. In any case, it is necessary either to express the diversion mathematically, in terms of the "week" of the year, or to provide a set of data cards, one for each week. The coding in this work is set up for the periodic curve of six harmonics.

Releases depend upon the volume of water in the reservoir, or the associated depth, and any other additional constraints imposed by watershed management. Releases are made according to operating rules which are, in turn, dependent upon reservoir uses.

The five types of reservoirs, classified according to use, are described below.

Type I Reservoir. The multiple purpose reservoir is classed as Type I. The uses include, but are not limited to, flood control, low flow augmentation, recreation, diversions and fisheries. The generalized operating rules are:

(1) All spillway overflows are released.
(2) All storage in the flood control pool is released at a fixed rate which will not damage any downstream property or facilities.
(3) A minimum release rate shall be maintained. Releases will be made at this minimum release rate for all pool levels between the flood control pool and the minimum pool. Store all net inflow in excess of this minimum release rate when the water level is between the flood control pool and the minimum pool.
(4) When the water level is at or below the minimum, no releases or diversions are made.
(5) Diversions are made according to schedule, excepting when the water level is at or below minimum.



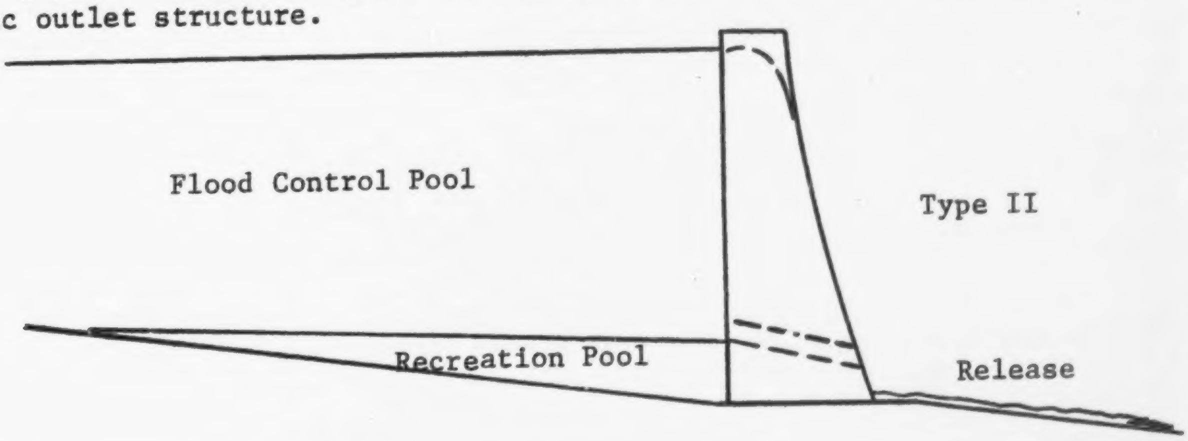
Type II Reservoir. The single purpose flood control and the dual purpose flood control (primary use) and recreation (secondary use) projects are classed as Type II. The generalized operating rules are:

(1) All volume in excess of the minimum or recreation pool is released according to formula rate or at net inflow rate, whichever is the lesser. In many flood control projects, the releases are made automatically through open conduits through the dam. The conduits are sized to restrict the flow. The conduits act essentially as an orifice, releasing water as a function of upstream depth, according to the relation:

$$Q = CH^d \quad . . . . [Eq. A4.72]$$

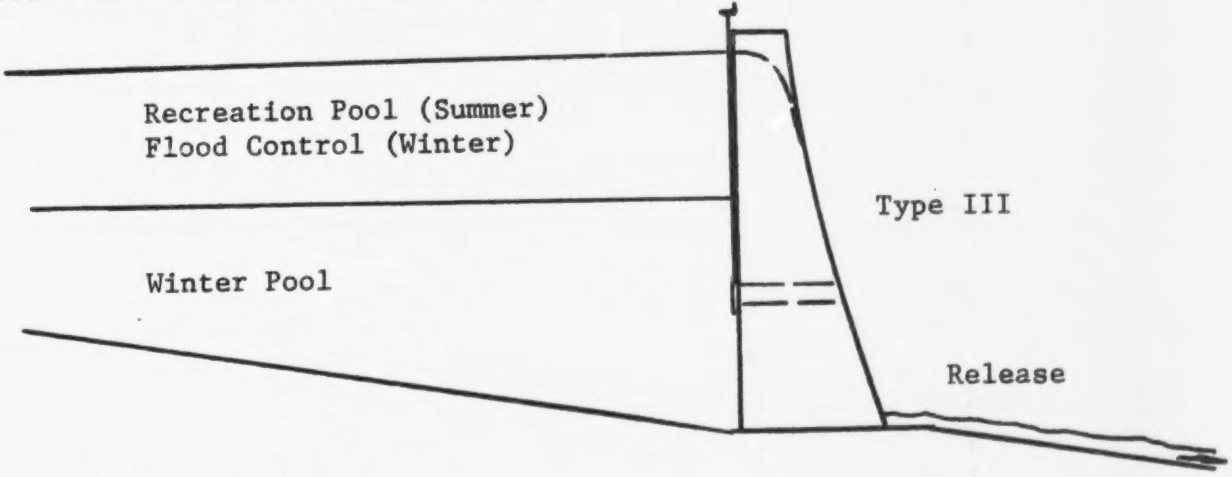
where Q is the discharge, H is the upstream head on the discharge conduit,

C is a constant depending on conduit size and inlet configuration and d is a constant usually having a value of about 0.5.
(2) All net inflow is held when the reservoir level is below the automatic outlet structure.



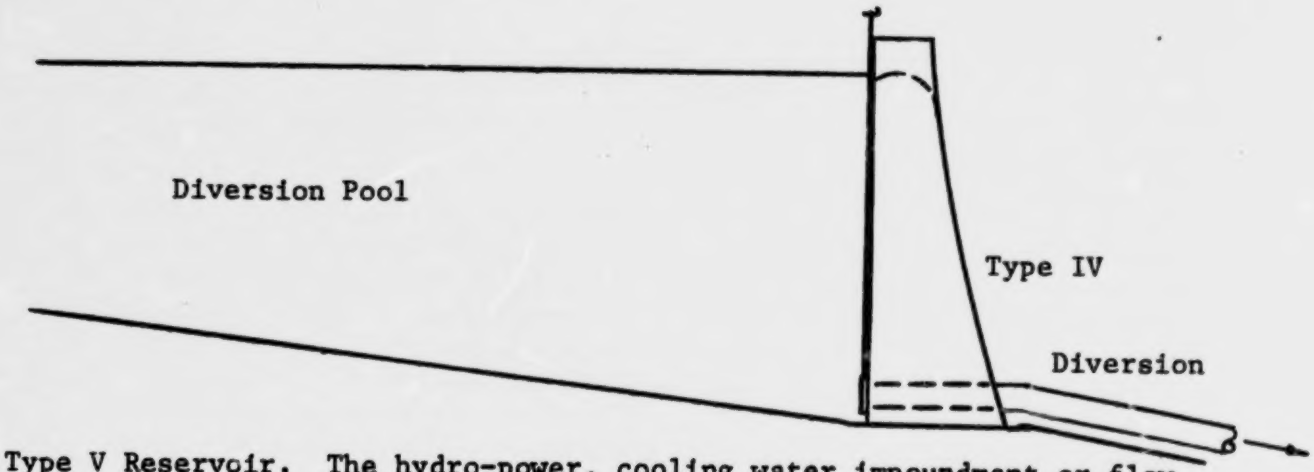
Type III Reservoir. The Type III reservoir is the lake or impoundment, having a controlled outlet, which is used primarily for recreation in the summer, but in the winter the level is drawn down to protect the shoreline facilities from winter ice damage and to afford some incidental spring flood protection. The operating rules are dependent upon the time of the year. The schedule set forth below is one that might be typical in New England. If the operating schedule is different for another location, one has only to change the "week" indices to alter it as desired. The operating rules are:

(1) Starting October 1, release all net inflow plus that volume in excess of the winter storage, the excess volume releases to be spread out over the four week period between October 1 and November 1.
(2) From November 1 to April 1, release all water in excess of the winter storage volume.
(3) From April 1 to October 1, hold all net inflow excepting that which causes overflow of the summer storage level.



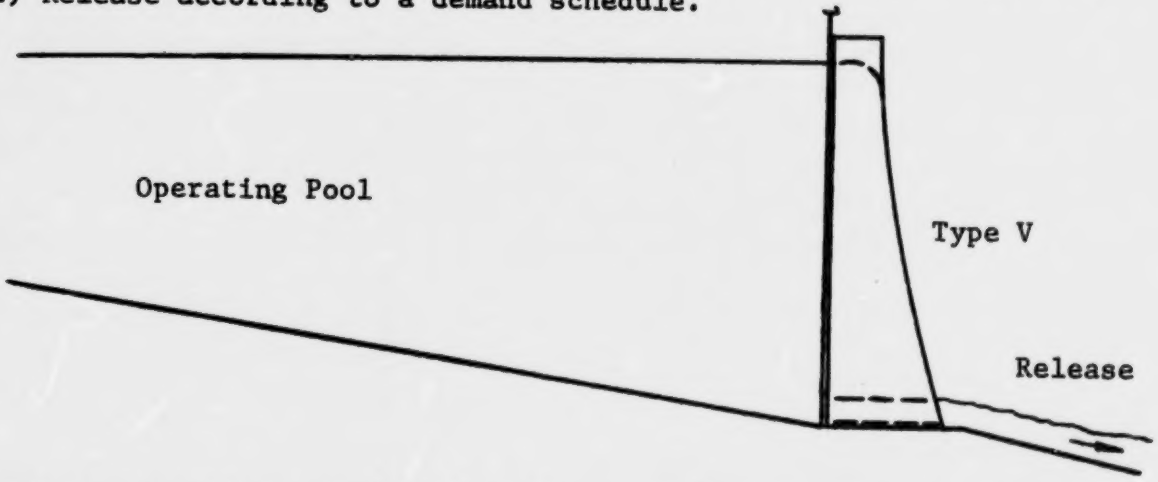
Type IV Reservoir. A Type IV reservoir is a water supply reservoir where water is impounded for diversion to a location outside the watershed or to a point downstream which is not adjacent to the dam. The operating rules are:

(1) Hold all net inflow excepting that which causes overflow of the spillway.
(2) Divert according to a demand schedule.



Type V Reservoir. The hydro-power, cooling water impoundment or flow augmentation projects are classed as Type V reservoirs. These impound water for release to the adjacent downstream reach and subsequent flow down the normal watercourse. In this type, the operating rules are:

(1) Hold all net inflow excepting that which overflows the spillway.
(2) Release according to a demand schedule.



As the computation proceeds down the watershed according to the internal sequence, the regulating effect of each reservoir, flow discharge or withdrawal upstream is added algebraically to the unregulated flow to determine the regulated flow:

$$QREG_i = QNAT_i + \sum_{j=1}^{NR} DQ_j + \sum_{k=1}^{N} IQ_k \quad . . . . . \text{[Eq. A4.73]}$$

$QREG_i$ = The regulated flow in reach i.
$QNAT_i$ = The unregulated flow in reach i.
DQ_j = The regulating effect of reservoir j.
NR = Number of reservoirs upstream of reach i.
IQ_k = The irrigation withdrawals or waste discharges, k.
N = Number of irrigation withdrawals or waste discharges upstream of reach i.

The third operation in the simulation is to compute the water quality data. In this study, only the oxygen demand and dissolved oxygen concentration parameters are considered. The subroutines QUAL and RQUAL make the necessary computations to simulate water quality in a reach which does not contain a reservoir and one containing a reservoir, respectively.

The mathematical relationships used in the subroutines QUAL and RQUAL are developed in Section 6.6 and the results are repeated here for ready reference. Modified Streeter-Phelps equations are used to compute BOD and dissolved oxygen values at each reach point in the watershed. The BOD equation is:

$$\underline{L}_i = [\bar{L}_i - \frac{r}{K_1}]\, e^{-K_1 t} + \frac{r}{K_1} \quad . . . . . . \text{[Eq. A4.74]}$$

where:

$\underline{L}_i$ = the BOD at the downstream end of reach i.
$\bar{L}_i$ = the BOD at the upstream end of reach i.
r = the M-P (Moreau-Pyatt) (5) BOD error term.
K_i = the deoxygenation velocity constant, days^{-1}.
t = time, days.
e = the natural logarithm base.

The dissolved oxygen deficit equation is:

$$\underline{D}_i = \frac{K_1 L - r}{K_2 - K_1}\,[e^{-K_1 t} - e^{-K_2 t}] + \frac{r+s}{K_2}\,(1 - e^{-K_2 t}) + \bar{D}_i e^{-K_2 t} \quad . \text{[Eq. A4.75]}$$

where:

$\underline{D}_i$ = the dissolved oxygen deficit at the downstream end of reach i.
$\bar{D}_i$ = the dissolved oxygen deficit at the upstream end of reach i.
K_i = the reoxygenation velocity constant.
s = the M-P reoxygenation error term, and
other variables are as defined above.

Where $K_1 = K_2$, the BOD equation is unchanged, but:

$$\underline{D}_i = [K_1 t(\bar{L}_i - \frac{r}{K_1}) + \bar{D}_i + \frac{r+s}{K_1}]\, e^{-K_1 t} - \frac{r+s}{K_1} \quad \ldots\ldots \text{[Eq. A4.76]}$$

where all variables are as defined above.

Where the reach is a reservoir, the assumption is made that there is complete mixing in the reservoir. Based upon the development in Section 6.6, the BOD and dissolved oxygen deficit equations are, respectively:

$$\underline{L}_{i,j} = \frac{1}{At}(\underline{L}_{i,j-1} - \frac{Z\bar{L}_{i,j}}{A})(1 - e^{-At}) + \frac{Z\bar{L}_{i,j}}{A} \quad \ldots . \text{[Eq. A4.77]}$$

and:

$$\begin{aligned}\underline{D}_{i,j} = \frac{1}{t}\Bigg(\bigg(\frac{1}{B}\Big(\underline{D}_{i,j-1} - \frac{Z\bar{D}_{i,j}}{B}\Big) + \frac{K_1}{AB}(\underline{L}_{i,j-1} + \frac{Z\bar{L}_{i,j}}{AB}(A+B)\bigg) \\ + \frac{e^{-Bt}}{B}\Big(\frac{Z\bar{D}_{i,j}}{B} - \underline{D}_{i,j-1} + \frac{K_1}{A-B}\Big(\frac{Z\bar{L}_{i,j}}{B} - \underline{L}_{i,j-1}\Big)\Big) \\ \frac{Zt}{B}\Big(\bar{D}_{i,j} + \frac{K_1\bar{L}_{i,j}}{A}\Big) + \frac{K_1 e^{-At}}{A(B-A)}\Big(\frac{Z\bar{L}_{i,j}}{A} - \underline{L}_{i,j-1}\Big)\Bigg) \quad . \text{[Eq. A4.78]}\end{aligned}$$

where:

A = $K_1 + W + K_3$.
W = Qout/volume of reservoir.
Z = QIN/volume of reservoir.
K_3 = a velocity constant for settlement of organic solids.
B = $K_2 + w$.
$\underline{L}_{i,j}$ = BOD concentration leaving reservoir i, during the jth time frame.
$\underline{L}_{i,j-1}$ = BOD concentration leaving reservoir i, during the (j-1)th time frame.
$\bar{L}_{i,j}$ = BOD concentration in water entering reservoir i during time frame j.
$\underline{D}_{i,j}$ = DO concentration in water leaving reservoir i, during jth time frame.
$\underline{D}_{i,j-1}$ = DO concentration in water leaving reservoir i, during (j-1)th time frame.
$\bar{D}_{i,j}$ = DO concentration in water entering reservoir i during ith time frame.
K_1 = deoxygenation velocity constant in reservoir.
K_2 = reoxygenation velocity constant in reservoir.
t = time.
e = natural logarithm base.

Mass balance principles are used in the computation of the BOD, DO deficit and K_1 of the water entering the reach or reservoir. The equations are:

$$\bar{L}_i = \frac{\sum_{j=1}^{N} L_{i,j} Q_{i,j} + L_{wi} Q_{wi}}{\sum_{j=1}^{N} Q_{i,j} + Q_{wi}} \quad \text{. . . . . [Eq. A4.79]}$$

$$\bar{D}_i = \frac{\sum_{j=1}^{N} D_{i,j} Q_{i,j} + D_{wi} Q_{wi}}{\sum_{j=1}^{N} Q_{i,j} + Q_{wi}} \quad \text{. . . . . . [Eq. A4.80]}$$

$$K_{1i} = \frac{\sum_{j=1}^{N} K_{1i,j} Q_{i,j} + K_{1wi} Q_{wi}}{\sum_{j=1}^{N} Q'_{i,j} + Q_{wi}} \quad \text{. . . . . [Eq. A4.81]}$$

where:

$\bar{L}_i$ = incoming BOD concentration in reach i.

$L_{i,j}$ = BOD concentration in lower end of tributary j which flows into the upstream end of reach i.

$Q_{i,j}$ = flow entering reach i from tributary j.

N = number of tributaries converging at the upper end of reach i.

$L_{w,i}$ = BOD concentration in waste entering upper end of reach i.

$Q_{w,i}$ = flow rate of waste entering upper end of reach i.

$\bar{D}_i$ = incoming DO deficit in reach i.

$D_{i,j}$ = DO deficit in lower end of tributary j which flows into the upstream end of reach i.

$K_{1,i}$ = the deoxygenation velocity constant in reach i.

$K_{1,i,j}$ = the deoxygenation velocity constant in reach j which is tributary to the upper end of reach i.

K_{1wi} = the deoxygenation velocity constant of the waste discharged at the upper end of reach i.

Q'_{ij} = corrected flow (see below).

Note that $Q'_{ij} = 0$ if $K_{1ij} = 0$ for tributary j; i.e., additional flow does not "dilute" the value of K_1, as it does for BOD and DO deficit.

An important factor in water quality considerations is temperature. As described in Section 6.3, temperature variations can be characterized by a sine curve having a single period of one year. In general, records of river water temperature at several points in a watershed often are not available. For this reason, and because river systems which do not contain large summer-stratified reservoirs have nearly constant temperatures throughout their length, this simulation model assumes that temperature is constant throughout the system length for any given time frame. Temperature is varied from time-frame to time-frame according to a sine curve:

$$T(L) = \bar{T} + A \sin(L+C) + R\,\sigma_T \quad \ldots\ldots \quad \text{[Eq. A4.82]}$$

where:

$T(L)$ = the temperature during the Lth week of the year.
$\bar{T}$ = the mean at all historical temperature observations.
A = a constant.
L = the week of the year; when used in the argument of the sine function, L is expressed as a time angle.
C = phase constant.
R = standard normal random deviate $N(0,1)$.
σ_T = standard deviation of historical temperature data.

The values of A and C are developed by a least squares fit of the historical data to the sine curve. The program is set up to receive data items $\bar{T}$, A, C and σ_T. The random numbers are generated internally.

In locations where temperature data are available and indicate that more precision is possible by computing temperature variations by region or reach, it is possible to modify the program as set forth here to make use of the data available.

Another important factor in considerations of oxygen balance in a stream is the reoxygenation velocity constant, K_2. The source of oxygen for replenishing oxygen used in the water is from the air, at the air-water interface. The value of K_2 depends upon the mechanism for transport of this reoxygenated surface water into the depths. The reoxygenation velocity constant is directly proportional to the velocity of water flow in the river and inversely proportional to the depth. The constant is also temperature dependent.

If there is available a considerable fund of water quality data for a stream, along with corresponding flow data, it is possible to develop K_2-flow-temperature relationships. One must be able to determine the value of K_1 by other means. It is suspected that in only a few of the more thoroughly studied river systems has there been enough data obtained to develop a relationship in which confidence may be placed. The work of Langbein and Durum (6) is an alternate source of this information. They developed the empirical formula:

$$K_2 = \frac{3.3\,v}{h^{1.33}} \quad \text{. . . . . [Eq. A4.83]}$$

where v is the mean velocity of flow in the stream and h is the mean depth. This formula gives the value of K_2 at 20°C. The program coding is set up to compute the value of K_2 at 20°C using the Langbein and Durum formula.

If the Langbein and Durum formula is used, there is the problem of determining the values of v and h to use. Of course, measurements can be made in each reach in sufficient number to be able to develop a flow-velocity-depth relationship, or if cross sections, slopes and roughness data are available, one of the open-channel hydraulic formulas can be used. Refer to Section 6.3. Again, there are available empirical formulas which relate the velocity and depth to the flow rate. These were developed by Leopold and Maddock (7) and are:

$$w = aQ^b \quad \text{. . . . . [Eq. A4.84]}$$

$$h = cQ^f \quad \text{. . . . . [Eq. A4.85]}$$

$$v = kQ^m \quad \text{. . . . . [Eq. A4.86]}$$

where:

w = stream width, ft.
h = mean depth of water in the stream, ft.
v = mean velocity of flow, ft./second.
Q = the flow rate in the stream, in cfs and a,b,c,f,k, and m are constants which must be determined.

Thus, even using these empirical relationships, it is necessary to make at least a few measurements at cross sections in order to evaluate the constants. The program coding is set up to compute h and v from corresponding values of Q for input values of c,f,k, and m.

The temperature dependence of K_2 is given by the formula:

$$K_{2_T} = K_{2_{20}} e^{0.24(T-20)} \quad \text{. . . . . [Eq. A4.87]}$$

where:

K_{2_T} = the reoxygenation velocity constant at temperature T.
$K_{2_{20}}$ = the reoxygenation velocity constant at 20°C.
T = the temperature in °C.
e = the natural logarithm base.

The value of the deoxygenation velocity constant also is temperature dependent as given by a similar formula:

$$K_{1_T} = K_{1_{20}} e^{0.046(T-20)} \quad . . . . [Eq. A4.88]$$

The dissolved oxygen concentration at a stream location is given by the difference between the dissolved oxygen saturation level, also temperature dependent, and the dissolved oxygen deficit at that location. The dissolved oxygen saturation concentration used in the program is given by:

$$DO_{SAT} = 14.65 - 0.41T + 0.008T^2 - 0.00008T^3 \quad . . . [Eq. 4.89]$$

which has been rounded off from the formula set forth by the ASCE Committee on Sanitary Engineering Research (8). T is the temperature in degrees centigrade.

A4.5.2 Program Components.

The simulation program, called WASP, is made up of a controlling program, WASP-MAIN and several subroutines, each of which contributes to the overall program. A list of the subroutines and their length, in bytes, along with the lengths of functions and common blocks, follows.

WASP MAIN	906	TWASTE	522
SIM	1544	TGEN	2772
REG	3660	TRAN	2852
QUAL	5004	IREACH	412
RQUAL	1730	UPGAGE	1162
RAN	460	S	8580
RRN	396	GTRAN	536
STD	490	GFLOW	456
RDATA	1256	TRES	514
DIVREL	1220		
Functions	20936		
Common	49480		

Total program length = 104,888 bytes.

These lengths are based on a program which reads gage flow data from a magnetic tape produced by FLASH.

A4.5.2.1 WASP-MAIN.

The simulation program is controlled by WASP-MAIN. This subroutine sets up the ten common blocks needed to transfer variable values from subroutine to subroutine, calls subroutines in the required order and reads input of number of years to be simulated and the number of gages (with gage numbers) to be used in the simulation. The gage numbers used and their weight coefficients are written. Subroutines TGEN and TRAN are called to set up the reach indices and computation sequence and to compute the transformation matrix used to convert gage data to reach flow data. Subroutine RDATA is called to read in data needed to regulate the streamflow in the watershed. The final subroutine called is SIM which carries out the simulation process.

A4.5.2.2 SIM.
Subroutine SIM reads in constants and data needed to compute the temperature and waste parameter values, computes the deterministic component of the temperature equation for each week of the year and, through two "do loops" one nested in the other, performs the simulation by calling various subroutines. The first of the "do loops" covers from the first year of the simulation period to NYR, the number of years of simulation desired. This "do loop" also calls RAN which generates the random deviates for the temperature equation and then calls GFLOW which reads in gage data for one year at a time.

The second "do loop" covers from "week" one through 48 "weeks" of the year. Inside this "do loop," subroutines QTRAN, REG and QUAL are called, successively, to convert gage flows to unregulated flows and to compute the regulated flows and the water quality values at reach point in the stream. These operations are described below in more detail as individual subroutines. Weekly values of the simulation results are written out within the "do loop." When the simulation is complete, the computer is returned to WASP-MAIN for termination.

A4.5.2.3 Subroutines RAN and RRN.
Subroutines RAN and RRN are the same random number generating subroutines used in programs CHKDATA and FLASH, which have been described previously.

A4.5.2.4 Subroutine GFLOW.
Subroutine GFLOW reads the generated (or historical, if desired) gage data. GFLOW is called in the yearly "do loop" to read gage data for one year for each gage being used in the simulation. The data read are transferred to COMMON/FLOW 7 for use by other subroutines.

A4.5.2.5 Subroutine QTRAN.
QTRAN is a short subroutine which receives gage data through COMMON/FLOW 7 and, using the transformation matrix developed by subroutine TRAN and the matrix multiplication operation of subroutine S, converts gage data to streamflow data. The streamflow data are designated QNAT(I), to indicate the natural or unregulated flow at the upper end of reach I. These values of QNAT(I) are also entered in COMMON/FLOW 7 for use by other subroutines linked through this common block.

A4.5.2.6 Subroutine S.
S is a general subroutine which performs various matrix operations. It was described in detail in A4.4.2.8.

A4.5.2.7 REG. Subroutine REG is called each week during the simulation. Each time it is called, it tests the reaches in the internal computation sequence for reaches having reservoirs. When REG finds a reservoir, it calls subroutine TRES which checks through the list of reservoirs read in through subroutine RDATA to determine that data for the reservoir are available in storage. Assured that reservoir data are available, REG then computes the depth and area of water corresponding to the inventory in this reservoir for the most recent time frame. This allows correction of the inventory for evaporation. The storage inventory is then brought up to date by adding in the flow and subtracting the evaporation loss. Having the current inventory, the subroutine checks the appropriate operating rule governing releases and diversions and, with data supplied by subroutine DIVREL, it determines the rates of release and diversion. Following this, the inventory computation is completed and the value is stored for use in the next time frame.

Finally, REG computes the amount of regulation afforded by the reservoir; that is, QREG-QNAT (regulated flow-natural flow). All flows in reaches are corrected by the sum of all upstream regulations. For instance, if the QNAT in reach 12 is 300 cfs and reservoir A, located upstream of reach 12, holds back 15 cfs while reservoir B, also upstream of reach 12, discharges 30 cfs more than its net inflow, then the regulated flow in reach 12 is QNAT(12) + (Regulations upstream) = 300 - 15 + 30 = 315 cfs = QREG(12). The values of QREG(I) are made available to other subroutines through COMMON/FLOW 7.

A4.5.2.8 Subroutine TRES.
This subroutine checks that, when in the pass through the reaches in a given time frame, a reservoir reach is found, operating data for that reservoir are available in machine storage. If not, TRES calls EXIT and the program stops.

A4.5.2.9 Subroutine DIVREL.
Subroutine DIVREL determines the type of reservoir encountered, then proceeds to compute the basic rates of release and/or diversion. These rates either are established by the use of the reservoir or are computed by demand formulas developed through external analysis of historical or projected data. The basic rates of release and/or diversion are sent to REG, which modifies them as necessary to correspond to the operating rule, for use in computing reservoir inventory and regulated flows.

A4.5.2.10 Subroutine RDATA.
Although subroutine RDATA is not strictly a part of REG, it is used to read in much of the data that are required by REG for its operation. Consequently, the portions of RDATA that relate to regulation of flows and reservoirs will be included here.

Subroutine RDATA reads in the data needed to operate the reservoir and to maintain the reservoir inventory. Data needed are:

(1) number of reservoirs and their reach location.
(2) the reservoir type and the capacity of the various pools.
(3) constants for release equations.
(4) constants for evaporation equation.
(5) constants for the diversion equations.
(6) irrigation demands by reach and time of year.

These data requirements are described in A4.5.1.

If the reservoirs are in existence, it is usually possible to obtain the maps, soundings and/or area-depth capacity curves needed to prepare the input data. If a non-existent reservoir is placed in a reach to simulate its effect on the watershed, it will be necessary to make a preliminary design sufficient in detail to obtain a suitable set of data for use in REG.

A4.5.2.11 Subroutine QUAL.
After regulated flows in each reach are computed, subroutine QUAL is called to compute the water quality values. First, QUAL completes the computation of the temperature for the time-frame. It should be remembered that QUAL is called in "do loop" operating on the "weekly" cycle which is, in turn, a "do loop" for the number of years of simulation desired. QUAL sets up a third "do loop" which cycles over the number of reaches, NR. Thus, for each "week," QUAL is called to compute the quality values for each reach for one "week" before it returns to its calling subroutine, SIM. The computations are made, starting at the upstream reaches, according to the sequence of computation set up by subroutine TGEN.

For each reach, QUAL initializes variables and proceeds to make the computations necessary to evaluate the incoming BOD and DO concentrations and the K_1 velocity constant. It then checks to determine if the current reach is a reservoir, and, if so, RQUAL is called. If the current reach is not a reservoir, QUAL computes the value of K_2, corrects K_1 and K_2 for temperature, computes the time of flow in the reach and, finally, the BOD and DO concentrations in the water leaving the reach. Note that if the computed values of K_1 and K_2 are equal, the program switches to the special formula used in that situation. QUAL then checks to determine if the DO deficit has reached a maximum within the reach, in which case the minimum DO concentration will have occurred within the reach. This is done by computing TCRIT, the critical time of flow, and comparing it to the time of flow in the reach. If TCRIT < TIME, a minimum DO concentration has occurred in the reach. TCRIT is then substituted for TIME and the value of DEFOUT is computed and used in turn to compute the minimum DO concentration, XMINDO, in the current reach. QUAL then writes out the water quality and related values in an array.

A4.5.2.12 Subroutine RQUAL.
Subroutine RQUAL is called by subroutine QUAL when a check of reaches indicates the current reach is a reservoir. Thus, RQUAL is operating in the same set of "do loops" as is QUAL. RQUAL immediately calls subroutine TRES (see A4.5.2.8) for a check of the reservoir data for the current reach. RQUAL computes the values of average BOD and DO deficit in the reservoir and the BOD and DO deficit in the water being released and/or diverted from the reservoir. The appropriate quality values are transferred back to QUAL which writes them in the same array with the other values computed for the stream.

A4.5.2.13 Subroutine TGEN.
Subroutine TGEN is called by WASP-MAIN to set up the reach indexing and the computing sequence. TGEN is described in Section A4.3.

A4.5.2.14 Subroutine TRAN.
Subroutine TRAN also is called by WASP-MAIN to develop the transformation matrix which is used to convert gage data to stream flows at reach points in the stream. TRAN is described in detail in Section A4.3.

A4.5.2.15 Subroutine DIVREL.
Subroutine DIVREL is auxiliary to subroutine REG and is called by REG to compute the reservoir releases and/or diversions. Depending upon the reservoir type, the release or diversion is computed and sent to REG for use in computing regulated flows and reservoir inventories.

A4.5.2.16 Subroutine TWASTE.
Subroutine TWASTE is a checking subroutine which does for a reach having a waste load what subroutine TRES does for a reach that is a reservoir. TWASTE checks that there is a set of waste load data corresponding to a reach number where there is scheduled a waste discharge. If the data are not available for the reach, TWASTE calls EXIT.

A4.5.3 Program Input.
Data are read into the simulation program, WASP, through subroutines WASP-MAIN, SIM, RDATA, GFLOW, and TGEN only.

(1) For WASP-MAIN:
Card #1 (2I5)
NYR = the number of years of simulation to be carried out.
NGT = the number of gages for which data are used.

Card #2 (10I8)
IGT(I) = the gage numbers used.
I = 1...NGT.
Maximum of ten.

(2) For SIM:
Card #1 (3F8.0, I5,5F8.0)
D = constant in temperature equation.
C = lag constant in temperature equation.
TMEAN = mean annual temperature.
ISTART = initial number for random number generator.
XK = constant in velocity equation.
XM = constant in velocity equation.
CC = constant in depth equation.
F = constant in depth equation.
SIGMAT = standard deviation of the temperature data.

Card #2 (4F10.0,I1)
RLNTH(I) = the length of reach (I), feet.
RCON(I) = the Moreau-Pyatt dexoygenation error term, r, for reach I.
SCON(I) = the Moreau-Pyatt reoxygenation error term, s, for reach I.
XK120W(I) = the deoxygenation constant K1 at 20°C for the waste which is introduced into reach I.
IWASTE(I) = 1 if there is a waste load introduced in reach I,
= 0 for no waste load introduced in reach I.

Card #3 There are 48 data cards, one for each "week," for each reach where a waste is introduced (3F10.0).
QWASTE(I,J) = the rate of discharge of waste, in cfs for each I and week J.
BODWST(I,J) = the concentration, in mg/l, of BOD in the waste being discharged into reach I during week J.
DOWST(I,J) = the concentration, in mg/l, of dissolved oxygen in the waste being discharged into reach I during week J.

Card #4 There is one card for each reservoir in the system (4F10.0).
XK22OR(I) = the value of the reoxygenation velocity constant K_2 at 20°C for reservoir I.
XK3(I) = the value of K_3 for reservoir I.
BODSTO(I) = the initial BOD, mg/l, in reservoir I.
DEFSTO(I) = the initial DO, mg/l, in reservoir I.

(3) For RDATA:
Card #1 (I5)
NRES = the number of reservoirs in the system.

Card #2 There is one card for each 16 reaches. The reservoir number is punched in an I5 field in the field number corresponding to the reach in which the reservoir occurs. For example, if the first reservoir, number 101, is in reach 12, the reservoir number is punched as follows: 1 in space 58, 0 in space 59, and 1 in space 60 of the first card.

(16I5)IRES(I) = the reservoir number, a convenient number for identification of the reservoir; may be up to 5 integers.

Card #3 There is one card for each reservoir in the system (2I5,4F10.2)
JRES(J) = the reservoir number; same as IRES(I) above.
ITYPE(J) = the type classification for reservoir J.
CAP(J) = the capacity of reservoir J, volume in 10^6 cubic feet at spillway level.
SMIN(J) = the volume, in 10^6 cubic feet, of the minimum pool, if applicable, for reservoir J.
TOPLEV(J) = the volume below the flood control pool, in 10^6 cubic feet, in a Type I reservoir, reservoir number J.

Card #4 One card contains the values of ACON and BCON for four reservoirs in an 8F10.4 field. The value of ACON for the first reservoir, I=1, is placed in the first 10 spaces, BCON for the first reservoir is placed in the 11-20 space, ACON for the second reservoir, I=2, is placed in the 21-30 space and so on. If, for the particular reservoir, ACON and/or BCON are zero, leave blank the space reserved for them. It takes 1 card for each 4 reservoirs (8F10.4).
ACON(I) = a constant used in computing the release from reservoir I.
BCON(I) = also a constant used in computing the release from reservoir I.

Card #5 One card for each reservoir in the system (8F10.2)
YMEAN(L) = the mean annual diverted flow, reservoir L, used in the diversion formula.
ACAP(L) = constant in the depth-capacity equation for reservoir L.
BCAP(L) = constant in the depth-capacity equation for reservoir L.
DCAP(L) = constant in the depth-capacity equation for reservoir L.
ADAP(L) = constant in the depth-area equation for reservoir L.
BDEP(L) = constant in the depth-area equation for reservoir L.
CDEP(L) = constant in the depth-area equation for reservoir L.

Card #6 (3F10.0)
AVAP = constant in the evaporation equation.
BVAP = constant in the evaporation equation.
CVAP = constant in the evaporation equation.

Card #7 Two cards are required for each reservoir (6F10.0).
TAU(I,J) = the lag constant for harmonic J in the diversion equation, for reservoir I.
CCON(I,J) = the coefficient for harmonic J in the diversion equation, for reservoir J.
TAU(1,1) is placed in spaces 1-10; CCON(1,1) is placed in spaces 11-20; TAU(1,2) is placed in spaces 21-30; CCON(1,2) is placed in spaces 31-40, and so on.

Card #8 (I5)
NADJST = the number of data cards for flow adjustments. The number equals the number of reaches where flow adjustment is required multiplied by the number of weeks of adjustment.

Card #9 NADJST cards, one for each reach for each week of adjustment.
(I5,I8,F14.4)L = the week of the year that flow adjustment is made.
NSTA = the reach number where the flow adjustment is made.
XIRRIG(NSTA,L) = the amount of flow adjustment in reach NSTA and for week L.

(4) For GFLOW: The generated gage data (or historical gage data, if desired are read by GFLOW. The format must be adjusted to the format of the data to be read. Normally, the data will be on magnetic tape. The program as set forth herein reads the data from magnetic tape as QG(I,J), (I=1,NG)(J=1,48); i.e., it reads one year of data for each station, within a 1, NYR "do loop."

(5) For TGEN:
Card #1 (2I5)
NR = number of reaches.
NG = number of gages.

Card #2 One card for each reach (4I5, 6F5.0).
NOR(I) = the reach number, reach I.
NUR(I,J) = the reach numbers J upstream of reach I; there may be 0, 1, 2 or 3 J numbers depending upon the stream configuration above reach I.
DAU(I,J) = the drainage area upstream of reach I; one number for each J, equal to the drainage area of tributary J upstream of reach I.
FL(I) = length of reach I.
SLOPE(I) = average slope, hydraulic gradient, in reach I.
ROUGH(I) = Manning's roughness, n for reach I.

Card #3 One card for each gage (I8,I5,F5.0).
NGAGE(I) = identifying number of gage I.
NGR(I) = number of the reach in which gage I is located.
DAG(I) = the drainage area upstream of gage I.

A4.5.4 Program Output.
The program output from WASP consists of three arrays from TGEN and TRAN, which are printed out once each run, and five arrays which are printed out for each week of the simulation.

The three arrays from TGEN and TRAN are identical to the first three arrays of output for TFLOW described in A4.3.4 above.

The five arrays of output printed for each week of simulation are described as follows:

(1) Array 1: The values of the unregulated flow, QNAT, in each reach, are written in horizontal rows of ten numbers per row. The array writes, through subroutine QTRAN using a 10F8.0 FORMAT, the values of QNAT(I) for the current week. I is the reach number. The QNAT values for the first ten reaches are in the first row, for 11th through 20th reaches in the second row, and so on. The number of lines printed will be NR/4 or the next larger integer.

(2) Array 2: The reservoir identifying number, the external index and the weekly evaporation correction are written in the second array. The order of appearance in the array is the order in which the computation took place.

(3) Array 3: This array is NR rows by two columns in which the external reach numbers and corresponding regulated flows, QREG, for the current week are written. The order in the array is the order of computation. The array has been folded for writing to decrease the number of lines of output. The number of folds can be arranged for each project to use a minimum number of lines.

(4) Array 4: The reservoir inventory data are contained in Array 4. The array consists of NRES rows, one for each reservoir, with rows headed Reservoir, Storage, Release and Diversion. "Reservoir" indicates the reservoir identifying number, "Storage" indicates the volume (in 10^6 cubic feet) in the reservoir at the end of the current time frame, "Release" indicates the average release rate (in cfs) during the time frame and "Diversion" indicates the volume (in 10^6 cubic feet) diverted during the current time frame.

(5) Array 5: The water quality data and results are contained in Array 5. The array is NR rows, one for each reach, by ten columns. The columns are as listed below:

(a) TIME - the average time of flow in the reach or, if the reach is a reservoir, the average detention time in the reservoir if the detention time is less than 30 days. The value entered will be 30 days if the detention time is greater than 30 days.

(b) QWASTE - the quantity of waste discharged, in cfs.

(c) DOWST - the dissolved oxygen concentration, in mg/l, in the waste discharge.

(d) BODWST - the biochemical oxygen demand concentration, in mg/l, in the waste discharge.

(e) XK_1 - the value of the deoxygenation velocity constant at T°C for the waste discharge, in days^{-1}. T°C is the average water temperature in the current time frame.

(f) XK_2 - the value of the reoxygenation velocity constant in the corresponding reach, at T°C where T°C is the temperature during the current time frame.

(g) XMINDO - the value of the minimum dissolved oxygen concentration, in mg/l, in the corresponding reach during the current time frame. XMINDO is the difference between the DO deficit and the saturated DO concentration, both computed for current time frame.

(h) BODOUT - the value of the BOD concentration in mg/l, at the downstream end of the corresponding reach.

(i) DEFOUT - the value of the DO concentration, in mg/l, at the downstream end of the corresponding reach.

(j) REACH - the identifying external reach number.

The order of appearance of the data in Array 5 is the order in which the data were computed and corresponds to the computation sequence set up by the subroutine TGEN.

A4.5.5 Dictionary of Variables.

Following is a list of the variables used in WASP and a brief definition of each:

A	Intermediate constant.
ACAP(I)	Constant, reservoir depth-capacity equation, reservoir I.
ACON(I)	Release or diversion constant, reservoir I.
ADEP(I)	Constant, reservoir depth-area equation, reservoir I.
AREA(I)	Reservoir surface area, reservoir I.
AVAP	Evaporation formula constant.
AVGSTO	Average storage, current time and in previous time frame.
AVW(I)	Average waste load, Ith reach (used for periodic load function).
AW(I)	Amplitude of waste load (used for periodic load function).
B	Intermediate constant.
BCAP(I)	Constant, reservoir depth-capacity equation, reservoir I.
BCON(I)	Release or diversion constant, reservoir I.

BDEP(I)	Constant, reservoir depth-area equation, reservoir I.
BODIN(I)	BOD concentration, upstream end reach I.
BODOUT (I)	BOD concentration, leaving reach I.
BODSTO(I)	BOD concentration in reservoir I.
BODWST(I)	BOD concentration in waste load, reach I, week J.
BVAP	Evaporation formula constant.
C	Constant, temperature equation.
CC	Constant, depth equation.
CCAP(I)	Constant reservoir depth-capacity equation, reservoir I.
CCON(I,J)	Constant, periodic diversion formula, reservoir I, harmonic J.
CDEP(I)	Constant, reservoir depth-area equation, reservoir I.
CN	Correction value, flow in upstream reach.
CNI	Intermediate variable.
CQREG(I)	Corrected regulated flow, reach I.
CVAP	Evaporation formula constant.
DAG(I)	Drainage area upstream of gage I.
DAU(I,J)	Drainage area upstream of tributary J upstream of reach I.
DCAP(I)	Constant, reservoir depth-capacity equation, reservoir I.
DEFIN(I)	DO deficit concentration, upstream end of reach I.
DEFOUT(I)	DO deficit concentration, leaving reach I.
DEFSTO(I)	DO deficit concentration in reservoir I.
DEP(I)	Reservoir water depth, reservoir I.
DIV(I)	Diversion rate, reservoir I.
DOS	Dissolved oxygen saturation concentration.
DOWST(I,J)	DO deficit concentration in waste load, reach I, week J.
DQ(I)	Amount of flow regulation reach I.
EVAP	Evaporation rate.
F	Constant, depth equation.
FL(I)	Length of reach, reach I.
HOLD	Reach index, auxiliary.
IDR(I)	Index of reach downstream of reach NOR(I).
IGT(I)	Gage identifying number, gage I.
II(I)	Temporary index.
IR	Reservoir index.
IRES(I)	Reservoir number in reach I.
IRRIG(I,J)	Irrigation withdrawal, from reach I, for week J.
ISTART	Starting number, random number generator.
ITYPE(I)	Type of reservoir, reservoir I.
IWASTE(I)	Equals 1 for a waste load in reach I.
JR(I)	Downstream index.
JRES(J)	Reservoir number in reach J.
L	Week index.
NADJST	Number of flow adjustments.

NG	Number of gages.
NGAGE(I)	Identifying number, gage I.
NGR(I)	Number of reach containing gage I.
NGT(I)	Total number of basis gages.
NOR(I)	Number of reach I.
NR	Number of reaches.
NRES	Number of reservoirs.
NUR(I,J)	Number of reach J upstream of reach I.
NWASTE	Number of waste loads.
NYR	Number of years.
PMW(I)	Peak waste load, reach, I (used for periodic load function).
PSTO(I)	Previous storage volume, reservoir I.
OG(I,J)	Weekly generated flow, station I, week J.
QIN	Rate of flow into reservoir, CFS.
QINV	Volume of flow into reservoir, in one time frame.
QNAT(I)	Natural (unregulated) flow, reach I.
QREG(I,J)	Regulated flow, reach I, week J.
QSUM	Sum of incoming flows, corrected-waste loaded reaches only.
QSUM2	Sum of incoming flows, total.
QWASTE(I,J)	Rate of waste discharge, reach I, week J.
R(I)	Random number, normal distribution, time frame I.
RCON(I)	Deoxygenation error constant, reach I.
RLNTH(I)	Length of reach I.
ROUGH(I)	Channel roughness factor, reach I.
RREL(I)	Release rate, reservoir I.
RRELV	Volume of flow released from reservoir in one time frame.
RRN(I)	Temporary variable, random number generator, time frame I.
RRV	Intermediate variable.
RTEMP	Intermediate variable.
SCON(I)	Reoxygenation error constant, reach I.
SDIF	Intermediate variable.
SIGMAT	Standard deviation, temperature data.
SLOPE(I)	Mean slope, hydraulic grade line, reach I.
SMIN(I)	Minimum storage level, reservoir I.
SPRD1	Summing variable - K_1 computation.
SPRD2	Summing variable - BOD computation.
SPRD3	Summing variable - DO deficit computation.
STO(I)	Current storage volume, reservoir I.
T(L)	Temperature, week L.
TAU(I,J)	Lag constant, periodic diversion formula, reservoir I, harmonic J.
TCRIT	Critical time of flow, time to critical DO condition.
TDA(I)	Drainage area upstream of reach I.

THETA	Argument angle, diversion equation.
TIME	Time of flow in reach.
TMEAN	Mean temperature.
TOPLEV(I)	Volume below flood control pool, Type I reservoir.
TT	Temperature at current time frame.
V	Mean velocity of flow in reach.
W	Intermediate constant, reservoir oxygen balance equations.
WT(I,J)	Weight coefficient for reach I, gage J.
XH	Mean depth of water in reach.
XK	Constant, depth equation.
XK120(I)	Deoxygenation velocity constant, reach I, at 20°C.
XK2	Reoxygenation velocity constant, ambient temperature.
XK220	Reoxygenation velocity constant at 20°C.
XK220R(I)	Reoxygenation velocity constant, 20°C, reservoir I.
XK3(I)	Sedimentation velocity constant, reservoir I.
XKK(I)	Dissolved oxygen concentration, violation condition.
XLRAD	Intermediate variable, argument in evaporation equation.
XM	Constant, velocity equation.
XMINDO	Minimum dissolved oxygen concentration.
YMEAN(I)	Mean diversion, reservoir I.
Z	Constant, temperature equation.
Z	Intermediate constant, reservoir oxygen balance equation.

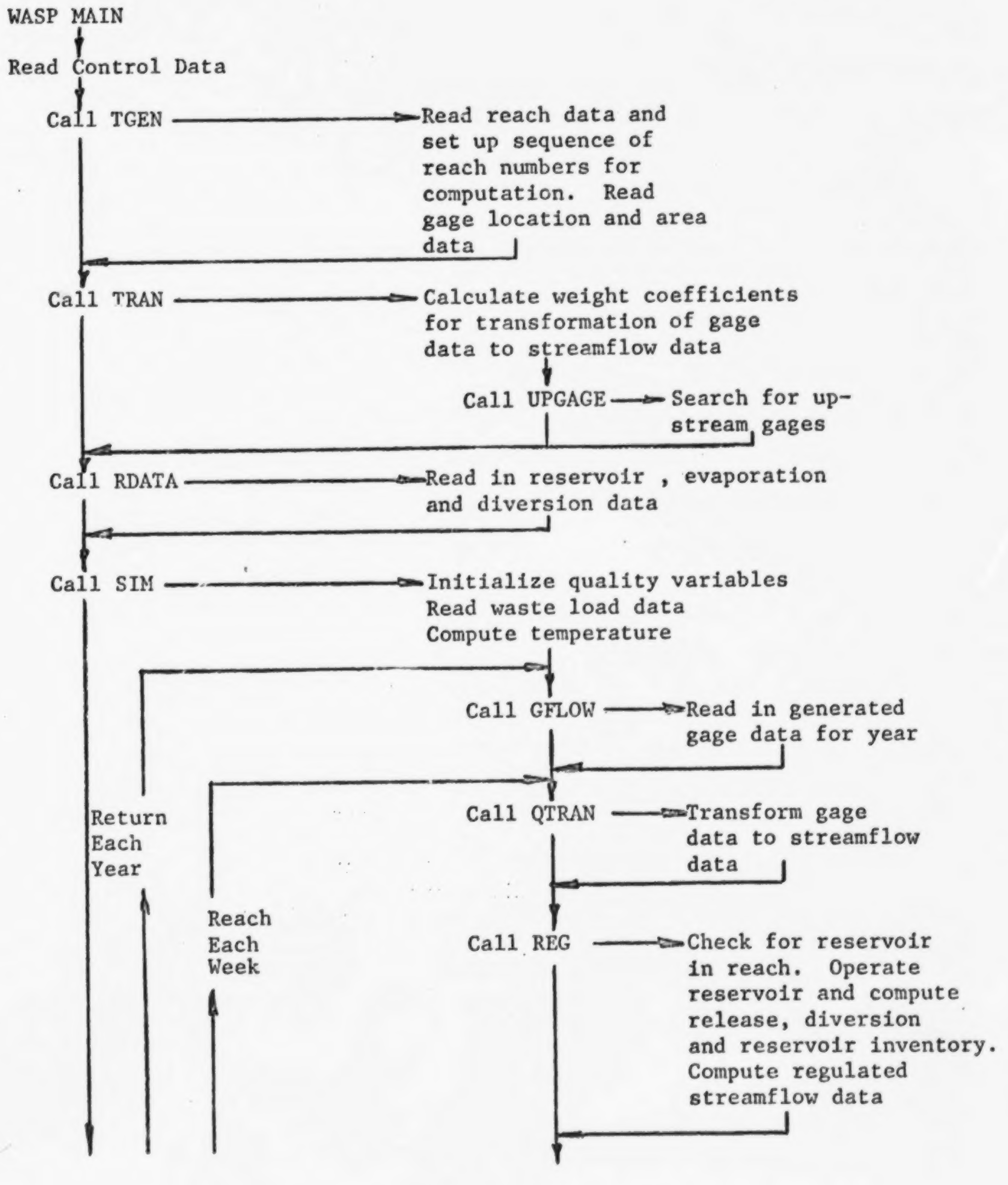
A4.5.6 Program Logic.

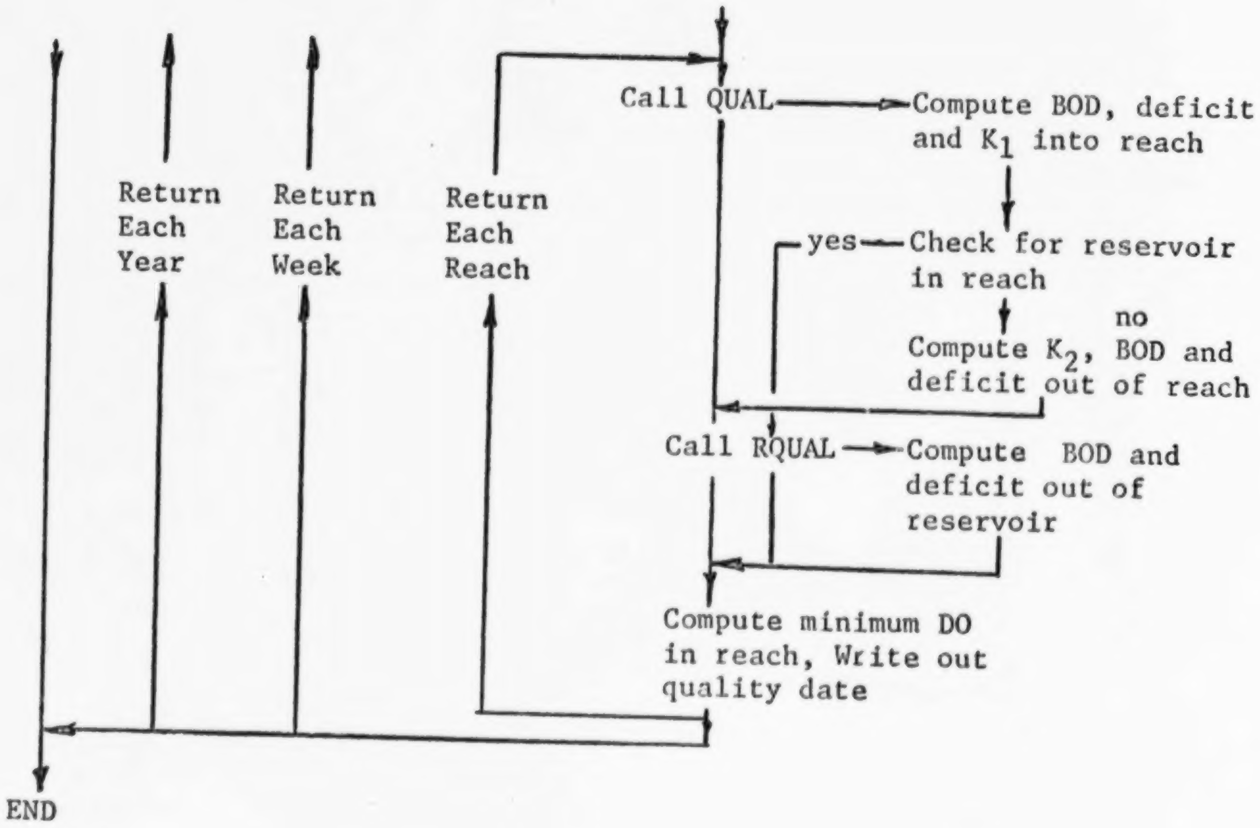
Figure A4-9 is a diagram of program logic for WASP.

A4.5.7 Program Coding.

The program coding for WASP follows.

FIGURE A4-9
PROGRAM LOGIC - WASP


WASP MAIN
Read Control Data
Call TGEN
Read reach data and set up sequence of reach numbers for computation. Read gage location and area data
Call TRAN
Calculate weight coefficients for transformation of gage data to streamflow data
Call UPGAGE
Search for upstream gages
Call RDATA
Read in reservoir , evaporation and diversion data
Call SIM
Initialize quality variables
Read waste load data
Compute temperature
Call GFLOW
Read in generated gage data for year
Return Each Year
Call QTRAN
Transform gage data to streamflow data
Reach Each Week
Call REG
Check for reservoir in reach. Operate reservoir and compute release, diversion and reservoir inventory. Compute regulated streamflow data


Call QUAL
Compute BOD, deficit and K_1 into reach
Return Each Year
Return Each Week
Return Each Reach
yes
Check for reservoir in reach
no
Compute K_2, BOD and deficit out of reach
Call RQUAL
Compute BOD and deficit out of reservoir
Compute minimum DO in reach, Write out quality date
END

```
//WASP1     JOB (1143,47,035,30,2000),'ALEMAN                ',CLASS
                                                               =M
// EXEC FORTRAN
//SOURCE    DD *
C WATER QUALITY SIMULATION PROGRAM (WASP)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),CAU(50,3),
     1       TCA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      COMMON/FLOW4/ATIME(50),BTIME
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                               ),
     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW6/NWASTE,IWASTE(50),JWASTE(50),AVW(50),AW(50),PM
                                                               W(50),
     1 FK1,WASTE,XFK1
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
      COMMON/FLOW8/AVTEMP,ATEMP,TPEAK,TEMP,DTEMP
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                               (10,6),
     1       YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2       TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                               10),
     3       BDEP(10),CDEP(10)
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(50),ISTART,XK,XM
                                                               ,
     1      CC,F,XK120W(50),BODWST(50,48),
     2       DOWST(50,48),R(48),RCON(50),SCON(50)
      COMMON/FLOW12/XK220R(10),XK3(10),BODSTO(10),DEFSTO(10)
      COMMON/FLOW13/QREG(50,50),DEFIN(50,50),DEFOUT(50,50),BODIN(
                                                               50,50),
     1BODOUT(50,50),XMINQ(50,30),LLX(50,30),XKK(50,50)
      DIMENSION IGT(10)
      READ(5,5001)NYR,NGT
 5001 FORMAT(2I5)
      READ(5,5002)(IGT(I),I=1,NGT)
 5002 FORMAT(10I8)
      CALL TGEN
      CALL TRAN
      WRITE(6,6103)(NGAGE(I),I=1,NG)
      WRITE(6,6105)
      DO 31 I=1,NR
   31 WRITE(6,6104) NOR(I),(WT(I,J),J=1,NG)
 6103 FORMAT('1',9X,'EXTERNAL'/
     112X,'REACH',9X,'COEFFICIENT OF FLOW AT GAGE'/
     211X,'NUMBER',10I10)
 6105 FORMAT(1X)
 6104 FORMAT(I15,6X,10F10.3)
      WRITE(6,6000)
 6000 FORMAT(1H1)
      CALL RDATA
      CALL SIM
      END
      SUBROUTINE SIM
C   $$$$$$$$$$$$$$$$$$$$$$$$
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1       TCA(50),NGAGE(10),NGR(10),CAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      COMMON/FLOW4/ATIME(50),BTIME
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                            ),
     1       DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW6/NWASTE,IWASTE(50),JWASTE(50),AVW(50),AW(50),PM
                                                            W(50),
     1 FK1,WASTE,XFK1
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
      COMMON/FLOW8/AVTEMP,ATEMP,TPEAK,TEMP,DTEMP
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                            (10,6),
     1       YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2       TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                            10),
     3       BDEP(10),CDEP(10)
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(50),ISTART,XK,XM
                                                            ,
     1      CC,F,XK12OW(50),BODWST(50,48),
     2       DOWST(50,48),R(48),RCON(50),SCON(50)
      COMMON/FLOW12/XK22OR(10),XK3(10),BODSTO(10),DEFSTO(10)
      COMMON/FLOW13/QREG(50,50),DEFIN(50,50),DEFOUT(50,50),BODIN(
                                                            50,50),
     1BODOUT(50,50),XMINQ(50,30),LLX(50,30),XKK(50,50)
      DO 8 I=1,10
      BODSTO(I)=0.0
      DEFSTO(I)=0.0
      DO 8 J=1,50
    8 QG(I,J)=0.0
      DO 9 I=1,NR
      DO 9 J=1,48
      DEFIN(I,J)=0.0
      DEFOUT(I,J)=0.0
      BODIN(I,J)=0.0
      BODOUT(I,J)=0.0
      BODWST(I,J)=0
      DOWST(I,J)=0
    9 QWASTE(I,J)=0
      DO 7 I=1,50
      DO 7 J=1,30
      XMINQ(I,J)=0.0
    7 LLX(I,J)=0
      REWIND 9
      READ(5,700)Z,C,TMEAN,ISTART,XK,XM,CC,F,SIGMAT
```

```
  700 FORMAT(3F8.0,I5,5F8.0)
      READ(5,701)(RLNTH(I),RCON(I),SCON(I),XK120W(I),IWASTE(I),I=
                                                              1,NR)
  701 FORMAT(4F10.0,I1)
      DO 1 I=1,NR
    1 IF(IWASTE(I).NE.0)READ(5,703)(QWASTE(I,J),BODWST(I,J),DOWST
                                                             (I,J),
     1J=1,48)
  703 FORMAT(5X,3F10.0)
      READ(5,702)(XK220R(I),XK3(I),BODSTO(I),DEFSTO(I),I=1,NRES)
  702 FORMAT(4F10.0)
C     COMPUTE MEAN TEMPERATURE FOR LTH WEEK OF THE YEAR
      DO 800 L=1,48
      XL=L
      ARG=(7.5*XL+C)/57.3
      T(L)=Z*SIN(ARG)+TMEAN
  800 CONTINUE
      DO 11 K=1,NYR
      CALL RAN(ISTART,48,R)
      CALL QFLOW(K)
      DO 10 L=1,48
      CALL QTRAN(L)
      CALL REG(L,K)
      CALL QUAL(L)
   10 CONTINUE
      KOUNT=0
      WRITE(6,400) K
  400 FORMAT('1',10X,'SUMMARY OF VIOLATIONS FOR YEAR ',I2//)
      DO 600 I=1,NR
      DO 600 L=1,48
      IF(XKK(JR(I),L).EQ.0.) GO TO 600
      WRITE(6,500) JR(I),L,XKK(JR(I),L)
  500 FORMAT('0','VIOLATION IN REACH ',I2,' DURING WEEK ',I2,T45,
                                                            'MINIM
     1UM DO = ',F8.4//)
      KOUNT=KOUNT+1
  600 CONTINUE
  601 IF(KOUNT.EQ.0) WRITE(6,620)
  620 FORMAT('0','THERE WERE NO VIOLATIONS DURING THIS YEAR'//)
   11 CONTINUE
      DO 12 I=1,NR
      WRITE(6,20) JR(I)
   20 FORMAT('0','SUMMARY OF MINIMUM FLOWS FOR REACH ',I2//,T20,
     1'YEAR',T35,'FLOW',T45,'WEEK'//)
      DO 12 K=1,NYR
   12 WRITE(6,30)K,XMINQ(JR(I),K),LLX(JR(I),K)
   30 FORMAT(T20,I2,T30,F8.0,T45,I2)
      REWIND 9
      RETURN
      END
      SUBROUTINE QUAL(L)
      COMMON/FLOW1/NR,NG,NDR(50),NUR(50,3),DAU(50,3),
     1      TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                               ),
     1       DIV(10),RREL(10)
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
      COMMON/FLOW13/QREG(50,50),DEFIN(50,50),DEFOUT(50,50),BODIN(
                                                             50,50),
     1BODOUT(50,50),XMINQ(50,30),LLX(50,30),XKK(50,50)
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(50),ISTART,XK,XM
                                                               ,
     1      CC,F,XK120W(50),BODWST(50,48),
     2       DOWST(50,48),R(48),RCON(50),SCON(50)
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                             (10,6),
     1       YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2       TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                             10),
     3       BDEP(10),CDEP(10)
      DIMENSION XK120(50),CQREG(50)
C     COMPUTE TEMPERATURE AT CURRENT TIME FRAME
      WRITE(6,620)
  620 FORMAT(1H1,'    TIME  QWASTE   DOWST  BODWST      XK1      XK
                                                          2  XMIN
     1DO  BODOUT  DEFOUT  REACH')
    3 TT=T(L)+R(L)*SIGMAT
      IF(TT.LT.0)TT=0
      DO 10 I=1,NR
C     IS THERE AN UPSTREAM REACH
C     ARE THERE ANY BRANCHES
      QSUM=0
      QSUM2=0
      SPRD1=0
      SPRD2=0
      SPRD3=0
      IF(NUR(JR(I),1).LE.0)GO TO 41
      J=1
      IF(NUR(JR(I),2).GT.0)J=2
      IF(NUR(JR(I),3).GT.0)J=3
      DO 12 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   12 QSUM=QSUM+QREG(LL,L)
      CNI=QREG(JR(I),L)
      IF(QSUM.GT.CNI)QSUM=QREG(JR(I),L)
      CN=(QREG(JR(I),L)-QSUM)/J
      QSUM=0
      DO 11 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
      CQREG(LL)=QREG(LL,L)+CN
      SPRD1=SPRD1+XK120(LL)*CQREG(LL)
      SPRD2=SPRD2+BODOUT(LL,L)*CQREG(LL)
      SPRD3=SPRD3+DEFOUT(LL,L)*CQREG(LL)
      IF(XK120(LL).EQ.0)GO TO 111
      QSUM=QSUM+CQREG(LL)
  111 QSUM2=QSUM2+CQREG(LL)
   11 CONTINUE
C     COMPUTE THE DEOXYGENATION VELOCITY CONSTANT OF THE MIXED
C       FLOW AT THE UPPER END OF EACH REACH
   41 HOLD=IDR(JR(I))
      IF(QSUM.EQ.0)QSUM=.005
      IF(QSUM2.EQ.0)QSUM2=.5
      XK120(JR(I))=(SPRD1+XK120W(JR(I))*QWASTE(JR(I),L))/(QSUM+
     1          QWASTE(JR(I),L))
C     CORRECT XK120 FOR TEMPERATURE
C     COMPUTE DISSOLVED OXYGEN SATURATION VALUE
      DOS=14.65-.41*TT+.008*TT**2-.00008*TT**3
   42 ARG=.046*(TT-20)
      XK1=XK120(JR(I))*EXP(ARG)
C     COMPUTE BOD + DEFICIT AT THE UPPER END OF THE CURRENT REACH
      IF(QREG(JR(I),L).EQ.0) GO TO 50
      BODIN(JR(I),L)=(SPRD2+BODWST(JR(I),L)*QWASTE(JR(I),L))/
     1 (QREG(JR(I),L)+QWASTE(JR(I),L))
      DEFIN(JR(I),L)=(SPRD3+(DOS-DOWST(JR(I),L))*QWASTE(JR(I),L))
                                                             /
     1 (QREG(JR(I),L)+QWASTE(JR(I),L))
      GO TO 51
   50 BODIN(JR(I),L)=0.
      DEFIN(JR(I),L)=0.
C     IS THERE A RESERVOIR AT THIS REACH
   51 IF(IRES(JR(I)).GT.0)CALL RQUAL(L,TT,XK1,BODIN,DEFIN,QSUM2,
     1TIME,I,BODOUT,DEFOUT,XK2,+100)
      IF(IRES(JR(I)).GT. 0) GO TO 100
C     COMPUTE VELOCITY OF FLOW
 2222 V=XK*(QREG(JR(I),L)+QWASTE(JR(I),L))**XM
      IF(V.EQ.0.0)V=XK*(QREG(HOLD,L)/2.)**XM
      IF(V.EQ.0)GO TO 43
C     COMPUTE TIME OF FLOW
      TIME=(RLNTH(JR(I))/V)/86400
C     COMPUTE DEPTH OF FLOW
      XH=CC*(QREG(JR(I),L)+QWASTE(JR(I),L))**F
      IF(XH.EQ.0.0)XH=CC*(QREG(HOLD,L)/2.)**F
C     COMPUTE REAERATION VELOCITY CONSTANT
      XK220=(1.3*V)/(XH**1.33)
C     CORRECT THE REAERATION VELOCITY CONSTANT FOR TEMPERATURE
      ARG=.024*(TT-20)
      XK2=XK220*EXP(ARG)
      GO TO 44
C     NO FLOW IN REACH
   43 TIME=7.6
```

```
      XK2=XK1
   44 IF(XK120(JR(I)).NE.0)GO TO 142
      BODOUT(JR(I),L)=0.0
      DEFOUT(JR(I),L)=0.0
      XK1=0
      GO TO 100
  142 ARG=-XK1*TIME
      BODOUT(JR(I),L)=(BODIN(JR(I),L)-RCON(JR(I))/XK1)*EXP(ARG)+R
                                                              CON(JR(
     1I))/XK1
      IF(XK1.EQ.XK2)GO TO 16
      ARG1=-XK1*TIME
      ARG2=-XK2*TIME
      DEF=((XK1*BODIN(JR(I),L))-RCON(JR(I)))/(XK2-XK1)
      DEF=DEF*(EXP(ARG1)-EXP(ARG2))
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK2)*(1.0-EXP(ARG2))
      DEFOUT(JR(I),L)=DEF+(DEFIN(JR(I),L)*EXP(ARG2))
      ARG=XK2/XK1-(XK2*DEFIN(JR(I),L)-RCON(JR(I))-SCON(JR(I)))*(X
                                                              K2-XK1)
     1/(XK1*(XK1*BODIN(JR(I),L)-RCON(JR(I))))
      IF(ARG.LE.0)GO TO 100
      TCRIT=(1./(XK2-XK1))*ALOG(ARG)
      IF(TCRIT.GT.TIME)GO TO 100
C     CRITICAL DEFICIT OCCURS IN THIS REACH
      ARG1=-XK1*TCRIT
      ARG2=-XK2*TCRIT
      DEF=((XK1*BODIN(JR(I),L))-RCON(JR(I)))/(XK2-XK1)
      DEF=DEF*(EXP(ARG1)-EXP(ARG2))
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK2)*(1.0-EXP(ARG2))
      DEFOUT(JR(I),L)=DEF+(DEFIN(JR(I),L)*EXP(ARG2))
      GO TO 100
   16 ARG=-XK1*TIME
      DEF=(XK1*TIME*BODIN(JR(I),L))-(TIME*RCON(JR(I)))
      DEF=DEF+DEFIN(JR(I),L)
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK1)
      DEF=DEF*EXP(ARG)
      DEFOUT(JR(I),L)=DEF-(RCON(JR(I))+SCON(JR(I)))/XK1
      TCRIT=(1./XK1)-DEFIN(JR(I),L)/(XK1*BODIN(JR(I),L)-RCON(JR(I
                                                              )))
     1  +(RCON(JR(I))+SCON(JR(I)))/(XK1*(XK1*BODIN(JR(I),L)-RCON(
                                                              JR(I)))
     2)
      ARG1=-XK1*TCRIT
      IF(TCRIT.GT.TIME)GO TO 100
C     OTHERWISE RECOMPUTE DEFOUT
      DEFOUT(JR(I),L)=(XK1*(BODIN(JR(I),L)-RCON(JR(I))/XK1)*TCRIT
                                                              +
     1   DEFIN(JR(I),L)+(RCON(JR(I))+SCON(JR(I)))/XK1)*EXP(ARG1)
     2   -(RCON(JR(I))+SCON(JR(I)))/XK1
C     COMPUTE MINIMUM DO
  100 IF(DEFOUT(JR(I),L).LT.DEFIN(JR(I),L)) XMINDO=DOS-DEFIN(JR(I
                                                                ),L)
    IF(DEFOUT(JR(I),L).GE.DEFIN(JR(I),L)) XMINDO=DOS-DEFOUT(JR(
                                                                I),L)
    WRITE(6,600)TIME,QWASTE(JR(I),L),DOWST(JR(I),L),BODWST(JR(I
                                                                ),L),
   1      XK1,XK2,XMINDO,BODOUT(JR(I),L),DEFOUT(JR(I),L),JR(I)
600 FORMAT(1H ,9F8.4,5X,I2)
150 XKK(JR(I),L)=0.
    IF(XMINDO.LE.4.0) XKK(JR(I),L)=XMINDO
 10 CONTINUE
    RETURN
    END
    SUBROUTINE RQUAL(L,TT,XK1,BODIN,DEFIN,QSUM2,
   1TIME,I,BODOUT,DEFOUT,XK2,*)
    COMMON/FLOW3/JR(50),WT(50,10)
    COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                                ),
   1       DIV(10),RREL(10),PSTO(10)
    COMMON/FLOW12/XK220R(10),XK3(10),BODSTO(10),DEFSTO(10)
    DIMENSION DEFIN(50,50),DEFOUT(50,50),BODIN(50,50),BODOUT(50
                                                                ,50)
    CALL TRES(IRES(JR(I)),IR)
    ARG=(.024*(TT-20.))
    XK2=XK220R(IR)*EXP(ARG)
    AVGSTO=(PSTO(IR)+STO(IR))/2.
    IF(AVGSTO.LE.0)GO TO 100
    IF(STO(IR).EQ.0.AND.RREL(IR).NE.0)GO TO 100
    Z=(QSUM2*3600.*24.)/(AVGSTO*10**6)
    W=((RREL(IR)+DIV(IR))*3600.*24.)/(AVGSTO*10**6)
    XFK3=(XK3(IR)*AVGSTO)/CAP(IR)
    A=XK1+W+XFK3
    B=XK2+W
    TIME=11.57*STO(IR)/QSUM2
    IF(TIME.GT.30.0)TIME=30.
    ARG=-A*TIME
    IF(BODSTO(IR).LT..001)BODSTO(IR)=0.0
    DEF9=BODSTO(IR)-Z*BODIN(JR(I),L)/A
    DEF9=DEF9*(1-EXP(ARG))/(A*TIME)
    BODOUT(JR(I),L)=DEF9+Z*BODIN(JR(I),L)/A
    ARG2=-B*TIME
    IF(DEFIN(JR(I),L).LT..001) GO TO 98
    DEF1=(DEFSTO(IR)/B)-(Z*DEFIN(JR(I),L)/(B*B))
    DEF1=DEF1+(XK1*BODSTO(IR)/(A*B))-(XK1*Z*BODIN(JR(I),L)*(A+B
                                                                )
   1/(A*B)**2)
    DEF2=(Z*DEFIN(JR(I),L)/B)-DEFSTO(IR)
    DEF2=DEF2+(XK1/(A-B))*((Z*BODIN(JR(I),L)/B)-BODSTO(IR))
    DEF2=DEF2*EXP(ARG2)/B
    DEF3=(Z*TIME/B)*(DEFIN(JR(I),L)+(XK1*BODIN(JR(I),L))/A)
    DEF4=(Z*BODIN(JR(I),L)/A)-BODSTO(IR)
    DEF4=DEF4*XK1*EXP(ARG)/(A*(B-A))
```

```
      DEFOUT(JR(I),L)=(DEF1+DEF2+DEF3+DEF4)/TIME
      GO TO 102
   98 DEFOUT(JR(I),L)=0.0
  102 IF(DEFSTO(IR).LT..001)GO TO 99
      DEFSTO(IR)=DEFSTO(IR)*EXP(ARG2)-Z*DEFIN(JR(I),L)*EXP(ARG2)/
                                                              B+
     1XK1*BODSTO(IR)*EXP(ARG2)/(A-B)-XK1*Z*BODIN(JR(I),L)*EXP(ARG
                                                          2)/(B*
     2(A-B))+XK1*BODSTO(IR)*EXP(ARG)/(B-A)-XK1*Z*BODIN(JR(I),L)*E
                                                         XP(ARG)
     3/(A*(B-A))+Z*DEFIN(JR(I),L)/B+Z*XK1*BODIN(JR(I),L)/(A*B)
      GO TO 103
   99 DEFSTO(IR)=0.0
  103 BODSTO(IR)=BODSTO(IR)*EXP(ARG)+(Z*BODIN(JR(I),L)*(1.-EXP(AR
                                                          G)))/A
      GO TO 101
  100 BODOUT(JR(I),L)=BODIN(JR(I),L)
      DEFOUT(JR(I),L)=DEFIN(JR(I),L)
      BODSTO(IR)=0
      DEFSTO(IR)=0
      Z=0
      W=0
      XFK3=0
      TIME=0
      A=0
      B=0
  101 RETURN
      END
      SUBROUTINE REG(L,KYR)
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1       TCA(50),NGAGE(10),NGR(10),CAG(10),IDR(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                              ),
     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
      COMMON/FLOW13/QREG(50,50),DEFIN(50,50),DEFOUT(50,50),BODIN(
                                                         50,50),
     1BODOUT(50,50),XMINQ(50,30),LLX(50,30),XKK(50,50)
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                         (10,6),
     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                         10),
     3        BDEP(10),CDEP(10)
      DIMENSION DQ(50),AREA(10),DEP(10)
      XL=L
      XLRAD=(XL*7.5+BVAP)/57.3
      EVAP=AVAP*SIN(XLRAD)+CVAP
      DO 100 I=1,NR
      QNAT(JR(I))=QNAT(JR(I))-XIRRIG(JR(I),L)
      IF(QNAT(JR(I)).LT.0)QNAT(JR(I))=0
C  THE INDEX OF THE CURRENT REACH IS JR(I)
C  IS THERE A REACH UPSTREAM
      IF(NUR(JR(I),1).GT.0) GO TO 10
C  NO-IS THERE A RESERVOIR AT THIS REACH
      IF(IRES(JR(I)).GT.0) GO TO 20
C  NO
      DQ(JR(I))=0.
      QREG(JR(I),L)=QNAT(JR(I))
      GO TO 499
C  THERE IS A REACH UPSTREAM
C  IS THIS A RESERVOIR
   10 IF(IRES(JR(I)).GT.0) GO TO 30
C  NO-COUNT UPSTREAM REACHES
      J=1
      IF(NUR(JR(I),2).GT.0) J=2
      IF(NUR(JR(I),3).GT.0) J=3
      DQ(JR(I))=0.
      DO 11 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   11 DQ(JR(I))=DQ(JR(I))+DQ(LL)
      QREG(JR(I),L)=             QNAT(JR(I))+DQ(JR(I))
      GO TO 499
C  THIS IS RESERVOIR , NO UPSTREAM REACH
   20 QIN=QNAT(JR(I))
      GO TO 40
C  THIS IS RESERVOIR, REACH UPSTREAM
C  COUNT REACHES
   30 J=1
      IF(NUR(JR(I),2).GT.0) J=2
      IF(NUR(JR(I),3).GT.0) J=3
      QIN=QNAT(JR(I))
      DO 31 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   31 QIN=QIN+DQ(LL)
      QREG(JR(I),L)=QIN
C  OPERATE RESERVOIR
   40 CALL TRES(IRES(JR(I)),IR)
      PSTO(IR)=STO(IR)
      DEP(IR)=(ACAP(IR)+(BCAP(IR)*STO(IR)+CCAP(IR))**.5)/DCAP(IR)
      AREA(IR)=ADEP(IR)+BDEP(IR)*DEP(IR)+CDEP(IR)*DEP(IR)**2
      QVAP=AREA(IR)*EVAP
      QINV=QIN*.657-QVAP
      CALL DIVREL(IR,L,RRELV,DEP)
      STO(IR)=STO(IR)+(QINV-DIV(IR))
      IF(STO(IR).GT.CAP(IR))GO TO 300
      IF(ITYPE(IR).GT.3)GO TO 310
      IF(STO(IR).GT.TOPLEV(IR).AND.ITYPE(IR).EQ.1)GO TO 302
      IF(STO(IR).GT.SMIN(IR))GO TO 303
```

```
C     NO RELEASES-NO DIVERSIONS
  301 RRELV=0
      STO(IR)=STO(IR)+DIV(IR)
      IF(STO(IR).LE.0)GO TO 307
      GO TO 310
C     RESERVOIR EMPTY
  307 STO(IR)=0
      GO TO 310
  300 IF(ITYPE(IR).EQ.1)GO TO 400
      RRELV=STO(IR)-CAP(IR)
      GO TO 310
  400 RRV=20*RRELV
      RTEMP=STO(IR)-CAP(IR)
      IF(RTEMP.GT.RRV)GO TO 401
      RRELV=RRV
      GO TO 310
  401 RRELV=RTEMP
      GO TO 310
  303 IF(ITYPE(IR).NE.3)GO TO 403
C     IS THE TIME BETWEEN OCT. 1 + NOV.1
      IF(L.GE.37 .AND. L.LE.40)GO TO 305
C     NO-IS THE TIME BETWEEN APR. 1 + OCT. 1
      IF(L.LT.13 .OR.L.GT. 36)GO TO 308
C     YES-TIME IS BETWEEN APR. 1 + OCT.1
      IF(STO(IR).LE.CAP(IR))RRELV=0
      GO TO 310
C     TIME IS BETWEEN OCTOBER 1 + NOV. 1
  305 SDIF=STO(IR)-SMIN(IR)
      RRELV=SDIF/(41-L)
      GO TO 310
  302 RRV=20*RRELV
      RTEMP=STO(IR)-TOPLEV(IR)
      IF(RTEMP.LE.RRV)GO TO 402
      RRELV=RRV
      GO TO 310
  402 IF(RTEMP.LT.RRELV)GO TO 310
      RRELV=RTEMP
      GO TO 310
  403 IF(ITYPE(IR).EQ.1)GO TO 309
      RRV=STO(IR)-SMIN(IR)
      IF(RRELV.LE.RRV)GO TO 310
      RRELV=RRV
      GO TO 310
  309 STO(IR)=STO(IR)+QINV-RRELV
      IF(STO(IR).GT.TOPLEV(IR))RRELV=STO(IR)-TOPLEV(IR)
      GO TO 312
  308 RRELV=STO(IR)-SMIN(IR)
  310 STO(IR)=STO(IR)-RRELV
  312 RREL(IR)=RRELV/.657
      DQ(JR(I))=RREL(IR)-QNAT(JR(I))
  499 IF(L.GT.1) GO TO 500
```

```
      XMINQ(JR(I),KYR)=QREG(JR(I),L)
      LLX(JR(I),KYR)=L
  500 IF(XMINQ(JR(I).KYR).GT.QREG(JR(I),L)) GO TO 502
      GO TO 99
  502 XMINQ(JR(I),KYR)=QREG(JR(I),L)
      LLX(JR(I),KYR)=L
   99 IF(QREG(JR(I),L).LT.0.0)QREG(JR(I),L)=0.0
  100 CONTINUE
      JR(44)=0
      QREG(JR(44),L)=0.0
      WRITE(6,600) L,KYR
  600 FORMAT(1H0,'REGULATED FLOWS FOR WEEK',I3,' OF YEAR',I4//,2X
                                                                  ,
     14('REACH',2X,'REG. FLOW',7X))
      DO 50 I=1,11
      I1=I+11
      I2=I+22
      I3=I+33
      WRITE(6,610)JR(I),QREG(JR(I),L),JR(I1),QREG(JR(I1),L),
     1JR(I2),QREG(JR(I2),L),JR(I3),QREG(JR(I3),L)
  610 FORMAT(' ',2X,4(I3,4X,F8.0,8X))
   50 CONTINUE
      WRITE(6,800)(JRES(I),STO(I),RREL(I),DIV(I),I=1,NRES)
  800 FORMAT(1H0,'AT THIS TIME THE CONTENTS OF OUR RESERVOIRS ARE
                                                            AS FOL
     1LOWS',/' RESERVOIR   STORAGE    RELEASE    DIVERSION',/(
     1        1H ,I9,F10.3,F11.3,F13.3))
      RETURN
      END
      SUBROUTINE RAN(IX,N,R)
      DIMENSION R(1)
      DO 10 I=1,N
      R(I)=0.
      DO 20 J=1,12
      R(I)=R(I)+RRN(IX)
   20 CONTINUE
   10 R(I)=R(I)-6.
      RETURN
      END
      FUNCTION RRN(IX)
      IX=IX*65539
      IF(IX)5,6,6
    5 IX=IX+2147483647+1
    6 RRN=IX
      RRN=RRN*.4656613E-9
      RETURN
      END
      FUNCTION STD(T)
      DIMENSION T(48)
      TBAR=0
      DO 1 I=1,48
    1 TBAR=TBAR+T(I)
      TBAR=TBAR/48.
      TSUM=0
      DO 2 I=1,48
    2 TSUM=TSUM+(T(I)-TBAR)**2
      STD=SQRT(TSUM/47.)
      RETURN
      END
      SUBROUTINE RDATA
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1      TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                                 ),

     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                               (10,6),

     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                                10),

     3        BDEP(10),CDEP(10)
      DO 30 I=1,NR
      DO 30 J=1,48
   30 XIRRIG(I,J)=0
      READ(5,500)NRES,(IRES(I),I=1,NR)
  500 FORMAT(I5/(16I5))
      READ(5,501)(JRES(J),ITYPE(J),CAP(J),STO(J),SMIN(J),TOPLEV(J
                                                                 ),

     1      J=1,NRES)
  501 FORMAT(2I5,4F10.2)
      READ(5,504)(ACON(I),BCON(I),I=1,NRES)
  504 FORMAT(8F10.4)
      READ(5,505)(YMEAN(L),ACAP(L),BCAP(L),CCAP(L),DCAP(L),
     1      ADEP(L),BDEP(L),CDEP(L),L=1,NRES)
  505 FORMAT(8F10.2)
      READ(5,503)AVAP,BVAP,CVAP,((TAU(I,J),CCON(I,J),J=1,6),I=1,N
                                                               RES)

  503 FORMAT(3F10.0,/(6F10.0))
      READ(5,502)NADJST,(L,NSTA,XIRRIG(NSTA,L),
     1    I=1,NADJST)
  502 FORMAT(I5/(I5,I8, F14.4))
      RETURN
      END
      SUBROUTINE DIVREL(IR,L,RRELV,DEP )
C     ****************************
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                                 ),

     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW9/XIRRIG(50,48),QWASTE(50,48),AVAP,BVAP,CVAP,TAU
                                                               (10,6),
     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                                  10),
     3        BDEP(10),CDEP(10)
      DIMENSION DEP(10)
      ITYP=ITYPE(IR)
      XL=L
      GO TO (1,2,3,4,5),ITYP
    1 RRELV=ACON(IR)*0.657
      THETA=(XL*2.*3.1416)/48.
      DIV(IR)=YMEAN(IR)
      DO 10 I=1,6
      ARG=THETA-TAU(IR,I)
   10 DIV(IR)=DIV(IR)+CCON(IR,I)*COS(ARG)
      GO TO 101
    2 BCONN=BCON(IR)
      RRELV=0.657*ACON(IR)*DEP(IR)**BCONN
      DIV(IR)=0
      GO TO 101
    3 RRELV=0
      DIV(IR)=0
      GO TO 101
    4 RRELV=0
      THETA=(XL*2.*3.1416)/48.
      DIV(IR)=YMEAN(IR)
      DO 40 I=1,6
      ARG=THETA-TAU(IR,I)
   40 DIV(IR)=DIV(IR)+CCON(IR,I)*COS(ARG)
      GO TO 101
    5 THETA=(XL*2.*3.1416)/48.
      RRELV=YMEAN(IR)
      DO 50 I=1,6
      ARG=THETA-TAU(IR,I)
   50 RRELV=RRELV    +CCON(IR,I)*COS(ARG)
      DIV(IR)=0
  101 RETURN
      END
      SUBROUTINE S(KT,NN,A,B,C,IM,JM,KM,DET)
      DIMENSION A(KT,KT),B(KT,KT),C(KT,KT),IN(100),EMP(100)
      IMAX=IM
      JMAX=JM
      KMAX=KM
      GOTO(30,32,34,36,38,40,42,44,46,50,52),NN
   30 DO 31 I=1,IMAX
      DO31J=1,JMAX
   31 A(I,J)=B(I,J)+C(I,J)
      GO TO 805
   32 DO33I=1,IMAX
      DO33J=1,JMAX
   33 A(I,J)=B(I,J)-C(I,J)
      GO TO 805
   34 DO101I=1,IMAX
      DO35J=1,KMAX
      EMP(J)=0.
     DO35K=1,JMAX
  35  EMP(J)= EMP(J)+B(I,K)*C(K,J)
     DO101K=1,KMAX
 101 A(I,K)= EMP(K)
     GO TO 805
  36 DO37I=1,IMAX
     DO37J=1,IMAX
  37 A(I,J)=B(I,J)
  59 IN(1)=0
     IMAXO=IMAX-1
     TEMP=A(1,1)
     DO70I=2,IMAX
     IF(ABS (TEMP)-ABS (A(I,1)))71,70,70
  71 IN(1)=I
     TEMP=A(I,1)
  70 CONTINUE
     IF(IN(1))73,72,73
  73 IS=IN(1)
     DO74J=1,IMAX
     TEMP=A(1,J)
     A(1,J)=A(IS,J)
  74 A(IS,J)=TEMP
  72 IF(A(1,1))98,99,98
  98 DO75I=2,IMAX
  75 A(I,1)=A(I,1)/A(1,1)
     DO100I=2,IMAX
     IPO=I+1
     IMO=I-1
     DO80L=1,IMO
  80 A(I,I)=A(I,I)-(A(L,I)*A(I,L))
     TEMP=A(I,I)
     IF(I-IMAX)55,83,55
  55 IN(I)=0
     DO81IS=IPO,IMAX
     DO85L=1,IMO
  85 A(IS,I)=A(IS,I)-A(L,I)*A(IS,L)
     IF(ABS (TEMP)-ABS (A(IS,I)))82,81,81
  82 TEMP=A(IS,I)
     IN(I)=IS
  81 CONTINUE
     ISS=IN(I)
     IF(ISS)84,83,84
  84 DO886J=1,IMAX
     TEMP=A(I,J)
     A(I,J)=A(ISS,J)
 886 A(ISS,J)=TEMP
  83 IF(A(I,I))97,99,97
  97 IF(I-IMAX)54,100,54
  54 DO86IS=IPO,IMAX
  86 A(IS,I)=A(IS,I)/A(I,I)
```

```
      DO89JS=IPO,IMAX
      DO89L=1,IMO
   89 A(I,JS)=A(I,JS)-(A(L,JS)*A(I,L))
  100 CONTINUE
      DO600JP=1,IMAX
      J=IMAX+1-JP
      A(J,J)=1.0/A(J,J)
      IF(J-1)603,700,603
  603 DO600IP=2,J
      I=J+1-IP
      IPO=I+1
      TEMP=0.0
      DO602L=IPO,J
  602 TEMP=TEMP-A(I,L)*A(L,J)
  600 A(I,J)=TEMP/A(I,I)
  700 DO151J=1,IMAXO
      JPO=J+1
      DO151I=JPO,IMAX
      TEMP=0.0
      IMO=I-1
      DO154L=J,IMO
      IF(L-J)152,153,152
  152 TEMP=TEMP-A(I,L)*A(L,J)
      GO TO154
  153 TEMP=TEMP-A(I,L)
  154 CONTINUE
  151 A(I,J)=TEMP
      DO901I=1,IMAX
      DO900J=1,IMAX
       EMP(J)=0.0
      DO899N=I,IMAX
      IF(N-J)899,897,898
  898  EMP(J)= EMP(J)+A(I,N)*A(N,J)
      GO TO899
  897  EMP(J)= EMP(J)+A(I,N)
  899 CONTINUE
  900 CONTINUE
      DO901J=1,IMAX
  901 A(I,J)= EMP(J)
      DO500I=2,IMAX
      M=IMAX+1-I
      IF(IN(M))502,500,502
  502 ISS=IN(M)
      DO503L=1,IMAX
      TEMP=A(L,ISS)
      A(L,ISS)=A(L,M)
  503 A(L,M)=TEMP
  500 CONTINUE
      DET=0.
      GO TO 805
  120 DET=1.
```

```
 99 WRITE             (6,806 )
806 FORMAT(18HC  SINGULAR MATRIX)
805 RETURN
 38 DO39I=1,IMAX
    DO39J=1,IMAX
 39 A(I,J)=B(I,J)
    N=IMAX
    DET=1.
    I1=1
  1 I3=I1
    SUM=ABS (A(I1,I1))
    DO 3 I=I1,N
    IF(SUM-ABS (A(I,I1)))2,3,3
  2 I3=I
    SUM=ABS (A(I,I1))
  3 CONTINUE
    IF(I3-I1)4,6,4
  4 DO5J=1,N
    SUM=-A(I1,J)
    A(I1,J)=A(I3,J)
  5 A(I3,J)=SUM
  6 I3=I1+1
    DO7I=I3,N
  7 A(I,I1)=A(I,I1)/A(I1,I1)
    J2=I1-1
    IF(J2)8,11,8
  8 DO9J=I3,N
    DO9I=1,J2
  9 A(I1,J)=A(I1,J)-A(I1,I)*A(I,J)
 11 J2=I1
    I1=I1+1
    DO12I=I1,N
    DO12J=1,J2
 12 A(I,I1)=A(I,I1)-A(I,J)*A(J,I1)
    IF(I1-N)1,14,1
 14 I3=1
    J2=N/2
    IF(2*J2-N)15,16,15
 15 I3=0
    DET=A(N,N)
 16 DO17I=1,J2
    J=N-I+I3
 17 DET=DET*A(I,I)*A(J,J)
    GO TO 805
 40 IF(IMAX-JMAX)41,102,102
 41 IP=IMAX
    GO TO 103
102 IP=JMAX
103 DO106K=1,IP
    DO104I=K,IMAX
104  EMP(I)=B(I,K)
```

```
      DO105J=K,JMAX
  105 A(J,K)=B(K,J)
      DO106I=K,IMAX
  106 A(K,I)= EMP(I)
      GO TO 805
   42 DO43I=1,IMAX
      DO43J=1,JMAX
   43 A(I,J)=B(I,J)
      GO TO 805
   44 DO45I=1,IMAX
      DO45J=1,JMAX
      A(I,J)=0.
      B(I,J)=0.
   45 C(I,J)=0.
      GO TO 805
   46 ID=2
   20  READ (KMAX,47) IN(1),IN(5),EMP(1),IN(2),IN(6),EMP(2),
     1  IN(3),IN(7),EMP(3),IN(4),IN(8),EMP(4)
   47 FORMAT (4(I3,I3,E12.8))
      IF(IN(1))805,805,23
   23 GO TO(19,24),ID
   24 IM=IN(1)
      JM=IN(5)
      ID=1
   19 DO21I=1,4
      I1=IN(I)
      J1=IN(I+4)
      IF(I1)21,21,18
   18 A(I1,J1)= EMP(I)
   21 CONTINUE
      GO TO 20
   50 DO 62 IP=1,JMAX,7
      JPO=IP+6
      IF(JPO-JMAX)61,61,60
   60 JPO=JMAX
   61 WRITE              (KMAX,63)(J,J=IP,JPO)
      DO 62 I=1,IMAX
      WRITE              (KMAX,64)I,(A(I,J),J=IP,JPO)
   62 CONTINUE
      GO TO 805
   63 FORMAT(5H0 ROW7(8X,4HCOL.I3,1X))
   64 FORMAT(I4,4X,7E16.8)
   52 DO53I=1,IMAX
      DO53J=1,JMAX
   53 A(I,J)=B(I,J)*DET
      GO TO 805
      END
      SUBROUTINE TRAN
      COMMON/FLOW1/NR,NG,NCR(50),NLR(50,3),DAU(50,3),
     1       TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION JGU(10),NIU(50),NID(50)
C  ****************************
C  COMPUTE FLOW IN LAST REACH DOWNSTREAM
C  ****************************
      I = JR(NR)
C  IS THERE A GAGE IN THIS REACH
      J = 0
      DO 5 JJ=1,NG
      IF (NGR(JJ) - NOR(I)) 5,6,5
    6 J = JJ
    5 CONTINUE
      IF (J) 10,10,15
C  THERE IS A GAGE - CASE 1
   15 DO 16 JJ=1,NG
   16 WT(I,JJ) = 0.
      WT(I,J) = TDA(I)/DAG(J)
      GO TO 100
C  THERE IS NO GAGE - CASE 2
   10 CALL UPGAGE (I,NGU,JGU)
      IF (NGU) 21,21,22
   21 WRITE (6,6000)
 6000 FORMAT (10X,'NO GAGES')
      STOP
   22 DO 23 JJ=1,NG
   23 WT(I,JJ) = 0.
      GDA = 0.
      DO 25 JJ=1,NGU
   25 GDA = GDA+DAG(JGU(JJ))
      DO 24 JJ=1,NGU
      J = JGU(JJ)
   24 WT(I,J) = TDA(I)/GDA
C  ********************************
C  CONTINUE UPSTREAM
C  ********************************
  100 IU = I
      IB = 1
      NID(IB) = IU
C  IS THERE A REACH UPSTREAM
  105 NU = NUR(IU,1)
      IF (NU) 110,110,115
C  THERE IS NO REACH UPSTREAM
  110 IB = IB - 1
C  HAVE ALL REACHES BEEN COMPLETED
      IF (IB) 200,200,120
C  TRANSFORM IS COMPLETE
  200 RETURN
C  CONTINUE CALCULATIONS
  120 IU = NIU(IB)
      ID = NID(IB)
      GO TO 150
```

```
C  THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
  115 IF (NUR(IU,2)) 125,125,130
C  THERE IS NO BRANCH
  125 NU = NUR(IU,1)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS A BRANCH - ARE THERE TWO
  130 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      IF (NUR(IU,3)) 135,135,140
C  THERE IS ONLY ONE BRANCH
  135 NU = NUR(IU,2)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS ANOTHER BRANCH
  140 NU = NUR(IU,2)
      IB = IB + 1
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      NU = NUR(IU,3)
      ID = IU
      IU = IREACH(NU)
C  IS THERE A GAGE IN THIS REACH
  150 J = 0
      DO 151 JJ=1,NG
      IF (NOR(JJ)-NOR(IU)) 151,152,151
  152 J = JJ
  151 CONTINUE
      IF (J) 160,160,165
C  THERE IS A GAGE IN THIS REACH
  165 DO 166 JJ=1,NG
  166 WT(IU,JJ) = 0.
      WT(IU,J) = TDA(IU)/DAG(J)
      GO TO 105
C  THERE IS NO GAGE IN THIS REACH
  160 CALL UPGAGE (IU,NGU,JGU)
      IF (NGU) 180,180,161
  161 GDA = 0.
      DO 170 JJ=1,NGU
      J = JGU(JJ)
  170 GDA = GDA + DAG(J)
      DO 171 JJ=1,NG
  171 WT(IU,JJ) = 0.
      A1 = (TDA(ID) - TDA(IU))/(TDA(ID) - GDA)
      DO 172 JJ=1,NGU
      J = JGU(JJ)
  172 WT(IU,J) = A1*TDA(IU)/GDA
```

```
      A1 = (TDA(IU) - GDA)/(TDA(ID) - GDA)/TDA(ID)
      DO 173 J=1,NG
  173 WT(IU,J) = WT(IU,J) + A1*WT(ID,J)*TDA(IU)
      GO TO 105
C  THERE IS NO GAGE UPSTREAM - CASE 2
  180 A1 = TDA(IU)/TDA(ID)
      DO 181 J=1,NG
  181 WT(IU,J) = A1*WT(ID,J)
      GO TO 105
      END
      SUBROUTINE TGEN
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1        TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION IR(50)
      DIMENSION NNOR(50)
      READ (5,5001) NR,NG
 5001 FORMAT (2I5)
      DO 1 I=1,NR
      NNOR(I)=0
    1 IDR(I) = 0
      DO 5 I=1,NR
    5 READ (5,5002) NOR(I),(NUR(I,J),J=1,3),
     1       (DAU(I,J),J=1,3),FL(I),SLOPE(I),ROUGH(I)
 5002 FORMAT (4I5,6F5.0)
      DO 6 I=1,NG
    6 READ (5,5003) NGAGE(I),NGR(I),DAG(I)
 5003 FORMAT(I8,I5,F5.0)
C  DETERMINE SEQUENCE OF REACH NUMBERS
      DO 15 I=1,NR
   15 IR(I) = 0
      DO 20 N=1,NR
      I = 1
   23 IF (IR(I)) 21,21,22
   22 I = I + 1
      IF(I.GT.NR) GO TO 20
      GO TO 23
   21 K = 0
      DO 25 J=1,3
      IF (NUR(I,J)) 25,25,26
   26 NUP = NUR(I,J)
      L = IREACH(NUP)
      IF (IR(L)) 27,27,25
   27 K = 1
   25 CONTINUE
      IF (K) 30,30,22
   30 JR(N) = I
      IR(I) = 1
      DO 36 K=1,3
      IF (NUR(I,K)) 36,36,37
   37 ND = NUR(I,K)
      ID = IREACH(ND)
      IDR(ID) = I
   36 CONTINUE
   20 CONTINUE
      DO 45 I=1,NR
      TDA(I) = 0.
      DO 45J=1,3
   45 TDA(I) = TDA(I) + DAU(I,J)
C  *****************
      WRITE(6,6000)
 6000 FORMAT('1EXTERNAL',49X,'DOWNSTREAM REACH',6X,'TOTAL',15X,
     1'REACH COMPUTATION SEQUENCE'/
     13X,'REACH',4X,'UPSTREAM REACHES',3X,'UPSTREAM DRAINAGE AREA
                                                                S',
     1        3X,'INTERNAL  EXTERNAL',3X,'UPSTREAM',3X,'INTERNAL',
                                                               7X,
     1'INTERNAL  EXTERNAL'/2X,'NUMBER',7X,'1',4X,'2',4X,'3'8X,'1'
                                                              ,7X,
     1'2',7X,'3',6X,'INDEX',7X,'NUMBER',5X,'AREA',7X,'INDEX',8X,'
                                                           INDEX',
     17X,'NUMBER'/)
      DO 800 I=1,NR
      IF(IDR(I).NE.0)NNOR(IDR(I))=NOR(IDR(I))
  800 CONTINUE
      DO 46 I=1,NR
   46 WRITE(6,6001)NOR(I),(NUR(I,J),J=1,3),(DAU(I,J),J=1,3),IDR(I
                                                               ),
     1NNOR(IDR(I)),TDA(I),I,JR(I),NOR(JR(I))
 6001 FORMAT(I6,I10,I5,I5,F11.1,F8.1,F8.1,I8,I11,F12.1,I10,6X,I8,
                                                             I11)
      WRITE(6,6002)
 6002 FORMAT(///43X,'BASIC',7X,'REACH',7X,'AREA'/
     143X,'GAGE',5X,'CONTAINING',3X,'UPSTREAM',3X,'INTERNAL'/
     142X,'NUMBER',7X,'GAGE',7X,'OF GAGE',5X,'INDEX'/)
      DO 47 I=1,NG
   47 WRITE(6,6003) NGAGE(I),NGR(I),DAG(I),I
 6003 FORMAT(38X,I8,I12,F13.1,I10)
C  **************
      RETURN
      END
      SUBROUTINE UPGAGE (I,NGU,JGU)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1       TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION JGU(10),NIU(50)
C  DETERMINE THE NUMBER AND IDENTITY OF GAGES UPSTREAM
C  OF THIS REACH. DISCOUNT FURTHER SEARCH WHEN A GAGE
C  IS ENCOUNTERED. I IS CURRENT REACH, NGU IS NUMBER OF
C  GAGES UPSTREAM AND JGU ARE INDICIES OF THESE GAGES
```

```
      NGU = 0
      IB = 1
      IU = I
C   IS THERE A REACH UPSTREAM
    5 NU = NUR(IU,1)
      IF (NU) 10,10,15
C   THERE IS NO REACH UPSTREAM
   10 IB = IB - 1
C   IS THE SEARCH COMPLETE
      IF (IB) 100,100,20
C   SEARCH COMPLETE
  100 RETURN
C   CONTINUE SEARCH
   20 IU = NIU(IB)
      GO TO 24
C   THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
   15 IF (NUR(IU,2)) 25,25,30
C   THERE IS NO BRANCH
   25 IU = IREACH(NU)
C   IS THERE A GAGE
   24 J = 0
      DO 26 JJ=1,NG
      IF (NGR(JJ) - NOR(IU)) 26,27,26
   27 J = JJ
   26 CONTINUE
      IF (J) 35,35,40
C   THERE IS NO GAGE IN THIS REACH
   35 GO TO 5
C   THERE IS A GAGE IN THIS REACH
   40 NGU = NGU + 1
      JGU(NGU) = J
C   CONTINUE SEARCHING
      GO TO 10
C   THERE IS A BRANCH
   30 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB - 1) = IREACH(NU)
C   IS THERE A SECOND BRANCH
      IF (NUR(IU,3)) 45,45,50
C   THERE IS NO SECOND BRANCH
   45 NU = NUR(IU,2)
      GO TO 25
C   THERE IS A SECOND BRANCH
   50 IB = IB + 1
      NU = NUR(IU,2)
      NIU(IB-1) = IREACH(NU)
      NU = NUR(IU,3)
      GO TO 25
      END
      FUNCTION IREACH (NU)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1        TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      II = 1
    3 IF (NCR(II) - NU) 1,2,1
    1 II = II + 1
      IF (II.EQ.NR) GO TO 2
      GO TO 3
    2 IREACH = II
      RETURN
      END
      SUBROUTINE GFLOW(K)
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1        TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
    2 READ(9)((QG(I,J),I=1,NG),J=1,48)
   11 WRITE(6,600) K
  600 FORMAT('1',T40,'BEGINNING OF YEAR',I2////)
      RETURN
      END
      SUBROUTINE QTRAN(L)
      COMMON/FLOW1/NR,NG,NCR(50),NUR(50,3),DAU(50,3),
     1        TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      COMMON/FLOW7/QG(10,48),QNAT(50),NYR
      COMMON/FLOW13/QREG(50,50),DEFIN(50,50),DEFOUT(50,50),BODIN(
                                                               50,50),
     1BODOUT(50,50),XMINQ(50,30),LLX(50,30),XKK(50,50)
      CALL S (50,3,QNAT,WT,QG(1,L),NR,NG,1,0)
      K=0
      DO 10 I=1,16
      J=17-I
      K=J+1
      IF(QNAT(J).LT.QNAT(K)) GO TO 20
   10 CONTINUE
      GO TO 30
   20 DO 11 I=1,J
      QNAT(I)=QNAT(K)
   11 CONTINUE
   30 WRITE(6,600) L
  600 FORMAT(' ','NATURAL FLOWS FOR WEEK',I3/)
      WRITE(6,601)(QNAT(I),I=1,NR)
  601 FORMAT(1H ,10F8.0)
      RETURN
      END
C     $$$$$$$$$$$$$$$$$$$$$$$$$$
C     **************************
      SUBROUTINE TRES(N,I)
      COMMON/FLOW5/NRES,IRES(50),JRES(10),CAP(10),SMIN(10),STO(10
                                                             ),
     1        DIV(10),RREL(10),PSTO(10)
```

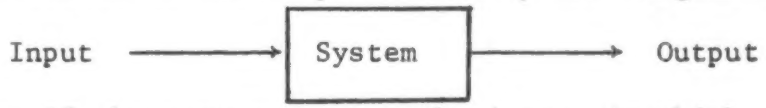
```
      J=0
    5 J=J+1
      IF(JRES(J).EQ.N) GO TO 10
      IF(J.LT.NRES) GO TO 5
      WRITE(6,6000)N,I,NRES,JRES
 6000 FORMAT(' RESERVOIR CANNOT BE LOCATED'/(20I5))
      CALL EXIT
   10 I=J
      RETURN
      END
/*
//GO.FT09F001 DD UNIT=TAPE9,VOLUME=SER=XXX,LABEL=(,BLP),DISP=(,PA
                                                              SS)
```

A4.6 AIJ

A4.6.1 Purpose

The AIJ program is designed to develop the transfer coefficients for BOD and DO deficit for each reach in the watershed for each "week" of the year. The coefficients are developed by simulation, using WASP to generate the data needed to compute them. The coefficients are dependent upon many factors, the effects of which are assumed to be either dependent upon the week of the year or upon the rate of streamflow. The weekly dependence is circumvented by computing the coefficients for each week and the flow dependence is accounted for by regressing the coefficients on the stream flow. The transfer coefficients are intended for use to predict the effects of adding a waste load in reach i on the BOD and DO deficit on a downstream reach j. The predictions can be used to gain insight into the effect of a set of loads on the watershed before computer time is expended to obtain a better prediction by simulation methods.

The idea of transfer functions and coefficients was obtained from the systems engineering field. In simple terms, a system is a "black box" which performs some change on the input information as it passes through. This is illustrated in its simplest form by the diagram shown.



For instance, if the system squares the input, in which case when the input is 4, the output would be 16. If the system function is constant, i.e., if the input is always squared, the system is called a stationary system. If the system function is time-dependent, the system is called non-stationary.

In the systems application to a watershed, the system is the river, the input is the BOD of a waste load and the output is a downstream BOD value, which is smaller by reason of the fact that the river has operated upon it. Similarly, a waste discharge causes a DO deficit which is operated upon by the river system, which now includes the BOD load, and the output DO deficit is the result of the system operation. It is obvious that a river is a non-stationary system, and likely, it is quite non-linear as are most natural systems.

In the consideration here, the simulation treats the river as a constant for each week, that is, the temperature is constant for a week and the flow is averaged for a week, so that the river system has been made stationary for intervals of a week. So, if the system function is considered constant for a week, the real system, which is continuously changing with time, is approximated by a system which is stationary for weekly intervals.

Since $BOD_j < BOD_i$ and $Q_i < Q_j$, and all values are positive, the value of a_{ij} must be

$$0 < a_{ij} < 1.0$$

A similar relationship can be worked out for the DO deficit. It should be noted that a value of zero for the deficit at the upstream end of reach i leads to a division by zero and an indeterminant form. It is necessary to assume the DO deficit has a positive finite value, an assumption which is proper because unpolluted natural waters rarely are in excess of 95 percent oxygen saturated. In the simulation program to develop the a_{ij} values, if the value of $DEFIN_i$ is zero, it is automatically set equal to 0.5 mg/l. Similarly, if the value of Q_i is zero, as it will be at all upstream reaches, Q_i is arbitrarily set equal to 1.0 cfs.

The program places a unit load at the upper end of reach i and by simulation, using WASP, the output BOD and DO deficit are computed for each downstream reach. This is done on a "weekly" basis for a given number of years to provide adequate sampling for each "week" of the year. This allows a linear regression analysis to be made of the transfer function, a_{ij}, on the streamflow ratio, r_{ij}. The program computes the constant coefficients of the regression equation. This is repeated for each reach remembering that, for reaches numbered consecutively upstream, when $i < j$, the values a_{ij} and r_{ij} have no meaning.

The result allows the prediction of any a_{ij} for week l having a flow ratio of r_{ijl} by the equation:

$$a_{ijl} = B_{ijl}\, r_{ijl} + A_{ijl} \quad \text{. . . . . . . [Eq. 4.97]}$$

where a_{ijl} is the BOD transfer coefficient from the upper end of reach i to the lower end of reach j during week of the year, l; B_{ijl} is the slope and A_{ijl} is the ordinate intercept of the regression equation for the transfer function i, j for week l and r_{ijl} is the flow ratio Q_j/Q_i for week, l. Similarly:

$$d_{ijl} = C_{ijl}\, r_{ijl} + D_{ijl} \quad \text{. . . . . . . [Eq. 4.98]}$$

for the DO deficit transfer coefficient d_{ijl}, C_{ijl}, and D_{ijl} are the slope and ordinate intercept constants in the regression equation and r_{ijl} is again Q_j/Q_i for week, l. The subscripts i, j and l have the same meaning as above.

The method of computing the values of the regression coefficients, B_{ijl} and C_{ijl} and the intercept constants A_{ijl} and D_{ijl} may be found in standard statistics text books (10).

RDATA	2,080	ITRAN	707
DIVREL	1,904	EIGEN	8404
S	14,670	FCOEF	1968
TRAN	4,434	SORT	2020
TGEN	4,184	NFIND	536
AIJ	53,796	COMMON BLOCKS	118,864
FUNCTIONS	22,864		

Total program length, for the program as applied to the 43 reaches of the Farmington River Basin, which has 505 i-j combinations, is 345,272 bytes.

The only subroutines not already described in the preceeding subsections of this appendix are SORT, NFIND and AIJ. These subroutines are described below. It should be noted that the subroutines in program AIJ are written to use the reach numbering scheme described in 6.3.1 and will not work unless this scheme is used.

A4.6.2.1 Subroutine SORT

The subroutine SORT uses the reach data read into subroutine TRAN, and the computational sequence established internally by TRAN, and computes the possible i-j combinations. For each i, that is, for each upstream reach, SORT, finds the reach numbers, j, of all reaches downstream. Then the subroutine continues, setting up an internal indexing system that indexes the possible i-j combinations. The i-i combinations are included. This indicates the stretch of river from the upstream end of reach i to the downstream end of reach i. SORT transfers the index numbers generated to subroutine AIJ through COMMON/SORT/1.

A4.6.2.2 Subroutine NFIND

Subroutine NFIND is called by subroutine SORT to search for reaches downstream. SORT, operating on a "do loop" through all reaches, assigns a reach number, in the order of the internal computational sequence (index JR(I)) and calls that number K. Then subroutine NFIND is called and, in a similar "do loop" but this time operating on the downstream reach index, IDR(I), a search is made through all reaches for reaches downstream from K. When a downstream reach is found, this reach number is transmitted back to SORT for indexing and printing.

A4.6.2.3 Subroutine AIJ

Subroutine AIJ computes the regression coefficients, B_{ijl} and C_{ijl} and the intercept constants A_{ijl} and D_{ijl} which can be used with a selected value of r_{ijl} to compute the transfer functions.

Subroutine AIJ is called by subroutine SIM (in program WASP) in a weekly "do loop" nested, in turn, in a yearly "do loop" running over the number of years of simulation desired. So, for each week, the subroutine AIJ

There is another problem in the analogy of the river to a system. There is variability in the river throughout its length. It is for this reason the river has been divided into reaches. The problem is that, if each reach is a system, then there is a chain of systems and, unless these system functions are linear, that is, the effects or changes of each system are additive, then the overall system effect becomes very complicated. The problem is overcome by assuming each reach system operation is linear and therefore the effects are additive. The problem of non-linearity plagues all systems work and a common means for handling it is to make this assumption of linearity and proceed, realizing the result is an approximation. Fortunately the BOD and DO deficit equations are linear excepting in the vicinity of the point of maximum DO deficit and, to the extent that the mathematical models of Streeter-Phelps describe the non-linear oxygen relationships in the stream, the assumption of linearity holds.

In systems engineering, the system is designed to perform a certain operation upon an input to produce the desired output. The system function is usually described by a differential equation. In the application to natural systems, it is often impractical or impossible to develop a mathematical expression for the system function, so the procedure sometimes used is to input a known signal and measure the output signal to determine the system function. This latter method is used in this work. A unit BOD load is placed successively at each reach point and, by simulation, the BOD and DO deficit at each downstream reach point are determined. The overall transfer coefficient determined in this manner contains the effect of the flow rates at the two reach points. If the overall transfer coefficient is α_{ij}, then:

$$\alpha_{ij} = a_{ij}\, r_{ij} \quad \ldots\ldots\ldots\ldots \text{[Eq. 4.93]}$$

where α_{ij} relates the BOD concentration at the lower end of reach j to a unit BOD loading at the upper end of reach i, r_{ij} relates the flow at the lower end of reach j to the upper end of reach i and a_{ij} is defined as the ratio of α_{ij}/r_{ij}. From these definitions,

$$\alpha_{ij} = \frac{BOD_j}{BOD_i} \quad \ldots\ldots\ldots\ldots \text{[Eq. 4.94]}$$

$$r_{ij} = \frac{Q_j}{Q_i} \quad \ldots\ldots\ldots\ldots \text{[Eq. 4.95]}$$

and

$$a_{ij} = \frac{\alpha_{ij}}{r_{ij}} = \frac{BOD_j Q_i}{BOD_i Q_j} \ldots\ldots\ldots\ldots \text{[Eq. 4.96]}$$

Since $BOD_j < BOD_i$ and $Q_i < Q_j$, and all values are positive, the value of a_{ij} must be

$$0 < a_{ij} < 1.0$$

A similar relationship can be worked out for the DO deficit. It should be noted that a value of zero for the deficit at the upstream end of reach i leads to a division by zero and an indeterminant form. It is necessary to assume the DO deficit has a positive finite value, an assumption which is proper because unpolluted natural waters rarely are in excess of 95 percent oxygen saturated. In the simulation program to develop the a_{ij} values, if the value of $DEFIN_i$ is zero, it is automatically set equal to 0.5 mg/l. Similarly, if the value of Q_i is zero, as it will be at all upstream reaches, Q_i is arbitrarily set equal to 1.0 cfs.

The program places a unit load at the upper end of reach i and by simulation, using WASP, the output BOD and DO deficit are computed for each downstream reach. This is done on a "weekly" basis for a given number of years to provide adequate sampling for each "week" of the year. This allows a linear regression analysis to be made of the transfer function, a_{ij}, on the streamflow ratio, r_{ij}. The program computes the constant coefficients of the regression equation. This is repeated for each reach remembering that, for reaches numbered consecutively upstream, when $i < j$, the values a_{ij} and r_{ij} have no meaning.

The result allows the prediction of any a_{ij} for week l having a flow ratio of r_{ijl} by the equation:

$$a_{ijl} = B_{ijl}\, r_{ijl} + A_{ijl} \qquad \text{. . . . . . . [Eq. 4.97]}$$

where a_{ijl} is the BOD transfer coefficient from the upper end of reach i to the lower end of reach j during week of the year, l; B_{ijl} is the slope and A_{ijl} is the ordinate intercept of the regression equation for the transfer function i, j for week l and r_{ijl} is the flow ratio Q_j/Q_i for week, l. Similarly:

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for the DO deficit transfer coefficient d_{ijl}, C_{ijl}, and D_{ijl} are the slope and ordinate intercept constants in the regression equation and r_{ijl} is again Q_j/Q_i for week, l. The subscripts i, j and l have the same meaning as above.

The method of computing the values of the regression coefficients, B_{ijl} and C_{ijl} and the intercept constants A_{ijl} and D_{ijl} may be found in standard statistics text books (10).

The use of these transfer coefficients is illustrated as follows. Given the program output of $A_{ij1} = 0.15$ and $B_{ij1} = 0.10$ for upstream reach i, downstream reach j, and week 1. What is the BOD at reach j for a flow ratio of $Q_j/Q_i = 4.5$ if the BOD loading at reach i is 10mg/l?

$$a_{ij1} = 0.10 \times 4.5 + 0.15 = 0.60$$

BOD at reach j = 0.60 x 10 = 6 mg/l.

A4.6.2 Program Components

The AIJ program is, in reality, a subroutine attached to and called by the simulation program, WASP when it is desired to determine the A_{ij1}, B_{ij1}, C_{ij1} and D_{ij1} values for the watershed. WASP supplies all the information necessary to compute these constants. The subroutine requires considerable storage capacity and an operator needs to ascertain, before attempting to use it, that the computer to be used has the required capacity.

Because the AIJ program requires considerable storage, and, for a large number of reaches, the volume of output will be great, the program is set to compute the a_{ij} and d_{ij} equation coefficients for only the critical summer months, weeks 25 through 33 (July and August). In addition, a sorting subroutine has been worked out to determine only those i-j combinations that are possible for a given watershed configuration. To reduce the storage required, equivalence statements are used wherever possible.

The simulation programs FLASH and WASP are modified slightly for use with AIJ. The modifications consist mainly of removing the subroutines which output the data generated by FLASH and WASP and the subroutines which compute the statistics. The generated data are used internally and the statistics are not needed. Program AIJ should not be used until it is reasonably assured that the data and parameter inputs produce accurate simulation.

A list of subroutines of the program AIJ and their length, in bytes, are as follows:

QUAL	11,648	MEAN	1102
RQUAL	5,316	UPGAGE	1542
FLASH	68,902	IREACH	452
MAIN	1,202	QTRAN	692
SIM	1,208	TRES	578
REG	6,948	GEN	1424
RAN	460	COREL	4538
RRN	396	TRANS	1684
STD	560	WFLOW	664

RDATA	2,080	ITRAN	707
DIVREL	1,904	EIGEN	8404
S	14,670	FCOEF	1968
TRAN	4,434	SORT	2020
TGEN	4,184	NFIND	536
AIJ	53,796	COMMON BLOCKS	118,864
FUNCTIONS	22,864		

Total program length, for the program as applied to the 43 reaches of the Farmington River Basin, which has 505 i-j combinations, is 345,272 bytes.

The only subroutines not already described in the preceeding subsections of this appendix are SORT, NFIND and AIJ. These subroutines are described below. It should be noted that the subroutines in program AIJ are written to use the reach numbering scheme described in 6.3.1 and will not work unless this scheme is used.

A4.6.2.1 Subroutine SORT

The subroutine SORT uses the reach data read into subroutine TRAN, and the computational sequence established internally by TRAN, and computes the possible i-j combinations. For each i, that is, for each upstream reach, SORT, finds the reach numbers, j, of all reaches downstream. Then the subroutine continues, setting up an internal indexing system that indexes the possible i-j combinations. The i-i combinations are included. This indicates the stretch of river from the upstream end of reach i to the downstream end of reach i. SORT transfers the index numbers generated to subroutine AIJ through COMMON/SORT/1.

A4.6.2.2 Subroutine NFIND

Subroutine NFIND is called by subroutine SORT to search for reaches downstream. SORT, operating on a "do loop" through all reaches, assigns a reach number, in the order of the internal computational sequence (index JR(I)) and calls that number K. Then subroutine NFIND is called and, in a similar "do loop" but this time operating on the downstream reach index, IDR(I), a search is made through all reaches for reaches downstream from K. When a downstream reach is found, this reach number is transmitted back to SORT for indexing and printing.

A4.6.2.3 Subroutine AIJ

Subroutine AIJ computes the regression coefficients, B_{ij1} and C_{ij1} and the intercept constants A_{ij1} and D_{ij1} which can be used with a selected value of r_{ij1} to compute the transfer functions.

Subroutine AIJ is called by subroutine SIM (in program WASP) in a weekly "do loop" nested, in turn, in a yearly "do loop" running over the number of years of simulation desired. So, for each week, the subroutine AIJ makes the desired computations for the indexed i-j combinations and adds them to the summing variables. The information for making the computations is obtained from program WASP (and FLASH) which is simulating in the same time sequence. Subroutine AIJ selects only the values it needs (BOD and DO into reach i, BOD and DO out of reach j and the ratio of flow in reach j to the flow in reach i) for the months of July and August. A check is made for the appropriate weeks (L=25, ---33) to assure the proper values are obtained. The number of weeks and week numbers may be changed as desired merely by changing the numbers in the appropriate statements of the subroutine AIJ coding.

The program is run for the number of years desired; thirty are recommended, to obtain an adequate number of points with which to form the regression equation. After this selected number of years has been traversed, subroutine AIJ is again called and, in the last pass through SIM, the regression coefficients and constants are computed and printed.

A4.6.3 Program Input

In setting up the coding for a FLASH-WASP-AIJ run, the operator should establish DIMENSION values that reflect exactly the number of reaches, basis gages, reservoirs, loads, i-j combinations, etc., so that the machine storage requirement is minimized.

The program inputs are identical to those required for the simulation programs FLASH and WASP excepting that a single unit load is placed at the upstream end of reach i. In the interest of savings in machine storage, FLASH should be used to produce a tape of parameters for gage data generation rather than to use the additional storage required to develop the parameters from historical data during the AIJ run.

A4.6.4 Program Output

The output in program AIJ consists in the following:
(1) a set of generated flow data for each year,
(2) two arrays which are identical to the first and third arrays in the output of TFLOW, see A4.3.4,
(3) an array showing, for each reach, the reaches in the system that are downstream thereof,
(4) a listing of the i-j combinations in the system and their corresponding index, and
(5) an array which lists the regression coefficients and constants along with their identifying week.

A4.6.5 Definition of Variables

Following is a list of variables used in AIJ and a brief definition of each. Variables associated with FLASH and WASP are defined in A4.4.5 and A4.5.5, respectively.

A	ALFA/R
AA(IDX, LM)	Regression constant, A_{ij}, for index IDX, week LW.
AD(IDX, LM)	Regression constant, D_{ij}, for index IDX, week LW
ALFA	Ratio, BODOUT/BODIN, for current index
BA(IDX, LM)	Regression coefficient, B_{ij}, for index IDX, week LW
BD(IDX, LM)	Regression coefficient, C_{ij}, for index IDX, week LM.
D	DELTA/R
DELTA	Ratio DO deficit out/DO deficit in, current index
IDR(I)	Downstream reach index
IDX	Index number
INDEX(M)	M^{th} index number
JJ	Convenience variable
JR(I)	Computational sequence index
JX	Convenience variable
K	Convenience variable
KK	Convenience variable
KOUNT(I)	Convenience variable
L	Convenience variable
LW	Week number
M	Convenience variable
MSORT	Number of i-j combinations
N	Convenience variable
NG	Convenience variable
NN	Convenience variable
R(I,J)	Ratio of flows, reach j/reach i.
SA(I,J)	Summing variable, sums A for i-j combination
SALFA(I)	Summing variable, sums ALFA for i-j combination
SD(I,J)	Summing variable, sums D for i-j combination
SDELTA(I,J)	Summing variable, sums DELTA for i-j combination
SR(I,J)	Summing variable, sums R for i-j combination
SRSQ(I,J)	Summing variable, sums R^2 for i-j combination

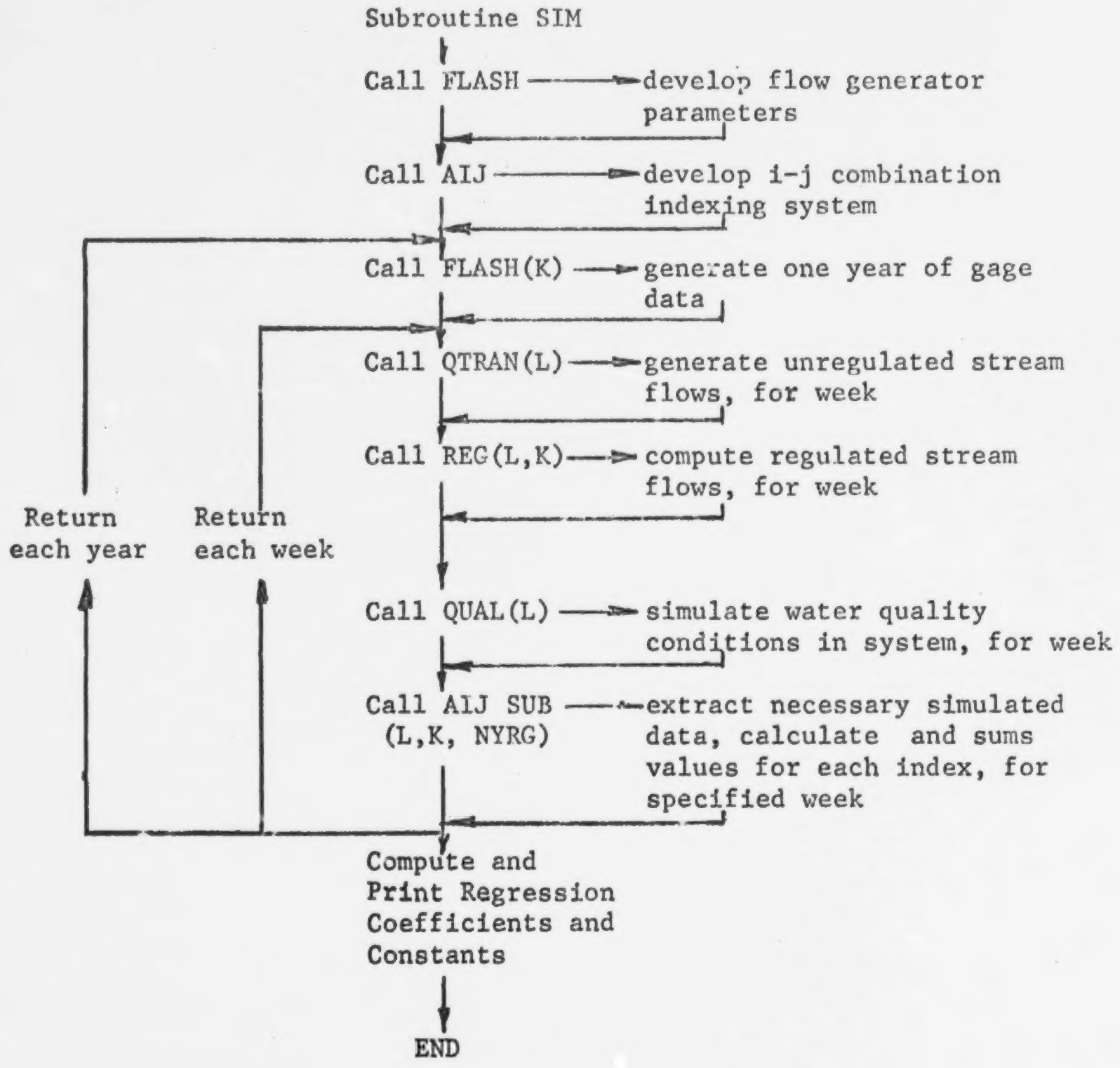
A4.6.6 Program Logic

Figure A4-10 is a diagram showing the program logic for AIJ.

A4.6.7 Program Coding

The program coding for AIJ follows.

FIGURE A4-10
PROGRAM LOGIC - AIJ


Subroutine SIM
Call FLASH
develop flow generator parameters
Call AIJ
develop i-j combination indexing system
Call FLASH(K)
generate one year of gage data
Call QTRAN(L)
generate unregulated stream flows, for week
Call REG(L,K)
compute regulated stream flows, for week
Return each year
Return each week
Call QUAL(L)
simulate water quality conditions in system, for week
Call AIJ SUB (L,K, NYRG)
extract necessary simulated data, calculate and sums values for each index, for specified week
Compute and Print Regression Coefficients and Constants
END

```
//WASP2    JOB (1143,47,040,11,5000),'ARROYO AMAURI A.      ',CLASS
                                                              =L
// EXEC FORTRAN
//SOURCE   DD *
      SUBROUTINE QUAL(L)
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                              ),
     1        DIV(10),RREL(10)
      COMMON/FLOW7/QQ(6,48),QNAT(43),NYRG
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(43),ISTART,XK,XM
                                                              ,
     1       CC,F,XK120W(43),BODWST(43,48),
     2        DOWST(43,48),R(48),RCON(43),SCON(43)
      COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                              (10,6),
     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                              10),
     3        BDEP(10),CDEP(10)
      COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                              43,48),
     1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
      DIMENSION XK120(43),CQREG(43)
C     COMPUTE TEMPERATURE AT CURRENT TIME FRAME
      IF(L.GT.36) GO TO 8
      IF(L.LT.22) GO TO 8
    8 TT=T(L)+R(L)*SIGMAT
      IF(TT.LT.0)TT=0
      DO 10 I=1,NR
C     IS THERE AN UPSTREAM REACH
C     ARE THERE ANY BRANCHES
      QSUM=0
      QSUM2=0
      SPRD1=0
      SPRD2=0
      SPRD3=0
      IF(NUR(JR(I),1).LE.0)GO TO 41
      J=1
      IF(NUR(JR(I),2).GT.0)J=2
      IF(NUR(JR(I),3).GT.0)J=3
      DO 12 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   12 QSUM=QSUM+QREG(LL,L)
      QNI=QREG(JR(I),L)
      IF(QSUM.GT.QNI)QSUM=QREG(JR(I),L)
      QN=(QREG(JR(I),L)-QSUM)/J
      QSUM=0
      DO 11 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
      CQREG(LL)=QREG(LL,L)+CN
      SPRD1=SPRD1+XK120(LL)*CQREG(LL)
      SPRD2=SPRD2+BODOUT(LL,L)*CQREG(LL)
      SPRD3=SPRD3+DEFOUT(LL,L)*CQREG(LL)
      IF(XK120(LL).EQ.0)GO TO 111
      QSUM=QSUM+CQREG(LL)
  111 QSUM2=QSUM2+CQREG(LL)
   11 CONTINUE
C     COMPUTE THE DEOXYGENATION VELOCITY CONSTANT OF THE MIXED
C       FLOW AT THE UPPER END OF EACH REACH
   41 HOLD=IDR(JR(I))
      IF(QSUM.EQ.0)QSUM=.005
      IF(QSUM2.EQ.0)QSUM2=.5
      XK120(JR(I))=(SPRD1+XK120W(JR(I))*QWASTE(JR(I),L))/(QSUM+
     1          QWASTE(JR(I),L))
C     CORRECT XK120 FOR TEMPERATURE
C     COMPUTE DISSOLVED OXYGEN SATURATION VALUE
      DOS=14.65-.41*TT+.008*TT**2-.00008*TT**3
   42 ARG=.046*(TT-20)
      XK1=XK120(JR(I))*EXP(ARG)
C     COMPUTE BOD + DEFICIT AT THE UPPER END OF THE CURRENT REACH
      IF(QREG(JR(I),L).EQ.0) GO TO 50
      BODIN(JR(I),L)=(SPRD2+BODWST(JR(I),L)*QWASTE(JR(I),L))/
     1 (QREG(JR(I),L)+QWASTE(JR(I),L))
      DEFIN(JR(I),L)=(SPRD3+(DOS-DOWST(JR(I),L))*QWASTE(JR(I),L))
                                                                 /

     1 (QREG(JR(I),L)+QWASTE(JR(I),L))
      GO TO 51
   50 BODIN(JR(I),L)=0.
      DEFIN(JR(I),L)=0.
C     IS THERE A RESERVOIR AT THIS REACH
   51 IF(IRES(JR(I)).GT.0)CALL RQUAL(L,TT,XK1,QSUM2,TIME,I,XK2,+1
                                                                00)

      IF(IRES(JR(I)).GT. 0.0) GO TO 100
C     COMPUTE VELOCITY OF FLOW
 2222 V=XK*(QREG(JR(I),L)+QWASTE(JR(I),L))**XM
      IF(V.EQ.0.0)V=XK*(QREG(HOLD,L)/2.)**XM
      IF(V.EQ.0)GO TO 43
C     COMPUTE TIME OF FLOW
      TIME=(RLNTH(JR(I))/V)/86400
C     COMPUTE DEPTH OF FLOW
      XH=CC*(QREG(JR(I),L)+QWASTE(JR(I),L))**F
      IF(XH.EQ.0.0)XH=CC*(QREG(HOLD,L)/2.)**F
C     COMPUTE REAERATION VELOCITY CONSTANT
      XK220=(1.3*V)/(XH**1.33)
C     CORRECT THE REAERATION VELOCITY CONSTANT FOR TEMPERATURE
      ARG=.024*(TT-20)
      XK2=XK220*EXP(ARG)
      GO TO 44
C     NO FLOW IN REACH
   43 TIME=7.6
      XK2=XK1
   44 IF(XK120(JR(I)).NE.0)GO TO 142
      BODOUT(JR(I),L)=0.0
      DEFOUT(JR(I),L)=0.0
      XK1=0
      GO TO 100
  142 ARG=-XK1*TIME
      BODOUT(JR(I),L)=(BODIN(JR(I),L)-RCON(JR(I))/XK1)*EXP(ARG)+R
                                                             CON(JR(

     1I))/XK1
      IF(XK1.EQ.XK2)GO TO 16
      ARG1=-XK1*TIME
      ARG2=-XK2*TIME
      DEF=((XK1*BODIN(JR(I),L))-RCON(JR(I)))/(XK2-XK1)
      DEF=DEF*(EXP(ARG1)-EXP(ARG2))
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK2)*(1.0-EXP(ARG2))
      DEFOUT(JR(I),L)=DEF+(DEFIN(JR(I),L)*EXP(ARG2))
      ARG=XK2/XK1-(XK2*DEFIN(JR(I),L)-RCON(JR(I))-SCON(JR(I)))*(X
                                                             K2-XK1)

     1/(XK1*(XK1*BODIN(JR(I),L)-RCON(JR(I))))
      IF(ARG.LE.0)GO TO 100
      TCRIT=(1./(XK2-XK1))*ALOG(ARG)
      IF(TCRIT.GT.TIME)GO TO 100
C     CRITICAL DEFICIT OCCURS IN THIS REACH
      ARG1=-XK1*TCRIT
      ARG2=-XK2*TCRIT
      DEF=((XK1*BODIN(JR(I),L))-RCON(JR(I)))/(XK2-XK1)
      DEF=DEF*(EXP(ARG1)-EXP(ARG2))
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK2)*(1.0-EXP(ARG2))
      DEFOUT(JR(I),L)=DEF+(DEFIN(JR(I),L)*EXP(ARG2))
      GO TO 100
   16 ARG=-XK1*TIME
      DEF=(XK1*TIME*BODIN(JR(I),L))-(TIME*RCON(JR(I)))
      DEF=DEF+DEFIN(JR(I),L)
      DEF=DEF+((RCON(JR(I))+SCON(JR(I)))/XK1)
      DEF=DEF*EXP(ARG)
      DEFOUT(JR(I),L)=DEF-(RCON(JR(I))+SCON(JR(I)))/XK1
      TCRIT=(1./XK1)-DEFIN(JR(I),L)/(XK1*BODIN(JR(I),L)-RCON(JR(I
                                                              )))

     1  +(RCON(JR(I))+SCON(JR(I)))/(XK1*(XK1*BODIN(JR(I),L)-RCON(
                                                            JR(I)))

     2)
      ARG1=-XK1*TCRIT
      IF(TCRIT.GT.TIME)GO TO 100
C     OTHERWISE RECOMPUTE DEFOUT
      DEFOUT(JR(I),L)=(XK1*(BODIN(JR(I),L)-RCON(JR(I))/XK1)*TCRIT
                                                             +

     1   DEFIN(JR(I),L)+(RCON(JR(I))+SCON(JR(I)))/XK1)*EXP(ARG1)
```

```
    2        -(RCON(JR(I))+SCON(JR(I))))/XK1
C      COMPUTE MINIMUM DO
  100 IF(DEFOUT(JR(I),L).LT.DEFIN(JR(I),L)) XMINDO=DOS-DEFIN(JR(I
                                                              ),L)
      IF(DEFOUT(JR(I),L).GE.DEFIN(JR(I),L)) XMINDO=DOS-DEFOUT(JR(
                                                              I),L)
   30 IF(L.LT.22) GO TO 150
      IF(L.GT.36) GO TO 150
      WRITE(6,600)TIME,CWASTE(JR(I),L),DOWST(JR(I),L),BODWST(JR(I
                                                              ),L),
     1        XK1,XK2,XMINDO,BODOUT(JR(I),L),DEFOUT(JR(I),L),JR(I)
  600 FORMAT(1H ,9F8.4,5X,I2)
  150 CONTINUE
   10 CONTINUE
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE RQUAL(L,T1,XK1,QSUM2,TIME,I,XK2,*)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                              ),
     1         DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW12/XK220R(10),XK3(10),BODSTO(10),DEFSTO(10)
      COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                              43,48),
     1BODOUT(43,48),XMINO(43,30),LLX(43,30),XKK(43,48)
      CALL TRES(IRES(JR(I)),IR)
      ARG=(.024*(TT-20.))
      XK2=XK220R(IR)*EXP(ARG)
      AVGSTO=(PSTO(IR)+STO(IR))/2.
      IF(AVGSTO.LE.0)GO TO 100
      IF(STO(IR).EQ.0.AND.RREL(IR).NE.0)GO TO 100
      Z=(QSUM2*3600.*24.)/(AVGSTO*10**6)
      W=((RREL(IR)+DIV(IR))*3600.*24.)/(AVGSTO*10**6)
      XFK3=(XK3(IR)*AVGSTO)/CAP(IR)
      A=XK1+W+XFK3
      B=XK2+W
      TIME=11.57*STO(IR)/QSUM2
      IF(TIME.GT.30.0)TIME=30.
      ARG=-A*TIME
      IF(BODSTO(IR).LT..001)BODSTO(IR)=0.0
      DEF9=BODSTO(IR)-Z*BODIN(JR(I),L)/A
      DEF9=DEF9*(1-EXP(ARG))/(A*TIME)
      BODOUT(JR(I),L)=DEF9+Z*BODIN(JR(I),L)/A
      ARG2=-B*TIME
      IF(DEFIN(JR(I),L).LT..001) GO TO 99
      DEF1=(DEFSTO(IR)/B)-(Z*DEFIN(JR(I),L)/(B*B))
      DEF1=DEF1+(XK1*BODSTO(IR)/(A*B))-(XK1*Z*BODIN(JR(I),L)*(A+B
                                                              )
     1/(A*B)**2)
      DEF2=(Z*DEFIN(JR(I),L)/B)-DEFSTO(IR)
      DEF2=DEF2+(XK1/(A-B))*((Z*BODIN(JR(I),L)/B)-BODSTO(IR))
      DEF2=DEF2*EXP(ARG2)/B
      DEF3=(Z*TIME/B)*(DEFIN(JR(I),L)+(XK1*BODIN(JR(I),L))/A)
      DEF4=(Z*BODIN(JR(I),L)/A)-BODSTO(IR)
      DEF4=DEF4*XK1*EXP(ARG)/(A*(B-A))
      DEFOUT(JR(I),L)=(DEF1+DEF2+DEF3+DEF4)/TIME
      GO TO 102
   98 DEFOUT(JR(I),L)=0.0
  102 IF(DEFSTO(IR).LT..001)GO TO 99
      DEFSTO(IR)=DEFSTO(IR)*EXP(ARG2)-Z*DEFIN(JR(I),L)*EXP(ARG2)/
                                                                 B+
     1XK1*BODSTO(IR)*EXP(ARG2)/(A-B)-XK1*Z*BODIN(JR(I),L)*EXP(ARG
                                                                 2)/(B*
     2(A-B))+XK1*BODSTO(IR)*EXP(ARG)/(B-A)-XK1*Z*BODIN(JR(I),L)*E
                                                                 XP(ARG)
     3/(A*(B-A))+Z*DEFIN(JR(I),L)/B+Z*XK1*BODIN(JR(I),L)/(A*B)
      GO TO 103
   99 DEFSTO(IR)=0.0
  103 BODSTO(IR)=BODSTO(IR)*EXP(ARG)+(Z*BODIN(JR(I),L)*(1.-EXP(AR
                                                                 G)))/A

      GO TO 101
  100 BODOUT(JR(I),L)=BODIN(JR(I),L)
      DEFOUT(JR(I),L)=DEFIN(JR(I),L)
      BODSTO(IR)=0
      DEFSTO(IR)=0
      Z=0
      W=0
      XFK3=0
      TIME=0
      A=0
      B=0
  101 RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE FLASH
C     FLORIDA SYNTHETIC HYDROLOGY MODEL(FLASH)
      COMMON/FLOW7/QG(6,48),QNAT(43),NYRG
      DIMENSION Q(21,4,12,6),QAV(4,12,6),AO(6),ASI(6,6),BCO(6,6),
                                                                 QEST
     1  (4,12,6),QVAR(12,6),S11(24,24),S12(24,24),S21(24,24),S22(
                                                                 24,24)
     2  , B(24,24,12),C(24,24,12),EVAL(24),QPR(24),QC(4,6,12)
      EQUIVALENCE (QG(1),QC(1)),(QAV(1),QEST(1)),(Q(1),B(1))
  200 READ(5,5000)NYR,NYRG,NSITES,NTRAN,IRAN,ISAVE,IPARAM,NPRINT,
                                                                 IHIST

 5000 FORMAT(9I5)
      CALL WFLOW(NYR,NSITES,Q,IHIST)
      N=4*NSITES
      CALL TRANS(NYR,NSITES,NTRAN,Q,QAV,AO,ASI,BCO,QEST,QVAR)
      DO 10 J=1,12
      CALL COREL(NYR,NSITES,Q,QVAR,S11,S12,S21,S22,J)
      CALL S(24,4,S22,S22,0,N,N,0,TAG)
      CALL S (24,3,B(1,1,J),S12,S22,N,N,N,0)
      CALL S(24,3,S22,B(1,1,J),S21,N,N,N,0)
      CALL S (24,2,C(1,1,J),S11,S22,N,N N,0)
      CALL EIGEN(24,C(1,1,J),EVAL,N,N)
      DO 10 I=1,N
      XL=0.
      IF(EVAL(I).LE.0.)GO TO 11
      XL=SQRT(EVAL(I))
   11 DO 10 II=1,N
   10 C(II,I,J)=C(II,I,J)*XL
      N=4*NSITES
      CALL RAN (IRAN,N,S11)
      CALL S(24,3,QPR,C(1,1,12),S11,N,N,1,0)
      RETURN
      ENTRY FLASH1(K)
      CALL GEN(IRAN,NSITES,QPR,B,C,GQ,NPRINT)
      CALL ITRAN(NSITES,NTRAN,GQ,QVAR,QEST)
      WRITE (6,6001) K,(((GQ(J,L,I),I=1,12),J=1,4),L=1,NSITES)
 6001 FORMAT (//1X,'GENERATED FLOWS FOR YEAR',I3/(1X,12G10.3))
  160 DO 101 MK=1,12
      DO 101 ML=1,4
      DO 101 MI=1,NSITES
      II=ML+4*(MK-1)
  101 QG(MI,II)=GQ(ML,MI,MK)
      RETURN 1
      DEBUG SUBCHK
      END
C WATER QUALITY SIMULATION PROGRAM (WASP)
      COMMON/FLOW1/NR,NC,NCR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW4/ATIME(43),BTIME
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STC(10
                                                                ),

     1       DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW6/NWASTE,IWASTE(43),JWASTE(43),AVW(43),AW(43),PM
                                                                W(43),

     1 FK1,WASTE,XFK1
      COMMON/FLOW7/QG(6,48),QNAT(43),NYRG
      COMMON/FLOW8/AVTEMP,ATEMP,TPEAK,TEMP,DTEMP
      COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                                (10,6),

     1       YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2       TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                                10),

     3       BDEP(10),CDEP(10)
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(43),ISTART,XK,XM
                                                                ,

     1       CC,F,XK120W(43),BODWST(43,48),
    2        DOWST(43,48),R(48),RCON(43),SCON(43)
      COMMON/FLOW12/XK22OR(10),XK3(10),BODSTO(10),DEFSTO(10)
      COMMON/SORT1/NDOWN(43,43),KOUNT(43),INDEX(505),MSORT
      COMMON/FLOW13/GREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                             43,48),

     1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
      DIMENSION IGT(10)
      READ(5,5001)NYRG,NGT
 5001 FORMAT(2I5)
      READ(5,5002)(IGT(I),I=1,NGT)
 5002 FORMAT(10I8)
      CALL TGEN
      CALL TRAN
      WRITE(6,6103)(NGAGE(I),I=1,NG)
      WRITE(6,6105)
      DO 31 I=1,NR
   31 WRITE(6,6104) NOR(I),(WT(I,J),J=1,NG)
 6103 FORMAT('1',9X,'EXTERNAL'/
     112X,'REACH',9X,'COEFFICIENT OF FLOW AT GAGE'/
     211X,'NUMBER',10I10)
 6105 FORMAT(1X)
 6104 FORMAT(I15,6X,10F10.3)
      WRITE(6,6000)
 6000 FORMAT(1H1)
      CALL RDATA
      CALL SIM
      DEBUG SUBCHK
      END
      SUBROUTINE SIM
C   $$$$$$$$$$$$$$$$$$$$$$$$
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAU(43,3),
     1        TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW4/ATIME(43),BTIME
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                              ),

     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW6/NWASTE,IWASTE(43),JWASTE(43),AVW(43),AW(43),PM
                                                             W(43),

     1 FK1,WASTE,XFK1
      COMMON/FLOW7/CG(6,48),CNAT(43),NYRG
      COMMON/FLOW8/AVTEMP,ATEMP,TPEAK,TEMP,DTEMP
      COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                             (10,6),

     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                             10),

     3        BDEP(10),CDEP(10)
      COMMON/FLOW11/T(48),SIGMAT,Z,C,TMEAN,RLNTH(43),ISTART,XK,XM
                                                              ,
 1       CC,F,XK120W(43),BODWST(43,48),
 2        DOWST(43,48),R(48),RCON(43),SCON(43)
  COMMON/FLOW12/XK220R(10),XK3(10),BODSTO(10),DEFSTO(10)
  COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                        43,48),
 1BODOUT(43,48),XMINC(43,30),LLX(43,30),XKK(43,48)
  COMMON/SORT1/NDOWN(43,43),KOUNT(43),INDEX(505),MSORT
  DO 8 I=1,6
  BODSTO(I)=0.0
  DEFSTO(I)=0.0
  DO 8 J=1,48
8 QG(I,J)=0.0
  DO 9 I=1,NR
  DO 9 J=1,48
  DEFIN(I,J)=0.0
  DEFOUT(I,J)=0.0
  BODIN(I,J)=0.0
  BODOUT(I,J)=0.0
  BODWST(I,J)=0
  DOWST(I,J)=0
9 QWASTE(I,J)=0
  DO 7 I=1,43
  DO 7 J=1,30
  XMINC(I,J)=0.0
7 LLX(I,J)=0
  REWIND 9
  READ(5,700)Z,C,TMEAN,ISTART,XK,XM,CC,F,SIGMAT
700 FORMAT(3F8.0,I5,5F8.0)
  READ(5,701)(RLNTH(I),RCON(I),SCON(I),XK120W(I),IWASTE(I),I=
                                                         1,NR)
701 FORMAT(4F10.0,I1)
  DO 1 I=1,NR
1 IF(IWASTE(I).NE.0)READ(5,703)(QWASTE(I,J),BODWST(I,J),DOWST
                                                        (I,J),
 1J=1,48)
703 FORMAT(5X,3F10.0)
  READ(5,702)(XK220R(I),XK3(I),BODSTO(I),DEFSTO(I),I=1,NRES)
702 FORMAT(4F10.0)
C    COMPLTE MEAN TEMPERATURE FOR LTH WEEK OF THE YEAR
  DO 800 L=1,48
  XL=L
  ARG=(7.5*XL+C)/57.3
  T(L)=Z*SIN(ARG)+TMEAN
800 CONTINUE
  CALL FLASH
  CALL AIJ
  DO 11 K=1,NYRG
  CALL RAN(ISTART,48,R)
  CALL FLASH1(K)
  DO 10 L=1,48
  CALL QTRAN(L)
```

```
      CALL REG(L,K)
      CALL QUAL(L)
      CALL AIJSUB(L,K,NYRG)
   10 CONTINUE
   11 CONTINUE
      REWIND 9
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE REG(L,KYR)
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                                ),
     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW7/QQ(6,48),QNAT(43),NYRG
      COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                           43,48),
     1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
      COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                           (10,6),
     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                              10),
     3        BDEP(10),CDEP(10)
      DIMENSION DQ(43),AREA(10),DEP(10)
      XL=L
      XLRAD=(XL*7.5+BVAP)/57.3
      EVAP=AVAP*SIN(XLRAD)+CVAP
      DO 100 I=1,NR
      QNAT(JR(I))=QNAT(JR(I))-XIRRIG(JR(I),L)
      IF(QNAT(JR(I)).LT.0)QNAT(JR(I))=0
C  THE INDEX OF THE CURRENT REACH IS JR(I)
C  IS THERE A REACH UPSTREAM
      IF(NUR(JR(I),1).GT.0) GO TO 10
C  NO-IS THERE A RESERVOIR AT THIS REACH
      IF(IRES(JR(I)).GT.0) GO TO 20
C  NO
      DQ(JR(I))=0.
      QREG(JR(I),L)=QNAT(JR(I))
      GO TO 499
C  THERE IS A REACH UPSTREAM
C  IS THIS A RESERVOIR
   10 IF(IRES(JR(I)).GT.0) GO TO 30
C  NO-COUNT UPSTREAM REACHES
      J=1
      IF(NUR(JR(I),2).GT.0) J=2
      IF(NUR(JR(I),3).GT.0) J=3
      DQ(JR(I))=0.
      DO 11 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   11 DQ(JR(I))=DQ(JR(I))+DQ(LL)
      QREG(JR(I),L)=              QNAT(JR(I))+DQ(JR(I))
      GO TO 499
C  THIS IS RESERVOIR , NO UPSTREAM REACH
   20 QIN=QNAT(JR(I))
      GO TO 40
C  THIS IS RESERVOIR, REACH UPSTREAM
C  COUNT REACHES
   30 J=1
      IF(NUR(JR(I),2).GT.0) J=2
      IF(NUR(JR(I),3).GT.0) J=3
      QIN=QNAT(JR(I))
      DO 31 K=1,J
      NU=NUR(JR(I),K)
      LL=IREACH(NU)
   31 QIN=QIN+DQ(LL)
      QREG(JR(I),L)=QIN
C  OPERATE RESERVOIR
   40 CALL TRES(IRES(JR(I)),IR)
      PSTO(IR)=STO(IR)
      DEP(IR)=(ACAP(IR)+(BCAP(IR)*STO(IR)+CCAP(IR))**.5)/DCAP(IR)
      AREA(IR)=ADEP(IR)+BDEP(IR)*DEP(IR)+CDEP(IR)*DEP(IR)**2
      QVAP=AREA(IR)*EVAP
      QINV=QIN*.657-QVAP
      CALL DIVREL(IR,L,RRELV,DEP)
      STO(IR)=STO(IR)+(QINV-DIV(IR))
      IF(STO(IR).GT.CAP(IR))GO TO 300
      IF(ITYPE(IR).GT.3)GO TO 310
      IF(STO(IR).GT.TOPLEV(IR).AND.ITYPE(IR).EQ.1)GO TO 302
      IF(STO(IR).GT.SMIN(IR))GO TO 303
C     NO RELEASES-NO DIVERSIONS
  301 RRELV=0
      STO(IR)=STO(IR)+DIV(IR)
      IF(STO(IR).LE.0)GO TO 307
      GO TO 310
C     RESERVOIR EMPTY
  307 STO(IR)=0
      GO TO 310
  300 IF(ITYPE(IR).EQ.1)GO TO 400
      RRELV=STO(IR)-CAP(IR)
      GO TO 310
  400 RRV=20*RRELV
      RTEMP=STO(IR)-CAP(IR)
      IF(RTEMP.GT.RRV)GO TO 401
      RRELV=RRV
      GO TO 310
  401 RRELV=RTEMP
      GO TO 310
  303 IF(ITYPE(IR).NE.3)GO TO 403
```

```
C      IS THE TIME BETWEEN OCT. 1 + NOV.1
       IF(L.GE.37 .AND. L.LE.40)GO TO 305
C      NO-IS THE TIME BETWEEN APR. 1 + OCT. 1
       IF(L.LT.13 .OR.L.GT. 36)GO TO 308
C      YES-TIME IS BETWEEN APR. 1 + OCT.1
       IF(STO(IR).LE.CAP(IR))RRELV=0
       GO TO 310
C      TIME IS BETWEEN OCTOBER 1 + NOV. 1
  305 SDIF=STO(IR)-SMIN(IR)
       RRELV=SDIF/(41-L)
       GO TO 310
  302 RRV=20*RRELV
       RTEMP=STO(IR)-TOPLEV(IR)
       IF(RTEMP.LE.RRV)GO TO 402
       RRELV=RRV
       GO TO 310
  402 IF(RTEMP.LT.RRELV)GO TO 310
       RRELV=RTEMP
       GO TO 310
  403 IF(ITYPE(IR).EQ.1)GO TO 309
       RRV=STO(IR)-SMIN(IR)
       IF(RRELV.LE.RRV)GO TO 310
       RRELV=RRV
       GO TO 310
  309 STO(IR)=STO(IR)+QINV-RRELV
       IF(STO(IR).GT.TOPLEV(IR))RRELV=STO(IR)-TOPLEV(IR)
       GO TO 312
  308 RRELV=STO(IR)-SMIN(IR)
  310 STO(IR)=STO(IR)-RRELV
  312 RREL(IR)=RRELV/.657
       DQ(JR(I))=RREL(IR)-QNAT(JR(I))
  499 CONTINUE
   99 IF(QREG(JR(I),L).LT.0.0)QREG(JR(I),L)=0.0
  100 CONTINUE
       RETURN
       DEBUG SUBCHK
       END
       SUBROUTINE RAN(IX,N,R)
       DIMENSION R(1)
       DO 10 I=1,N
       R(I)=0.
       DO 20 J=1,12
       R(I)=R(I)+RRN(IX)
   20 CONTINUE
   10 R(I)=R(I)-6.
       RETURN
       END
       FUNCTION RRN(IX)
       IX=IX*65539
       IF(IX)5,6,6
    5 IX=IX+2147483647+1
  6 RRN=IX
    RRN=RRN*.4656613E-9
    RETURN
    END
    FUNCTION STD(T)
    DIMENSION T(48)
    TBAR=0
    DO 1 I=1,48
  1 TBAR=TBAR+T(I)
    TBAR=TBAR/48.
    TSUM=0
    DO 2 I=1,48
  2 TSUM=TSUM+(T(I)-TBAR)**2
    STD=SQRT(TSUM/47.)
    RETURN
    DEBUG SUBCHK
    END
    SUBROUTINE RDATA
    COMMON/FLOW1/NR,NG,NCR(43),NUR(43,3),DAU(43,3),
   1      TDA(43),NGAGE( 6),NGR( 6),DAG( 6),ILR(43)
    COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                            ),
   1       DIV(10),RREL(10),PSTO(10)
    COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                            (10,6),
   1       YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
   2       TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                            10),
   3       BDEP(10),CDEP(10)
    DO 30 I=1,NR
    DO 30 J=1,48
 30 XIRRIG(I,J)=0
    READ(5,500)NRES,(IRES(I),I=1,NR)
500 FORMAT(I5/(16I5))
    READ(5,501)(JRES(J),ITYPE(J),CAP(J),STO(J),SMIN(J),TOPLEV(J
                                                            ),
   1    J=1,NRES)
501 FORMAT(2I5,4F10.2)
    READ(5,504)(ACON(I),BCON(I),I=1,NRES)
504 FORMAT(8F10.4)
    READ(5,505)(YMEAN(L),ACAP(L),BCAP(L),CCAP(L),DCAP(L),
   1    ADEP(L),BDEP(L),CDEP(L),L=1,NRES)
505 FORMAT(8F10.2)
    READ(5,503)AVAP,BVAP,CVAP,((TAU(I,J),CCON(I,J),J=1,6),I=1,N
                                                            RES)
503 FORMAT(3F10.0,/(6F10.0))
    READ(5,502)NADJST,(L,NSTA,XIRRIG(NSTA,L),
   1    I=1,NADJST)
502 FORMAT(I5/(I5,I8, F14.4))
    WRITE(6,600)
600 FORMAT(' DATA HAVE BEEN READ IN THRU RDATA')
```

```
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE DIVREL(IR,L,RRELV,DEP )
C   ***************************
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                                 ),
     1        DIV(10),RREL(10),PSTO(10)
      COMMON/FLOW9/XIRRIG(43,48),QWASTE(43,48),AVAP,BVAP,CVAP,TAU
                                                               (10,6),
     1        YMEAN(10),ACON(10),BCON(10),CCON(10,6),ITYPE(10),
     2        TOPLEV(10),ACAP(10),BCAP(10),CCAP(10),DCAP(10),ADEP(
                                                                10),
     3        BDEP(10),CDEP(10)
      DIMENSION DEP(10)
      ITYP=ITYPE(IR)
      XL=L
      GO TO (1,2,3,4,5),ITYP
    1 RRELV=ACON(IR)*0.657
      THETA=(XL*2.*3.1416)/48.
      DIV(IR)=YMEAN(IR)
      DO 10 I=1,6
      ARG=THETA-TAU(IR,I)
   10 DIV(IR)=DIV(IR)+CCON(IR,I)*COS(ARG)
      GO TO 101
    2 BCONN=BCON(IR)
      RRELV=0.657*ACON(IR)*DEP(IR)**BCONN
      DIV(IR)=0
      GO TO 101
    3 RRELV=0
      DIV(IR)=0
      GO TO 101
    4 RRELV=0
      THETA=(XL*2.*3.1416)/48.
      DIV(IR)=YMEAN(IR)
      DO 40 I=1,6
      ARG=THETA-TAU(IR,I)
   40 DIV(IR)=DIV(IR)+CCON(IR,I)*COS(ARG)
      GO TO 101
    5 THETA=(XL*2.*3.1416)/48.
      RRELV=YMEAN(IR)
      DO 50 I=1,6
      ARG=THETA-TAU(IR,I)
   50 RRELV=RRELV    +CCON(IR,I)*COS(ARG)
      DIV(IR)=0
  101 RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE S(KT,NN,A,B,C,IM,JM,KM,DET)
      DIMENSION A(KT,KT),B(KT,KT),C(KT,KT),IN(100),EMP(100)
      IMAX=IM
      JMAX=JM
      KMAX=KM
      GOTO(30,32,34,36,38,40,42,44,46,50,52),NN
   30 DO 31 I=1,IMAX
      DO31J=1,JMAX
   31 A(I,J)=B(I,J)+C(I,J)
      GO TO 805
   32 DO33I=1,IMAX
      DO33J=1,JMAX
   33 A(I,J)=B(I,J)-C(I,J)
      GO TO 805
   34 DO101I=1,IMAX
      DO35J=1,KMAX
       EMP(J)=0.
      DO35K=1,JMAX
   35  EMP(J)= EMP(J)+B(I,K)*C(K,J)
      DO101K=1,KMAX
  101 A(I,K)= EMP(K)
      GO TO 805
   36 DO37I=1,IMAX
      DO37J=1,IMAX
   37 A(I,J)=B(I,J)
   59 IN(1)=0
      IMAXO=IMAX-1
      TEMP=A(1,1)
      DO70I=2,IMAX
      IF(ABS (TEMP)-ABS (A(I,1)))71,70,70
   71 IN(1)=I
      TEMP=A(I,1)
   70 CONTINUE
      IF(IN(1))73,72,73
   73 IS=IN(1)
      DO74J=1,IMAX
      TEMP=A(1,J)
      A(1,J)=A(IS,J)
   74 A(IS,J)=TEMP
   72 IF(A(1,1))98,99,98
   98 DO75I=2,IMAX
   75 A(I,1)=A(I,1)/A(1,1)
      DO100I=2,IMAX
      IPO=I+1
      IMO=I-1
      DO80L=1,IMO
   80 A(I,I)=A(I,I)-(A(L,I)*A(I,L))
      TEMP=A(I,I)
      IF(I-IMAX)55,83,55
   55 IN(I)=0
      DO81IS=IPO,IMAX
      DO85L=1,IMO
   85 A(IS,I)=A(IS,I)-A(L,I)*A(IS,L)
      IF(ABS (TEMP)-ABS (A(IS,I)))82,81,81
```

```
 82 TEMP=A(IS,I)
    IN(I)=IS
 81 CONTINUE
    ISS=IN(I)
    IF(ISS)84,83,84
 84 DO886J=1,IMAX
    TEMP=A(I,J)
    A(I,J)=A(ISS,J)
886 A(ISS,J)=TEMP
 83 IF(A(I,I))97,99,97
 97 IF(I-IMAX)54,100,54
 54 DO86IS=IPO,IMAX
 86 A(IS,I)=A(IS,I)/A(I,I)
    DO89JS=IPO,IMAX
    DO89L=1,IMO
 89 A(I,JS)=A(I,JS)-(A(L,JS)*A(I,L))
100 CONTINUE
    DO600JP=1,IMAX
    J=IMAX+1-JP
    A(J,J)=1.0/A(J,J)
    IF(J-1)603,700,603
603 DO600IP=2,J
    I=J+1-IP
    IPO=I+1
    TEMP=0.0
    DO602L=IPO,J
602 TEMP=TEMP-A(I,L)*A(L,J)
600 A(I,J)=TEMP/A(I,I)
700 DO151J=1,IMAXO
    JPO=J+1
    DO151I=JPO,IMAX
    TEMP=0.0
    IMO=I-1
    DO154L=J,IMO
    IF(L-J)152,153,152
152 TEMP=TEMP-A(I,L)*A(L,J)
    GO TO154
153 TEMP=TEMP-A(I,L)
154 CONTINUE
151 A(I,J)=TEMP
    DO901I=1,IMAX
    DO900J=1,IMAX
     EMP(J)=0.0
    DO899N=I,IMAX
    IF(N-J)899,897,898
898  EMP(J)= EMP(J)+A(I,N)*A(N,J)
    GO TO899
897  EMP(J)= EMP(J)+A(I,N)
899 CONTINUE
900 CONTINUE
    DO901J=1,IMAX
```

```
901 A(I,J)= TEMP(J)
    DO500I=2,IMAX
    M=IMAX+1-I
    IF(IN(M))502,500,502
502 ISS=IN(M)
    DO503L=1,IMAX
    TEMP=A(L,ISS)
    A(L,ISS)=A(L,M)
503 A(L,M)=TEMP
500 CONTINUE
    DET=0.
    GO TO 805
120 DET=1.
 99 WRITE                 (6,806 )
806 FORMAT(18H0  SINGULAR MATRIX)
805 RETURN
 38 DO39I=1,IMAX
    DO39J=1,IMAX
 39 A(I,J)=B(I,J)
    N=IMAX
    DET=1.
    I1=1
  1 I3=I1
    SUM=ABS (A(I1,I1))
    DO 3 I=I1,N
    IF(SUM-ABS (A(I,I1)))2,3,3
  2 I3=I
    SUM=ABS (A(I,I1))
  3 CONTINUE
    IF(I3-I1)4,6,4
  4 DO5J=1,N
    SUM=-A(I1,J)
    A(I1,J)=A(I3,J)
  5 A(I3,J)=SUM
  6 I3=I1+1
    DO7I=I3,N
  7 A(I,I1)=A(I,I1)/A(I1,I1)
    J2=I1-1
    IF(J2)8,11,8
  8 DO9J=I3,N
    DO9I=1,J2
  9 A(I1,J)=A(I1,J)-A(I1,I)*A(I,J)
 11 J2=I1
    I1=I1+1
    DO12I=I1,N
    DO12J=1,J2
 12 A(I,I1)=A(I,I1)-A(I,J)*A(J,I1)
    IF(I1-N)1,14,1
 14 I3=1
    J2=N/2
    IF(2*J2-N)15,16,15
 15 I3=0
    DET=A(N,N)
 16 DO17I=1,J2
    J=N-I+I3
 17 DET=DET*A(I,I)*A(J,J)
    GO TO 805
 40 IF(IMAX-JMAX)41,102,102
 41 IP=IMAX
    GO TO 103
102 IP=JMAX
103 DO106K=1,IP
    DO104I=K,IMAX
104  EMP(I)=B(I,K)
    DO105J=K,JMAX
105 A(J,K)=B(K,J)
    DO106I=K,IMAX
106 A(K,I)= EMP(I)
    GO TO 805
 42 DO43I=1,IMAX
    DO43J=1,JMAX
 43 A(I,J)=B(I,J)
    GO TO 805
 44 DO45I=1,IMAX
    DO45J=1,JMAX
    A(I,J)=0.
    B(I,J)=0.
 45 C(I,J)=0.
    GO TO 805
 46 ID=2
 20  READ (KMAX,47) IN(1),IN(5),EMP(1),IN(2),IN(6),EMP(2),
   1  IN(3),IN(7),EMP(3),IN(4),IN(8),EMP(4)
 47 FORMAT (4(I3,I3,E12.8))
    IF(IN(1))805,805,23
 23 GO TO(19,24),ID
 24 IM=IN(1)
    JM=IN(5)
    ID=1
 19 DO21I=1,4
    I1=IN(I)
    J1=IN(I+4)
    IF(I1)21,21,18
 18 A(I1,J1)= EMP(I)
 21 CONTINUE
    GO TO 20
 50 DO 62 IP=1,JMAX,7
    JPO=IP+6
    IF(JPO-JMAX)61,61,60
 60 JPO=JMAX
 61 WRITE              (KMAX,63)(J,J=IP,JPO)
    DO 62 I=1,IMAX
    WRITE              (KMAX,64)I,(A(I,J),J=IP,JPO)
   62 CONTINUE
      GO TO 805
   63 FORMAT(5HO ROW7(8X,4HCOL.I3,1X))
   64 FORMAT(I4,4X,7E16.8)
   52 DO53I=1,IMAX
      DO53J=1,JMAX
   53 A(I,J)=B(I,J)*DET
      GO TO 805
      DEBUG SUBCHK
      END
      SUBROUTINE TRAN
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAL(43,3),
     1        TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      DIMENSION JGU(10),NIU(50),NID(50)
C   ************************
C   COMPUTE FLOW IN LAST REACH DOWNSTREAM
C   ************************
      I = JR(NR)
C   IS THERE A GAGE IN THIS REACH
      J = 0
      DO 5 JJ=1,NG
      IF (NGR(JJ) - NOR(I)) 5,6,5
    6 J = JJ
    5 CONTINUE
      IF (J) 10,10,15
C   THERE IS A GAGE - CASE 1
   15 DO 16 JJ=1,NG
   16 WT(I,JJ) = 0.
      WT(I,J) = TDA(I)/DAG(J)
      GO TO 100
C   THERE IS NO GAGE - CASE 2
   10 CALL UPGAGE (I,NGU,JGU)
      IF (NGU) 21,21,22
   21 WRITE (6,6000)
 6000 FORMAT (10X,'NO GAGES')
      STOP
   22 DO 23 JJ=1,NG
   23 WT(I,JJ) = 0.
      GDA = 0.
      DO 25 JJ=1,NGU
   25 GDA = GDA+DAG(JGU(JJ))
      DO 24 JJ=1,NGU
      J = JGU(JJ)
   24 WT(I,J) = TDA(I)/GDA
C   ********************************
C   CONTINUE UPSTREAM
C   ********************************
  100 IU = I
      I3 = 1
      NID(IB) = IU
C  IS THERE A REACH UPSTREAM
  105 NU = NUR(IU,1)
      IF (NU) 110,110,115
C  THERE IS NO REACH UPSTREAM
  110 IB = IB - 1
C  HAVE ALL REACHES BEEN COMPLETED
      IF (IB) 200,200,120
C  TRANSFORM IS COMPLETE
  200 RETURN
C  CONTINUE CALCULATIONS
  120 IU = NIU(IB)
      ID = NID(IB)
      GO TO 150
C  THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
  115 IF (NUR(IU,2)) 125,125,130
C  THERE IS NO BRANCH
  125 NU = NUR(IU,1)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS A BRANCH - ARE THERE TWO
  130 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      IF (NUR(IU,3)) 135,135,140
C  THERE IS ONLY ONE BRANCH
  135 NU = NUR(IU,2)
      ID = IU
      IU = IREACH(NU)
      GO TO 150
C  THERE IS ANOTHER BRANCH
  140 NU = NUR(IU,2)
      IB = IB + 1
      NIU(IB-1) = IREACH(NU)
      NID(IB-1) = IU
      NU = NUR(IU,3)
      ID = IU
      IU = IREACH(NU)
C  IS THERE A GAGE IN THIS REACH
  150 J = 0
      DO 151 JJ=1,NG
      IF (NGR(JJ)-NCR(IU)) 151,152,151
  152 J = JJ
  151 CONTINUE
      IF (J) 160,160,165
C  THERE IS A GAGE IN THIS REACH
  165 DO 166 JJ=1,NG
  166 WT(IU,JJ) = 0.
      WT(IU,J) = TDA(IU)/DAG(J)
      GO TO 105
C  THERE IS NO GAGE IN THIS REACH
  160 CALL UPGAGE (IU,NGU,JGU)
      IF (NGU) 180,180,161
  161 GDA = 0.
      DO 170 JJ=1,NGU
      J = JGU(JJ)
  170 GDA = GDA + DAG(J)
      DO 171 JJ=1,NG
  171 WT(IU,JJ) = 0.
      A1 = (TDA(ID) - TDA(IU))/(TDA(ID) - GDA)
      DO 172 JJ=1,NGU
      J = JGU(JJ)
  172 WT(IU,J) = A1*TDA(IU)/GDA
      A1 = (TDA(IU) - GDA)/(TDA(ID) - GDA)/TDA(ID)
      DO 173 J=1,NG
  173 WT(IU,J) = WT(IU,J) + A1*WT(ID,J)*TDA(IU)
      GO TO 105
C  THERE IS NO GAGE UPSTREAM - CASE 2
  180 A1 = TDA(IU)/TDA(ID)
      DO 181 J=1,NG
  181 WT(IU,J) = A1*WT(ID,J)
      GO TO 105
      DEBUG SUBCHK
      END
      SUBROUTINE TGEN
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      DIMENSION IR(43)
      DIMENSION NNOR(50)
      READ (5,5001) NR,NG
 5001 FORMAT (2I5)
      DO 1 I=1,NR
      NNOR(I)=0
    1 IDR(I) = 0
      DO 5 I=1,NR
    5 READ (5,5002) NOR(I),(NUR(I,J),J=1,3),
     1   (DAU(I,J),J=1,3),FL(I),SLOPE(I),ROUGH(I)
 5002 FORMAT (4I5,6F5.0)
      DO 6 I=1,NG
    6 READ (5,5003) NGAGE(I),NGR(I),DAG(I)
 5003 FORMAT(I8,I5,F5.0)
C  DETERMINE SEQUENCE OF REACH NUMBERS
      DO 15 I=1,NR
   15 IR(I) = 0
      DO 20 N=1,NR
      I = 1
   23 IF (IR(I)) 21,21,22
   22 I = I + 1
```

```
      IF(I.GT.NR) GO TO 20
      GO TO 23
   21 K = 0
      DO 25 J=1,3
      IF (NUR(I,J)) 25,25,26
   26 NUP = NUR(I,J)
      L = IREACH(NUP)
      IF (IR(L)) 27,27,25
   27 K = 1
   25 CONTINUE
      IF (K) 30,30,22
   30 JR(N) = I
      IR(I) = 1
      DO 36 K=1,3
      IF (NUR(I,K)) 36,36,37
   37 ND = NUR(I,K)
      ID = IREACH(ND)
      IDR(ID) = I
   36 CONTINUE
   20 CONTINUE
      DO 45 I=1,NR
      TDA(I) = 0.
      DO 45J=1,3
   45 TDA(I) = TDA(I) + DAL(I,J)
C     *****************
      WRITE(6,6000)
 6000 FORMAT('1EXTERNAL',49X,'DOWNSTREAM REACH',6X,'TOTAL',15X,
     1'REACH COMPUTATION SEQUENCE'/
     13X,'REACH',4X,'UPSTREAM REACHES',3X,'UPSTREAM DRAINAGE AREA
                                                                   S',
     1        3X,'INTERNAL  EXTERNAL',3X,'UPSTREAM',3X,'INTERNAL',
                                                                   7X,
     1'INTERNAL  EXTERNAL'/2X,'NUMBER',7X,'1',4X,'2',4X,'3'8X,'1'
                                                                  ,7X,
     1'2',7X,'3',6X,'INDEX',7X,'NUMBER',5X,'AREA',7X,'INDEX',8X,'
                                                                INDEX',
     17X,'NUMBER'/)
       DO 800 I=1,NR
       IF(IDR(I).NE.0)NNOR(IDR(I))=NOR(IDR(I))
  800 CONTINUE
      DO 46 I=1,NR
   46 WRITE(6,6001)NOR(I),(NUR(I,J),J=1,3),(DAL(I,J),J=1,3),IDR(I
                                                                   ),
     1NNOR(IDR(I)),TDA(I),I,JR(I),NOR(JR(I))
 6001 FORMAT(I6,I10,I5,I5,F11.1,F8.1,F8.1,I8,I11,F12.1,I10,6X,I8,
                                                                 I11)
      WRITE(6,6002)
 6002 FORMAT(///43X,'BASIC',7X,'REACH',7X,'AREA'/
     143X,'GAGE',5X,'CONTAINING',3X,'UPSTREAM',3X,'INTERNAL'/
     142X,'NUMBER',7X,'GAGE',7X,'OF GAGE',5X,'INDEX'/)
       DO 47 I=1,NG
   47 WRITE(6,6003) NGAGE(I),NGR(I),DAG(I),I
 6003 FORMAT(38X,I8,I12,F13.1,I10)
C     **************
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE MEAN(X,N,AV,SD,S3,S4)
      DIMENSION X(21)
      AV=0.
      SD=0.
      S3=0.
      S4=0.
      DO 10 I=1,N
      AV=AV+X(I)
      SD=SD+X(I)**2
      S3=S3+X(I)**3
   10 S4=S4+X(I)**4
      AV=AV/N
      SD=SD/N
      S3=S3/N
      S4=S4/N
      S4=S4-4*S3*AV+6*SD*AV**2-3*AV**4
      S3 = S3 - 3*SD*AV + 2*AV**3
      SD=SQRT(((SD-AV**2)*N)/(N-1))
      S3=S3/SD**3
      S4=S4/SD**4
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE UPGAGE (I,NGU,JGU)
      COMMON/FLOW1/NR,NG,NDR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      DIMENSION JGU(10),NIU(43)
C     DETERMINE THE NUMBER AND IDENTITY OF GAGES UPSTREAM
C     OF THIS REACH. DISCOUNT FURTHER SEARCH WHEN A GAGE
C     IS ENCOUNTERED. I IS CURRENT REACH, NGU IS NUMBER OF
C     GAGES UPSTREAM AND JGU ARE INDICIES OF THESE GAGES
      NGU = 0
      IB = 1
      IU = I
C     IS THERE A REACH UPSTREAM
    5 NU = NUR(IU,1)
      IF (NU) 10,10,15
C     THERE IS NO REACH UPSTREAM
   10 IB = IB - 1
C     IS THE SEARCH COMPLETE
      IF (IB) 100,100,20
C     SEARCH COMPLETE
  100 RETURN
```

```
C  CONTINUE SEARCH
   20 IU = NIU(IB)
      GO TO 24
C  THERE IS AN UPSTREAM REACH - IS THERE A BRANCH
   15 IF (NUR(IU,2)) 25,25,30
C  THERE IS NO BRANCH
   25 IU = IREACH(NU)
C  IS THERE A GAGE
   24 J = 0
      DO 26 JJ=1,NG
      IF (NGR(JJ) - NOR(IU)) 26,27,26
   27 J = JJ
   26 CONTINUE
      IF (J) 35,35,40
C  THERE IS NO GAGE IN THIS REACH
   35 GO TO 5
C  THERE IS A GAGE IN THIS REACH
   40 NGU = NGU + 1
      JGU(NGU) = J
C  CONTINUE SEARCHING
      GO TO 10
C  THERE IS A BRANCH
   30 IB = IB + 1
      NU = NUR(IU,1)
      NIU(IB - 1) = IREACH(NU)
C  IS THERE A SECOND BRANCH
      IF (NUR(IU,3)) 45,45,50
C  THERE IS NO SECOND BRANCH
   45 NU = NUR(IU,2)
      GO TO 25
C  THERE IS A SECOND BRANCH
   50 IB = IB + 1
      NU = NUR(IU,2)
      NIU(IB-1) = IREACH(NU)
      NU = NUR(IU,3)
      GO TO 25
      DEBUG SUBCHK
      END
      FUNCTION IREACH (NU)
      COMMON/FLOW1/NR,NG,NOR(43),NUR(43,3),DAU(43,3),
     1       TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW2/FL(43),ROUGH(43),SLOPE(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      II = 1
    3 IF (NOR(II) - NU) 1,2,1
    1 II = II + 1
      IF (II.EQ.NR) GO TO 2
      GO TO 3
    2 IREACH = II
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE QTRAN(L)
      COMMON/FLOW1/NR,NG,NCR(43),NUR(43,3),DAU(43,3),
     1        TDA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
      COMMON/FLOW3/JR(43),WT(43,6)
      COMMON/FLOW7/QG(6,48),QNAT(43),NYRG
      COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                            43,48),
     1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
      CALL S (43,3,QNAT,WT,QG(1,L),NR,NG,1,0)
      DO 10 I=1,16
      J=I+1
      IF(QNAT(I).LT.QNAT(J))QNAT(I)=QNAT(J)
   10 CONTINUE
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE TRES(N,I)
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$
C     ****************************
      COMMON/FLOW5/NRES,IRES(43),JRES(10),CAP(10),SMIN(10),STO(10
                                                             ),
     1        DIV(10),RREL(10),PSTO(10)
      J=0
    5 J=J+1
      IF(JRES(J).EQ.N) GO TO 10
      IF(J.LT.NRES) GO TO 5
      WRITE(6,6000)N,I,NRES,JRES
 6000 FORMAT(' RESERVOIR CANNOT BE LOCATED'/(20I5))
       CALL EXIT
   10 I=J
       RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE GEN(IRAN,NSITES,QPR,B,C,GQ,NPRINT)
      DIMENSION QPR(24),GQ(24,12),B(24,24,12),C(24,24,12),R(24)
      N=4*NSITES
      XN=N
      CALLS(24,10,QPR,0,0,N,1,6,0)
      DO 10 J=1,12
      CALL RAN(IRAN,N,R)
      DO 21 K=1,N
      GQ(K,J)=0.
      DO 20 M=1,N
   20 GQ(K,J)=GQ(K,J)+B(K,M,J)*QPR(M)+C(K,M,J)*R(M)
   21 GQ(K,J)=GQ(K,J)/XN
      DO 10 K=1,N
   10 QPR(K)=GQ(K,J)
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE COREL(NYR,NSITES,Q,QVAR,S11,S12,S21,S22,J)
      DIMENSION Q(21,4,12,6),S11(4,6,4,6),S12(4,6,4,6),S21(4,6,4,
                                                              6),
     1  S22(4,6,4,6),T11(4),T22(4),T12(7),QVAR(12,6),NM(7)
     1          ,SL12(7),SLL12(7),SUML1(4),SUMLL1(4),SUML2(4),SUML
                                                             L2(4)
      DATA NM/1,2,3,4,3,2,1/
      N=4*NSITES
      JJ=J-1
      IF(J.EQ.1)JJ=12
      DO 10 I=1,NSITES
      DO 10 II=1,NSITES
      DO 5 M=1,7
      SL12(M)=0
      SLL12(M)=0
    5 T12(M)=0.
      DO 6 M=1,4
      T11(M)=0.
      SUML1(M)=0
      SUMLL1(M)=0
      SUML2(M)=0
      SUMLL2(M)=0
    6 T22(M)=0.
      DO 20 L=1,4
      DO 20 LL=1,L
      M=L-LL+1
      DO 20 K=1,NYR
      T11(M)=T11(M)+Q(K,L,J,I)*Q(K,LL,J,II)
      SUML1(M)=SUML1(M)+Q(K,L,J,I)**2
      SUMLL1(M)=SUMLL1(M)+Q(K,LL,J,II)**2
      KK=K
      IF(JJ.LT.12)GO TO 21
      KK=K-1
      IF(KK.LT.1)KK=NYR
   21 T22(M)=T22(M)+Q(KK,L,JJ,I)*Q(KK,LL,JJ,II)
      SUML2(M)=SUML2(M)+Q(KK,L,JJ,I)**2
   20 SUMLL2(M)=SUMLL2(M)+Q(KK,LL,JJ,II)**2
      DO 25 L=1,4
      DO 25 LL=1,4
      M=L-LL+4
      DO 25 K=1,NYR
      KK=K
      IF(JJ.LT.12)GO TO 24
      KK=K-1
      IF(KK.LT.1)KK=NYR
   24 T12(M)=T12(M)+Q(K,L,J,I)*Q(KK,LL,JJ,II)
      SL12(M)=SL12(M)+Q(K,L,J,I)**2
   25 SLL12(M)=SLL12(M)+Q(KK,LL,JJ,II)**2
      DO 30 M=1,4
      T11(M)=T11(M)/SQRT(SUML1(M)*SUMLL1(M))
   30 T22(M)=T22(M)/SQRT(SUML2(M)*SUMLL2(M))
```

```
      DO 35 M=1,7
   35 T12(M)=T12(M)/SQRT(SL12(M)*SLL12(M))
      DO 40 L=1,4
      DO 40 LL=1,L
      M=L-LL+1
      S11(L,I,LL,II)=T11(M)
      S11(LL,I,L,II)=T11(M)
      S22(L,I,LL,II)=T22(M)
   40 S22(LL,I,L,II)=T22(M)
      DO 10 L=1,4
      DO 10 LL=1,4
      M=L-LL+4
   10 S12(L,I,LL,II)=T12(M)
      CALL S(24,6,S21,S12,0,N,N,0,0)
      RETURN
      DEBUG SUBCHK
      END
      SUBROUTINE TRANS(NYR,NSITES,NTRAN,Q,QAV,AO,ASI,BCO,QEST,QVA
                                                                R)
      DIMENSION Q(21,4,12,6),QAV(4,12,6),AO(6),ASI(6,6),BCO(6,6),
                                                                QEST
     1  (4,12,6),QVAR(12,6)
      DO 10 I=1,NSITES
      DO 5 J=1,12
      DO 5 L=1,4
      CALL MEAN(Q(1,L,J,I),NYR,AV,SD,S3,S4)
      IF(NTRAN-2)4,20,30
   20 DO 25 K=1,NYR
   25 Q(K,L,J,I)=ALOG(Q(K,L,J,I))
      GO TO 4
   30 DO 35 K=1,NYR
   35 Q(K,L,J,I)=SQRT(Q(K,L,J,I))
    4 CALL MEAN(Q(1,L,J,I),NYR, QAV(L,J,I),S5,S6,S7)
    5 CONTINUE
      CALL FCOEF(QAV(1,1,I),48,6,AO(I),ASI(1,I),BCO(1,I),QEST(1,1
                                                                ,I))
      DO 40 J=1,12
      DEV=0.
      DO 45 L=1,4
      DO 45 K=1,NYR
      X=Q(K,L,J,I)-QEST(L,J,I)
      DEV=DEV+X**2
   45 Q(K,L,J,I)=X
      FN=4*NYR
      QVAR(J,I)=SQRT(DEV/FN)
      DO 40 L=1,4
      DO 40 K=1,NYR
   40 Q(K,L,J,I)=Q(K,L,J,I)/QVAR(J,I)
   10 CONTINUE
      RETURN
      END
      SUBROUTINE WFLOW (NYR,NSITES,Q,IHIST)
      DIMENSION Q(21,4,12,6)
      IF(IHIST)4,4,14
    4 IO=12
      GO TO 15
   14 IO=5
   15 DO 20 I=1,NSITES
      DO 20 K=1,NYR
      DO 20 J=1,12
   20 READ(IO,5000)(Q(K,L,J,I),L=1,4)
 5000 FORMAT (14X,4F8.2)
      RETURN
      END
      SUBROUTINE ITRAN(NSITES,NTRAN,GQ,QVAR,QEST)
      DIMENSION GQ(4,6,12),QVAR(12,6),QEST(4,12,6)
      DO 10 I=1,NSITES
      DO 10 J=1,12
      DO 10 L=1,4
      GQ(L,I,J)=GQ(L,I,J)*QVAR(J,I)+QEST(L,J,I)
      IF(NTRAN-2)10,20,30
   20 GQ(L,I,J)=EXP(GQ(L,I,J))
      GO TO 10
   30 GQ(L,I,J)=GQ(L,I,J)**2
   10 CONTINUE
      RETURN
      END
        SUBROUTINE EIGEN(IDIM,A,EVAL,N,M)
C
C      EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC MATRIX
C
       DIMENSION A(IDIM,IDIM),B(24,24),EVAL(IDIM), S(24),C(24),
     1  C(24),IND(24),U(24)
       DOUBLE PRECISION ANORM,ANORM2,TAU,P,DIAG(24),VALU(24),VALL(
                                                             24),

     1  T1,T2,T,SUPERD(24),Q(24),DSI,DCO,BETA
C
C      CALCULATE NORM OF MATRIX
C
        MAXIT= 50*N*N
        IT=0
     3 ANORM2 = 0.
     4 DO 6 I=1,N
     5 DO 6 J=1,N
     6 ANORM2 = ANORM2 + A(I,J)**2
     7 ANORM =DSQRT (ANORM2)
C
C      GENERATE IDENTITY MATRIX
C
     9 IF (M) 10,45,10
    10 DO 40 I=1,N
    12 DO 40 J=1,N
   20 IF (I-J) 35,25,35
   25 B(I,J) = 1.
   30 GO TO 40
   35 B(I,J) = 0.
   40 CONTINUE
C
C     PERFORM ROTATIONS TO REDUCE MATRIX TO JACOBI FORM
C
   45 IEXIT = 1
   50 NN = N-2
   52 IF (NN) 890,170,55
   55 DO 160 I=1,NN
   60 II = I + 2
   65 DO 160 J=II,N
   70 T1 = A(I,I+1)
   75 T2 = A(I,J)
        IF(T2) 1910,160,1910
 1910   T=DSQRT(T1**2 +T2**2)
        CO=T1/T
         SI=T2/T
   90 DO 105 K=1,N
   95 T2 = CO *A(K,I+1)+SI *A(K,J)
  100 A(K,J) = CO *A(K,J)-SI *A(K,I+1)
  105 A(K,I+1) = T2
  110 DO 125 K=1,N
  115 T2 = CO *A(I+1,K) + SI *A(J,K)
  120 A(J,K) = CO *A(J,K) - SI *A(I+1,K)
  125 A(I+1,K) = T2
  128 IF (M) 130,160,130
  130 DO 150 K=1,N
  135 T2 = CO *B(K,I+1) + SI *B(K,J)
  140 B(K,J) = CO *B(K,J) - SI *B(K,I+1)
  150 B(K,I+1) = T2
  160 CONTINUE
C
C     MOVE JACOBI ELEMENTS AND INITIALIZE EIGENVALUE BOUNDS
C
  170 DO 200 I=1,N
  180 DIAG(I) = A(I,I)
  190 VALU(I) = ANORM
  200 VALL(I) = -ANORM
  210 DO 230 I=2,N
  220 SUPERD(I-1) = A(I-1,I)
  230 Q(I-1) = (SUPERD(I-1))**2
C
C     DETERMINE SIGNS OF PRINCIPAL MINORS
C
  235 TAU = 0.
  240 I = 1
  260 MATCH = 0
        IT=IT+1
  270 T2 = 0.
  275 T1 = 1.
  277 DO 450 J=1,N
  280 P = DIAG(J) - TAU
  290 IF (T2) 300,330,300
  300 IF (T1) 310,370,310
  310 T = P*T1 - Q(J-1)*T2
  320 GO TO 410
  330 IF (T1) 335,350,350
  335 T1 = -1.
  340 T = -P
  345 GO TO 410
  350 T1 = 1.
  355 T = P
  360 GO TO 410
  370 IF (Q(J-1)) 380,350,380
  380 IF (T2) 400,390,390
  390 T = -1.
  395 GO TO 410
  400 T = 1.
C
C     COUNT AGREEMENTS IN SIGN
C
  410 IF (T1) 425,420,420
  420 IF (T) 440,430,430
  425 IF (T) 430,440,440
  430 MATCH = MATCH + 1
  440 T2 = T1
  450 T1 = T
C
C      ESTABLISH TIGHTER BOUNDS ON EIGENVALUES
C
  460 DO 530 K=1,N
  465 IF (K - MATCH) 470,470,520
  470 IF (TAU - VALL(K)) 530,530,480
  480 VALL(K) = TAU
  490 GO TO 530
  520 IF (TAU - VALU(K)) 525,530,530
  525 VALU(K) = TAU
  530 CONTINUE
  540 IF (VALU(I) - VALL(I) - 5.0D-8) 570,570,550
  550 IF (VALU(I)) 560,580,560
  560 IF(DABS (VALL(I)/VALU(I) - 1.) - 5.0D-8) 570,570,580
  570 I = I + 1
        IT=0
  575 IF (I - N) 540,540,590
  580 TAU = (VALL(I) + VALU(I))/2.
        IF(IT-MAXIT) 260,260,581
  581  WRITE(6,6001)IT,I,VALL(I),VALU(I)
 6001  FORMAT(15H MAXIT EXCEEDED,2I10,2E20.8)
       GO TO 570
```

```
C
C     JACOBI EIGENVECTORS BY ROTATIONAL TRIANGULARIZATION
C
  590 IF (M) 593,890,593
  593 IEXIT = 2
  595 DO 610 I=1,N
  600 DO 610 J=1,N
  610 A(I,J) = 0.
  615 DO 850 I=1,N
  620 IF (I-1) 625,625,621
  621 IF (VALU(I-1) - VALU(I) - 5.0D-7) 730,730,622
  622 IF (VALU(I-1)) 623,625,623
  623 IF(DABS (VALU(I)/VALU(I-1) - 1.) - 5.0D-7) 730,730,625
  625 DCO=1.
  628 DSI=0.
  630 DO 700 J=1,N
  635 IF (J-1) 680,680,640
  640 T=DSQRT(T1**2+T2**2)
      DCO=T1/T
      DSI=T2/T
  650 S(J-1) =DSI
  660 C(J-1) =DCO
  670  D(J-1)= T1*DCO         +T2*DSI
  680 T1 = (DIAG(J) - VALU(I))*DCO - BETA*DSI
  690 T2 = SUPERD(J)
  700 BETA = SUPERD(J)*DCO
  710 D(N) = T1
  720 DO 725 J=1,N
  725 IND(J) = 0
  730 SMALLD = ANORM
  735 DO 780 J=1,N
  740 IF (IND(J) - 1) 750,780,780
  750 IF (ABS (SMALLD) - ABS (D(J))) 780,780,760
  760 SMALLD = D(J)
  770 NN = J
  780 CONTINUE
  790 IND(NN) = 1
  800 PRODS = 1.
  805 IF (NN-1) 810,850,810
  810 DO 840 K=2,NN
  820 II = NN + 1 - K
  830 A(II+1,I) = C(II)*PRODS
  840 PRODS = - PRODS*S(II)
  850 A(1,I) = PRODS
C
C     FORM MATRIX PRODUCT OF ROTATION MATRIX WITH JACOBI VECTOR M
                                                          ATRIX
C
  855 DO 885 J=1,N
  860 DO 865 K=1,N
  865 U(K) = A(K,J)
```

```
870 DO 885 I=1,N
875 A(I,J) = 0.
880 DO 885 K=1,N
885 A(I,J) = B(I,K)*U(K) + A(I,J)
890 GO TO 941
941      DO 945 I=1,N
945  EVAL(I)=  VALU(I)
       RETURN
      END
    SUBROUTINE FOCEF(X,NSP,NFREQ,AO,AS,BC,XEST)
    DIMENSION X(48),XEST(48),AS(6),BC(6)
    T=NSP
    W=2.*3.1416/T
    AO = 0.
    DO 5 I=1,NSP
  5 AO = AO + X(I)
    AO = AO/FLOAT(NSP)
    DO 10 M=1,NFREQ
    AS(M) = 0.
    BC(M) = 0.
    TA=0.
    TB=0.
    DO 15 I=1,NSP
    WT = W*FLOAT(I*M)
    STA=SIN(WT)
    STB=COS(WT)
    AS(M)=AS(M)+X(I)*STA
 15 BC(M)=BC(M)+X(I)*STB
    AS(M)=2.*AS(M)/T
 10 BC(M)=2.*BC(M)/T
    X2 = 0.
    DO 20 I=1,NSP
    X2 = X2 + X(I)**2
    XEST(I) = AO
    DO 20 M=1,NFREQ
    WT = W*FLOAT(I*M)
 20 XEST(I) = XEST(I) + AS(M)*SIN(WT)+BC(M)*COS(WT)
    RETURN
    DEBUG SUBCHK
    END
    SUBROUTINE SORT
    COMMON/FLOW1/NR,NC,NCR(43),NUR(43,3),DAU(43,3),
   1       TDA(43),NGADE( 6),NGR( 6),DAG( 6),IDR(43)
    COMMON/FLOW3/JR(43),WT(43,6)
    COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                           43,48),
   1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
    COMMON/SORT1/NDOWN(43,43),KOUNT(43),INDEX(505),MSORT
    MX=50
    WRITE(6,3)
  3 FORMAT('1',///T40,'AIJ RENUMBERING SYSTEM')
```

```
      WRITE(6,4)
    4 FORMAT('0',/T5,'JR(I)',3X,'DOWNSTREAM ELEMENT OF JR(I)',/T1
                                                              3,'01',
     12X,'02',2X,'03',2X,'04',2X,'05',2X,'06',2X,'07',2X,'08',2X,
                                                              '09',2X
     2,'10',2X,'11',2X,'12',2X,'13',2X,'14',2X,'15',2X,'16',2X,'1
                                                              7',2X,
     3'18',2X,'19',2X,'20',2X,'21',2X,'22',2X,'23',2X,'24',2X,'25
                                                              ')
      M=1
      N=1
      DO 6 I=1,NR
      DO 5 J=1,NR
    5 NDOWN(I,J)=0
    6 KOUNT(I)=0
      DO 10 I=1,NR
      K=JR(I)
      L=1
      NDOWN(K,L)=K
    7 CALL NFIND(K,NR,MX,IDR,JX,N$)
      IF(N$.EQ.0) GO TO 8
      L=L+1
      NDOWN(JR(I),L)=K
      IF(NDOWN(JR(I),L).EQ.NDOWN(JR(I),L-1)) L=L-1
      J$=IDR(JX-1)
      K=J$
      IF(K.EQ.0) GO TO 9
      GO TO 7
    8 K=K-1
      IF(K.EQ.0) GO TO 9
      GO TO 7
    9 KOUNT(JR(I))=L
   10 CONTINUE
      DO 15 I=1,NR
      J=JR(I)
   11 INDEX(M)=J*100+NDOWN(J,N)
      IF(N.GT.KOUNT(J)) GO TO 14
   12 N=N+1
   13 M=M+1
      GO TO 11
   14 N=1
   15 CONTINUE
      DO 20 I=1,NR
      JJ=JR(I)
      KK=KOUNT(JJ)
      WRITE(6,16)JJ,(NDOWN(JJ,LL),LL=1,KK)
   16 FORMAT(1H0,T6,I2,T13,25(I2,2X))
   20 CONTINUE
      WRITE(6,25)
   25 FORMAT('1',///T5,'INDEX',3X,'COMBINATION',T30,'INDEX',3X,'C
                                                              OMBINAT
```

```
  1ION',T60,'INDEX',3X,'COMBINATION',T90,'INDEX',3X,'COMBINATI
                                                            ON')
   DO 30 I=1,M
   II=I+126
   III=II+126
   IIII=III+126
   IF(IIII.GE.M) GO TO 40
   WRITE(6,28)I,INDEX(I),II,INDEX(II),III,INDEX(III),IIII,INDE
                                                           X(IIII)
28 FORMAT(' ',T5,I3,5X,I5,T30,I3,5X,I5,T60,I3,5X,I5,T90,I3,5X,
                                                           I5)
30 CONTINUE
40 RETURN
   END
   SUBROUTINE NFIND(K,NR,MX,IDR,JX,NC)
   INTEGER IDR(MX)
   DO 10 I=1,NR .
   N=IDR(I)
   IF(N.EQ.K) GO TO 5
   GO TO 10
 5 NC=1
   JX=I
   RETURN
10 CONTINUE
   NC=0
   JX=1
   RETURN
   END
   SUBROUTINE AIJ
   COMMON/FLOW1/NR,NG,NCR(43),NUR(43,3),DAU(43,3),
  1       TCA(43),NGAGE( 6),NGR( 6),DAG( 6),IDR(43)
   COMMON/FLOW3/JR(43),WT(43,6)
   COMMON/FLOW13/QREG(43,48),DEFIN(43,48),DEFOUT(43,48),BODIN(
                                                           43,48),
  1BODOUT(43,48),XMINQ(43,30),LLX(43,30),XKK(43,48)
   COMMON/SORT1/NDOWN(43,43),KOUNT(43),INDEX(505),MSORT
   DIMENSION SALFA(505,8),SDELTA(505,8),SR(505,8),SA(505,8),SD
                                                           (505,8)
  1,SRSQ(505,8),BA(505,8),BD(505,8),AA(505,8),AD(505,8)
   EQUIVALENCE (BA(1),SALFA(1),LLX(1)),(BD(1),SDELTA(1),XMINQ(
                                                           1)),
  1 (AA(1),SA(1)),(SD(1),AD(1))
   EQUIVALENCE(XKK(1),SRSQ(1))
   INTEGER CC
   CALL SORT
   WRITE(6,41)
   DO 1 I=1,MSORT
   DO 1 J=1,8
   SA(I,J)=0.
   SD(I,J)=0.
   SALFA(I,J)=0.
```

```
      SDELTA(I,J)=0.
      SR(I,J)=0.
    1 SRSQ(I,J)=0.
      RETURN
      ENTRY AIJSUB(L,NYR,CC)
      IF(L.LT.25)RETURN 1
      IF(L.GT.33)RETURN 1
      LW=L-24
      K=1
      I=1
      DO 10 IDX=1,MSORT
      II=JR(I)
      NN=NDOWN(II,K)
      IF(BODIN(II,L ).EQ.0.)BODIN(II,L )=1.0
      ALFA=BODOUT(NN,L )/BODIN(II,L )
      IF(DEFIN(II,L ).EQ.0.)BODIN(II,L )=0.5
      DELTA=DEFOUT(NN,L )/DEFIN(II,L )
      IF(QREG(II,L ).EQ.0)QREG(II,L )=1.0
      R=QREG(NN,L )/QREG(II,L )
      A=ALFA/R
      D=DELTA/R
      SALFA(IDX,LW)=SALFA(IDX,LW)+ALFA
      SDELTA(IDX,LW)=SDELTA(IDX,LW)+DELTA
      SR(IDX,LW)=SR(IDX,LW)+R
      SA(IDX,LW)=SA(IDX,LW)+A
      SD(IDX,LW)=SD(IDX,LW)+D
      SRSQ(IDX,LW)=SRSQ(IDX,LW)+R*R
      K=K+1
      IF(K.GT.KOUNT(II))GO TO 5
      GO TO 10
    5 K=1
      IF(I.LE.NR)I=I+1
   10 CONTINUE
   41 FORMAT(1H1,T40,'DESIRED OUTPUT FROM AIJ')
      IF(NYR.LT.CC) RETURN 1
      WRITE(6,42)
   42 FORMAT(T2,'YEAR',T7,'WEEK',T31,'BA',T51,'BD',T71,'AA',T91,'
                                                                AD',T16
     1,'STRETCH',/T14,'UPSTR.',2X,'DWNSTR.')
      K=1
      I=1
      DO 20 IDX=1,MSORT
      II=JR(I)
      NN=NDOWN(II,K)
      BA(IDX,LW)=(SALFA(IDX,LW)-SA(IDX,LW)*SR(IDX,LW)/NYR)/SRSQ(I
                                                               DX,LW)
      BD(IDX,LW)=(SDELTA(IDX,LW)-SD(IDX,LW)*SR(IDX,LW)/NYR)/SRSQ(
                                                              IDX,LW)
      AA(IDX,LW)=(SA(IDX,LW)/NYR)-BA(IDX,LW)*SR(IDX,LW)/NYR
      AD(IDX,LW)=(SD(IDX,LW)/NYR)-BD(IDX,LW)*SR(IDX,LW)/NYR
      WRITE(6,74)NYR,LW,II,NN,BA(IDX,LW),BD(IDX,LW),AA(IDX,LW),AD
```

```
                                                                  (IDX,LW
   1)
74 FORMAT(1H ,I4,I6,2I7,6F15.7)
   K=K+1
   IF(K.GT.KOUNT(II))GO TO 15
   GO TO 20
15 K=1
   IF(I.LE.NR)I=I+1
20 CONTINUE
   RETURN 1
   DEBUG SUBCHK
   END
```

A4.7 INTERF -- Interface Program

A4.7.1 Purpose

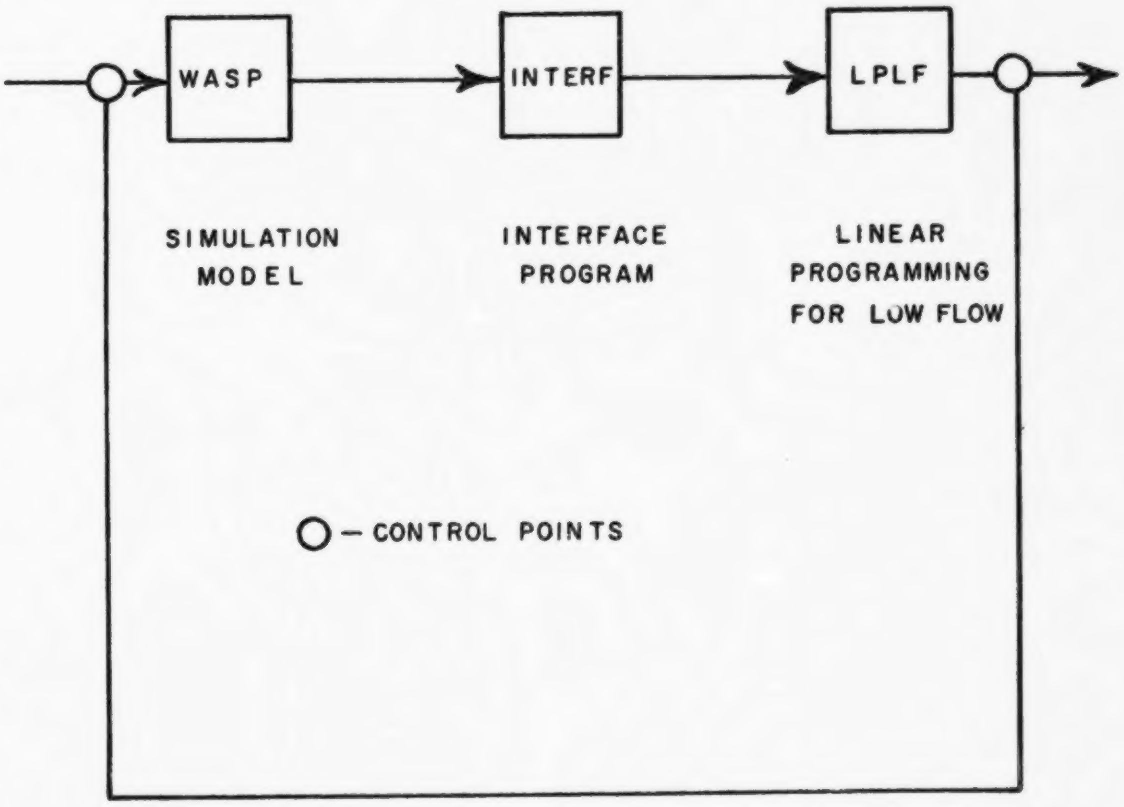
The interface program, INTERF, was written primarily for the important linkage between the two main models of this project. In this study the simulation model, programmed in Fortran IV, and the optimization model, programmed for IBM's Mathematical Programming System/360, are visualized as a closed loop information feedback system. This specific type of control system could not be possible without INTERF. As seen in Figure A4-11, the input to the interface program is the output from the simulation model. After receiving this input in the form of stream and wastewater data, the program will perform three major routines before generating input to the linear programming model. These routines are discussed in the Program Components section.

The overall objective of INTERF is, therefore, to accept specified output data from the simulation model and to generate the linear programming model in the format specified for the MPS/360 Processor. By computing and storing all of the elements in the linear programming tableau, INTERF will save the user considerable time developing the linear programming model in a multi-commodity network format. The program contains the flexibility of handling a region with up to 50 reaches and generating a new matrix in the MPS format for any changes in output data from the simulation model, e.g., when flow is augmented in the simulation model.

A4.7.2 Program Components

As mentioned previously, the interface program is composed of three major components, or routines. The first routine calculates all the parameters and right hand side elements (RHS) for the linear programming model. The parameters were derived from the biochemical oxygen demand (BOD) and dissolved oxygen (DO) deficit equations developed by Camp (10) and by Dobbins (11). These parameters are attenuation and amplification coefficients which describe the change in BOD and DO between the beginning and end of a reach. BOD and DO concentration in each reach, as well as the allowable amount of DO in the water, are converted to mass units in order to maintain a mass balance relationship in the network. The parameters and RHS with concentration units were also changed to mass units. The parameter values are calculated using the following equations:

FIGURE A4-11
OVERVIEW OF CLOSED LOOP
INFORMATION FEEDBACK SYSTEM



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$$\alpha_r = e^{-K_2 T_r} \quad \ldots\ldots\ldots\ldots \text{[Eq. A4.96]}$$

$$\lambda_r = e^{-(K_1+K_3)T_r} \quad \ldots\ldots\ldots \text{[Eq. A4.97]}$$

$$\mu_r = \left[(1 - \lambda_r) \quad \frac{R}{(K_1 + K_3)}\right] \times 8.34 \ .\text{[Eq. A4.98]}$$

$$\gamma_r = \frac{K_1(\lambda_r - \alpha_r)}{K_2 - (K_1 + K_3)} \quad \ldots\ldots \text{[Eq. A4.99]}$$

$$\rho_r = -\gamma_r \left(\frac{R}{K_1 + K_3}\right) + \frac{K_1}{K_2} \left(\frac{R}{K_1 + K_3} - \frac{M}{K_1}\right)(1-\alpha_r) \ \ldots\ldots \text{[Eq. A4.100]}$$

$$\Psi_r = [(CS_r)\ (1 - \alpha_r) - \rho_r] \times 8.34 \quad \text{[Eq. A4.101]}$$

where
$r = 1,2,\ldots,n$ is the reach number, and
$n \leq 50$.
RHS values for water, BOD, and DO are obtained as follows:

ROW NAME	RHS VALUE
WAT(I)B	QW(I)
WAT(I)P	QT(I)
BØD(I)B	QW(I)*BW(I)*8.34
BØD(I)P	QT(I)*BT(I)*8.34
DIØ(I)B	QW(I)*CW(I)*8.34
DIØ(I)P	QT(I)*CT(I)*8.34

where
r = (I)
B   represents wastewater,
P   represents tributary,
and the nodes are defined as
WAT(I)B - wastewater,
WAT(I)P - tributary for water,
BØD(I)B - BOD for wastewater,
BØD(I)P - BOD for tributary,
DIØ(I)B - DO for wastewater,
DIO(I)P - DO for tributary,
along with the parameter definitions of
QW(I) - wastewater flow,
QT(I) - tributary flow,
BW(I) - BOD concentration of wastewater discharged, and
BT(I) - BOD concentration of tributary flow.

The constant 8.34 represents the transformation of 1 mg/l to 8.34 lbs/MG.

The second routine was developed to determine the water quality standards of each reach in the region of interest for a watershed. This routine makes use of the minimum allowable D.O. in each reach, the wastewater and tributary flow in each reach, and the total reach flow to determine the water quality standard in mass units. The water quality routine will first determine the tributary and wastewater flow in the headwater and interior reaches. Then a sequential technique for determining the total flow in each reach and establishing water quality standards was incorporated. Specifically the sequential technique checks for any reaches, either headwater or interior above a specific reach, placing all reaches in the proper order for computational purposes. Starting with the upstream reaches in the region, the program will search its way down the tree diagram of the region until all flow values have been determined. These values will be summed along with the tributary and wastewater flows in the reach. The process for the headwater reaches is as follows:

$$QS(H) = QW(H) + QT(H) \quad \ldots\ldots \quad [\text{Eq. A4.102}]$$

$$QUAL(H)P = QS(H)*MDO(H)*8.34 \quad \ldots \quad [\text{Eq. A4.103}]$$

where

H = number of headwater reach,

QUAL(H)P = water quality standard for each headwater reach ,

QS(H) = sum of wastewater and tributary flows in headwater reach, and

MDO(H) = minimum allowable DO in headwater reach

The process for the interior reaches is:

$$QS(IN) = QW(IN) + QT(IN) \quad \ldots\ldots \quad [\text{Eq. A4.104}]$$

$$QUAL(IN)P = [\Sigma QS(HR) + \Sigma QS(IR)]* MDO(IN)*8.34 \quad \ldots \quad [\text{Eq. A4.105}]$$

where

HR = headwater reaches contributing to interior reach,

IR = interior reaches above and including the interior reach under consideration,

IN = number of interior reach,

QS(HR) = sum of wastewater and tributary flows in headwater reaches,

QS(IR) = sum of wastewater and tributary flows in interior reaches,

QUAL(IN)P = water quality standard for each interior reach, and

MDO(IN) = minimum allowable DO in interior reach.

The remaining routine takes all the LP parameters, RHS elements, and water quality standards, determined from the above routines, and structures the format for the MPS/360 Processor. All columns and rows are also placed in the fields specified by the MPS. Using the node-branch representation of the region in the network developed for the linear programming model, the rows are the nodes and the columns are the branches connecting the nodes. The three commodities being transported down the river are water, BOD, and DO. Subsequently,the rows and columns in the model are representative of these three commodities. The final output of this routine is a regional linear programming model in a multi-commodity network format.

A4.7.3 Program Input

The output data from the simulation may be printed, punched on cards or written on tape, depending on any final transformation in the data, and also depending on the control statements used, before becoming input to INTERF. Common, Read, and Dimension statements are incorporated in the program for the variables shared in both the simulation and interface program. Data statements are used to define variables or columns, row names, and array elements. The data statements appear at the beginning of the program, though this is not mandatory, in order that the initial skeleton of the LP tableau may begin being structured. In this case, the majority of the data statements are arrays dimensioned 20 by 8, (20,8), viz twenty rows down and eight columns across. For the literal data in the data statements, 4H format codes are used. The literal data specified by the data statements are the columns, rows, and parameters to be used in the LP tableau. The program specifies twenty-seven arrays for a 50 reach region, and actually uses only seven of these arrays for a hypothetical region of seven reaches. The number of arrays used for an n reach problem is dependent on the structure of the network format being used for a specific region in the watershed, not on the actual number of reaches. When a different region is considered in the watershed, the only changes which need to be made in the main program are in the data statements.

Three input data sets are required for the program. The first two data sets are in I format code and the third is in F format code. The first set includes the number of reaches (NR) and the number of days (NDAYS). The second set contains the numbering scheme used for the sequential technique incorporated in the program. The stream and wastewater data output from the simulation model are found in the third set and consist of the data in Section 9, Table 9-2.

A4.7.4 Program Output

The output from INTERF is in four parts:

1. The values of the six parameters shown in the Program Components section are given for each reach in the region.

2. The amount of flow in each reach, i.e., the sum of the wastewater and tributary flow in the reach.

3. A numerical diagram of the region under consideration so the user can actually sketch out or visualize the tree diagram of the river basin.

4. The last part of the output is the mathematical programming model structured in a multi-commodity network format. All columns, rows, RHS, and parameters are printed and punched according to the special format and field width in the MPS Processor.

A4.7.5 Definition of Program Variables

NR - Number of reaches, KK = 1, ..., NR

ALPHA(KK) or α_r	Parameter shown in Equation (A4.96).
XLAMDA(KK) or λ_r	Parameter shown in Equation (A4.97).
XMU(KK) or μ_r	Parameter shown in Equation (A4.98).
GAMMA(KK) or γ_r	Parameter shown in Equation (A4.99).
RHO(KK) or ρ_r	Parameter shown in Equation (A4.100).
PS1(KK) or Ψ_r	Parameter shown in Equation (A4.101).
TIME(KK) or T_r	Travel time of water to flow from beginning to end of a reach
R(KK) or R	Rate of addition of BOD along a stretch due to runoff and scour
XK1(KK) or K_1	Rate constant for deoxygenation
XK2(KK) or K_2	Rate constant for reaeration
XK3(KK) or K_3	Rate constant for sedimentation and absorption
A(KK) or M	Oxygen production or reduction due to plants and bottom deposits. A(KK) may be positive or negative.
WATIB(KK)	Wastewater node, related to value of wastewater flow
WATIP(KK)	Tributary node, related to value of tributary flow
BODIB(KK)	BOD node for wastewater, related to mass units of BOD in wastewater

BODIP(KK)	BOD node for tributary, related to mass units of BOD in tributary
DIOIB(KK)	DO node for wastewater, related to mass units of DO in wastewater
DIOIP(KK)	DO node for tributary, related to mass units of DO in tributary
QREG(KK)	Sum of wastewater and tributary flow
QUALIP(JR(KK))	Water quality standard for each reach.

A4.7.6 Program Logic

The basic flow chart shown in Figure A4-12 should be used as an aid to following the operation of INTERF.

FIGURE A4-12
FLOW CHART FOR INTERF

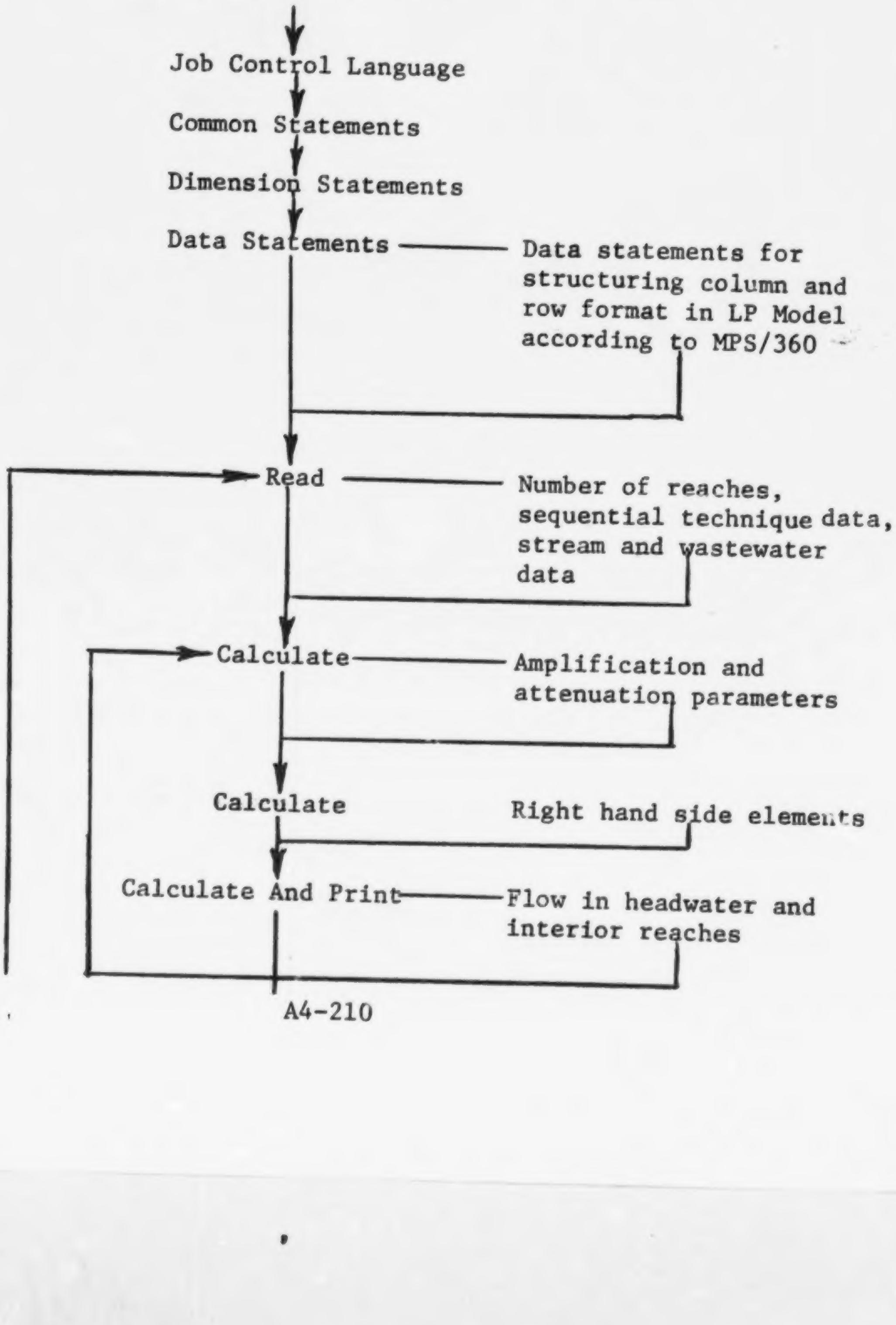
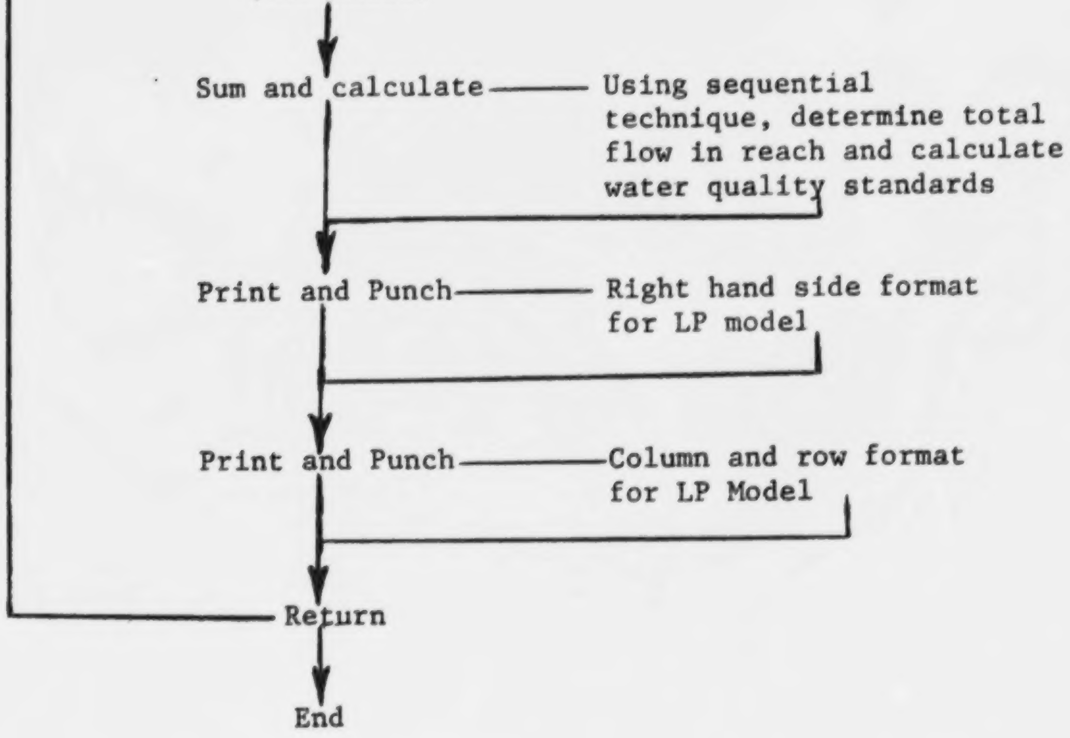

Job Control Language
Common Statements
Dimension Statements
Data Statements
Data statements for structuring column and row format in LP Model according to MPS/360
Read
Number of reaches, sequential technique data, stream and wastewater data
Calculate
Amplification and attenuation parameters
Calculate
Right hand side elements
Calculate And Print
Flow in headwater and interior reaches

FIGURE A4-12
(continued)


Sum and calculate
Using sequential technique, determine total flow in reach and calculate water quality standards
Print and Punch
Right hand side format for LP model
Print and Punch
Column and row format for LP Model
Return
End

A4.7.7 Program Coding

```
//INTERF   JOB (1143,47,003,03,3000),'CARTER                    ',CLASS
                                                                   =S
/*PASSWORD      1,LOFLOJOB
// EXEC F4GOXS      FORT G COMPILE (DECK), EXECUTE, CLASS S
//FORT.SYSIN DD *
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1       TCA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      DIMENSION ALPHA(50),XLAMDA(50),XMU(50),GAMMA(50),RHO(50),
     1          PSI(50),WATIB(50),WATIP(50),BODIB(50),BODIP(50),
     2           DIOIB(50),DIOIP(50),QUALIP(50)
      DIMENSION TIME(50),QWASTE(50),QSUM(50),XMXDEF(50),XMNDO(50)
     1           ,DOWST(50),DEFSUM(50),BODSUM(50),XK1(50),XK2(50),
     2          XK3(50),A(50),B(50),BODWST(50),R(50)
     3           ,DOS(50)
      DIMENSION GREG(50)
      DIMENSION X1(150,8),X2(50,8),X3(100,8),X4(50,8),X5(100,8),X
                                                                6(50,8)
     1  ,        X11(20,8),X12(20,8),X13(20,8),X14(20,8),X15(20,8)
                                                                 ,
     2           X16(20,8),X17(20,8),X18(10,8),X21(20,8),X22(20,8)
                                                                 ,
     3           X23(10,8),X31(20,8),X32(20,8),X33(20,8),X34(20,8)
                                                                 ,
     4           X35(20,8),X41(20,8),X42(20,8),X43(10,8),X51(20,8)
                                                                 ,
     5           X52(20,8),X53(20,8),X54(20,8),X55(20,8),X61(20,8)
                                                                 ,
     6           X62(20,8),X63(10,8)
C   * DATA STATEMENTS REQUIRED FOR THE COLUMN AND ROW FORMAT IN T
                                                              HE LP*
      DATA X11/4HQ1P3,4HQ1P3,4HQ1P3,4HQ2P3,4HQ2P3,4HQ2P3,4HQ3P5,4
                                                              HQ3P5,
     1          4HQ3P5,4HQ4P5,4HQ4P5,4HQ4P5,4HQ5P6,4HQ5P6,4HQ5P6,4
                                                              HQ6P7,
     2          4HQ6P7,4HQ6P7,4HQ7P8,4HQ7P8,4HP1  ,4HP1  ,4HP1  ,4
                                                              HP1  ,
     3          4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4
                                                              HP1  ,
     4          4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4HP1  ,4
                                                              HP1  ,
     5          4HWAT1,4HBOD3,4HQUAL,4HWAT2,4HBOD3,4HQUAL,4HWAT3,4
                                                              HBOD5,
     6          4HQUAL,4HWAT4,4HBOD5,4HQUAL,4HWAT5,4HBOD6,4HQUAL,4
                                                              HWAT6,
     7          4HBOD7,4HQUAL,4HWAT7,4HBOD8,4HP   ,4HP   ,4H1P  ,4
                                                              HP   ,
     8          4HP   ,4H2P  ,4HP   ,4HP   ,4H3P  ,4HP   ,4HP   ,4
                                                              H4P  ,
     9          4HP   ,4HP   ,4H5P  ,4HP   ,4HP   ,4H6P  ,4HP   ,4
```

```
                                                  HP    ,
A           1.0,4H    ,4H    ,   1.0,4H    ,4H    ,   1.0,4
                                                  H     ,
B        4H    ,   1.0,4H    ,4H    ,   1.0,4H    ,4H    ,
                                                      1.0,
C        4H    ,4H    ,   1.0,4H    ,4HWAT3,4HDIO3,4H    ,4
                                                  HWAT3,
D        4HDIO3,4H    ,4HWAT5,4HDIO5,4H    ,4HWAT5,4HDIO5,4
                                                  H     ,
E        4HWAT6,4HDIO6,4H    ,4HWAT7,4HDIO7,4H    ,4HWAT8,4
                                                  HDIO8,
F        4HP   ,4HP   ,4H    ,4HP   ,4HP   ,4H    ,4HP   ,4
                                                  HP    ,
G        4H    ,4HP   ,4HP   ,4H    ,4HP   ,4HP   ,4H    ,4
                                                  HP    ,
H        4HP   ,4H    ,4HP   ,4HP   ,   -1.0,4H    ,0.0   ,
                                                     -1.0,
I        4H    ,0.0   ,   -1.0,4H    ,0.0   ,   -1.0,4H    ,0
                                                  .0    ,
J           -1.0,4H    ,0.0    ,   -1.0,4H    ,0.0    ,   -1.0,4
                                                  H     /
 DATA X12/4HC7P8,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
1        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
2        4H    ,4H    ,4H    ,4H    ,4HP1  ,4H    ,4H    ,4
                                                  H     ,
3        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
4        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
5        4HQUAL,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
6        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
7        4H    ,4H    ,4H    ,4H    ,4H7P  ,4H    ,4H    ,4
                                                  H     ,
8        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
9        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
A        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
B        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
C        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
D        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
E        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                  H     ,
F         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
G         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
H         4H    ,4H    ,4H    ,4H    ,0.0   ,4H    ,4H    ,4
                                                    H     ,
I         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
J         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     /
  DATA X13/20*4H    /
  DATA X14/20*4H    /
  DATA X15/20*4H    /
  DATA X16/20*4H    /
  DATA X17/20*4H    /
  DATA X18/10*4H    /
  DATA X21/4HQ1B1,4HQ2B2,4HQ3B3,4HQ4B4,4HQ6B6,4HQ7B7,4HQ8P9,4
                                                    H     ,
1         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
2         4H    ,4H    ,4H    ,4H    ,4HP1  ,4HP1  ,4HP1  ,4
                                                    HP1   ,
3         4HP1  ,4HP1  ,4HB1  ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
4         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
5         4HWAT1,4HWAT2,4HWAT3,4HWAT4,4HWAT6,4HWAT7,4HWAT8,4
                                                    H     ,
6         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
7         4H    ,4H    ,4H    ,4H    ,4HB   ,4HB   ,4HB   ,4
                                                    HB    ,
8         4HB   ,4HB   ,4HP   ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
9         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
A           1.0,   1.0,   1.0,   1.0,   1.0,   1.0,   1.0,4
                                                    H     ,
B         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
C         4H    ,4H    ,4H    ,4H    ,4HWAT1,4HWAT2,4HWAT3,4
                                                    HWAT4,
D         4HWAT6,4HWAT7,4HWAT9,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
E         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
F         4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HB   ,4
                                                    H     ,
G         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                    H     ,
H         4H    ,4H    ,4H    ,4H    ,  -1.0,  -1.0,  -1.0,
                                                                  -1.0,
I          -1.0,  -1.0,  -1.0,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
J        4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     /
 DATA X22/20*4H     /
 DATA X23/10*4H     /
 DATA X31/4HQ1P3,4HQ1P3,4HQ2P3,4HQ2P3,4HQ3P5,4HQ3P5,4HQ4P5,4
                                                              HQ4P5,
1        4HQ5P6,4HQ5P6,4HQ6P7,4HQ6P7,4HQ7P8,4HQ7P8,4H     ,4
                                                              H     ,
2        4H     ,4H     ,4H     ,4H     ,4HP2   ,4HP2   ,4HP2   ,4
                                                              HP2   ,
3        4HP2   ,4HP2   ,4HP2   ,4HP2   ,4HP2   ,4HP2   ,4HP2   ,4
                                                              HP2   ,
4        4HP2   ,4HP2   ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
5        4HBOD1,4HDIO3,4HBOD2,4HDIO3,4HBOD3,4HDIO5,4HBOD4,4
                                                              HDIO5,
6        4HBOD5,4HDIO6,4HBOD6,4HDIO7,4HBOD7,4HDIO8,4H     ,4
                                                              H     ,
7        4H     ,4H     ,4H     ,4H     ,4HP    ,4HP    ,4HP    ,4
                                                              HP    ,
8        4HP    ,4HP    ,4HP    ,4HP    ,4HP    ,4HP    ,4HP    ,4
                                                              HP    ,
9        4HP    ,4HP    ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
A           1.0,4H     ,    1.0,4H     ,    1.0,4H     ,    1.0,4
                                                              H     ,
B           1.0,4H     ,    1.0,4H     ,    1.0,4H     ,4H     ,4
                                                              H     ,
C        4H     ,4H     ,4H     ,4H     ,4HBOD3,4HQUAL,4HBOD3,4
                                                              HQUAL,
D        4HBOD5,4HQUAL,4HBOD5,4HQUAL,4HBOD6,4HQUAL,4HBOD7,4
                                                              HQUAL,
E        4HBOD8,4HQUAL,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
F        4HP    ,4H1P   ,4HP    ,4H2P   ,4HP    ,4H3P   ,4HP    ,4
                                                              H4P   ,
G        4HP    ,4H5P   ,4HP    ,4H6P   ,4HP    ,4H7P   ,4H     ,4
                                                              H     ,
H        4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
I        4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     ,
J        4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4H     ,4
                                                              H     /
 DATA X32/20*4H     /
 DATA X33/20*4H     /
 DATA X34/20*4H     /
 DATA X35/20*4H     /
 DATA X41/4HQ1B1,4HQ2B2,4HQ3B3,4HQ4B4,4HQ6B6,4HQ7B7,4HQ8P9,4
                                                      H    ,
1         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
2         4H    ,4H    ,4H    ,4H    ,4HP2  ,4HP2  ,4HP2  ,4
                                                      HP2  ,
3         4HP2  ,4HP2  ,4HP2  ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
4         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
5         4HBOD1,4HBOD2,4HBOD3,4HBOD4,4HBOD6,4HBOD7,4HBOD8,4
                                                      H    ,
6         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
7         4H    ,4H    ,4H    ,4H    ,4HB   ,4HB   ,4HB   ,4
                                                      HB   ,
8         4HB   ,4HB   ,4HP   ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
9         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
A            1.0,   1.0,   1.0,   1.0,   1.0,   1.0,   1.0,4
                                                      H    ,
B         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
C         4H    ,4H    ,4H    ,4H    ,4HBOD1,4HBOD2,4HBOD3,4
                                                      HBOD4,
D         4HBOD6,4HBOD7,4HBOD9,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
E         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
F         4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HB   ,4
                                                      H    ,
G         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
H         4H    ,4H    ,4H    ,4H    ,  -1.0,  -1.0,  -1.0,
                                                        -1.0,
I           -1.0,  -1.0,  -1.0,4H    ,4H    ,4H    ,4H    ,4
                                                      H    ,
J         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                      H    /
 DATA X42/20*4H    /
 DATA X43/10*4H    /
 DATA X51/4HQ1P3,4HQ1P3,4HQ2P3,4HQ2P3,4HQ3P5,4HQ3P5,4HQ4P5,4
                                                      HQ4P5,
1         4HQ5P6,4HQ5P6,4HQ6P7,4HQ6P7,4HQ7P8,4HQ7P8,4H    ,4
                                                      H    ,
2         4H    ,4H    ,4H    ,4H    ,4HP3  ,4HP3  ,4HP3  ,4
                                                      HP3  ,
3         4HP3  ,4HP3  ,4HP3  ,4HP3  ,4HP3  ,4HP3  ,4HP3  ,4
                                                      HP3  ,
4         4HP3  ,4HP3  ,4H    ,4H    ,4H    ,4H    ,4H    ,4
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                                                                      H     ,
     5         4HDIO1,4HQUAL,4HDIO2,4HQUAL,4HDIO3,4HQUAL,4HDIO4,4
                                                                      HQUAL,
     6         4HDIO5,4HQUAL,4HDIO6,4HQUAL,4HDIO7,4HQUAL,4H     ,4
                                                                      H     ,
     7         4H    ,4H    ,4H    ,4H    ,4HP   ,4H1P  ,4HP   ,4
                                                                      H2P   ,
     8         4HP   ,4H3P  ,4HP   ,4H4P  ,4HP   ,4H5P  ,4HP   ,4
                                                                      H6P   ,
     9         4HP   ,4H7P  ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     A            1.0,4H    ,   1.0,4H    ,   1.0,4H    ,   1.0,4
                                                                      H     ,
     B            1.0,4H    ,   1.0,4H    ,   1.0,4H    ,4H    ,4
                                                                      H     ,
     C         4H    ,4H    ,4H    ,4H    ,4HDIO3,4H    ,4HDIO3,4
                                                                      H     ,
     D         4HDIO5,4H    ,4HDIO5,4H    ,4HDIO6,4H    ,4HDIO7,4
                                                                      H     ,
     E         4HDIO8,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     F         4HP   ,4H    ,4HP   ,4H    ,4HP   ,4H    ,4HP   ,4
                                                                      H     ,
     G         4HP   ,4H    ,4HP   ,4H    ,4HP   ,4H    ,4H    ,4
                                                                      H     ,
     H         4H    ,4H    ,4H    ,4H    ,4H    ,   0.0,4H    ,
                                                                         0.0,
     I         4H    ,   0.0,4H    ,   0.0,4H    ,   0.0,4H    ,
                                                                         0.0,
     J         4H    ,   0.0,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     /
      DATA X52/20*4H    /
      DATA X53/20*4H    /
      DATA X54/20*4H    /
      DATA X55/20*4H    /
      DATA X61/4HQ1B1,4HQ2B2,4HQ3B3,4HQ4B4,4HQ6B6,4HQ7B7,4HQ8P9,4
                                                                      H     ,
     1         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     2         4H    ,4H    ,4H    ,4H    ,4HP3  ,4HP3  ,4HP3  ,4
                                                                      HP3   ,
     3         4HP3  ,4HP3  ,4HB3  ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     4         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     5         4HDIO1,4HDIO2,4HDIO3,4HDIO4,4HDIO6,4HDIO7,4HDIO8,4
                                                                      H     ,
     6         4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                                      H     ,
     7         4H    ,4H    ,4H    ,4H    ,4HB   ,4HB   ,4HB   ,4
                                                                      HB    ,
    8        4HB   ,4HB   ,4HP   ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    9        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    A           1.0,   1.0,   1.0,   1.0,   1.0,   1.0,   1.0,
                                                           1.0,
    B        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    C        4H    ,4H    ,4H    ,4H    ,4HDI01,4HDI02,4HDI03,4
                                                        HDI04,
    D        4HDI06,4HDI07,4HDI09,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    E        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    F        4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HP   ,4HB   ,4
                                                        H     ,
    G        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    H        4H    ,4H    ,4H    ,4H    ,  -1.0,  -1.0,  -1.0,
                                                         -1.0,
    I          -1.0,  -1.0,  -1.0,4H    ,4H    ,4H    ,4H    ,4
                                                        H     ,
    J        4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4H    ,4
                                                        H     /
     DATA X62/20*4H    /
     DATA X63/10*4H    /
     DO 300 INDEX=1,20
     INDEX2=INDEX+20
     INDEX3=INDEX+40
     INDEX4=INDEX+60
     INDEX5=INDEX+80
     INDEX6=INDEX+100
     INDEX7=INDEX+120
     INDEX8=INDEX+140
     DO 301 NDEX=1,8
     X1(INDEX,NDEX)=X11(INDEX,NDEX)
     X1(INDEX2,NDEX)=X12(INDEX,NDEX)
     X1(INDEX3,NDEX)=X13(INDEX,NDEX)
     X1(INDEX4,NDEX)=X14(INDEX,NDEX)
     X1(INDEX5,NDEX)=X15(INDEX,NDEX)
     X1(INDEX6,NDEX)=X16(INDEX,NDEX)
     X1(INDEX7,NDEX)=X17(INDEX,NDEX)
     IF(INDEX8.GT.150)GO TO 303
     X1(INDEX8,NDEX)=X18(INDEX,NDEX)
303  X2(INDEX,NDEX)=X21(INDEX,NDEX)
     X2(INDEX2,NDEX)=X22(INDEX,NDEX)
     X4(INDEX,NDEX)=X41(INDEX,NDEX)
     X4(INDEX2,NDEX)=X42(INDEX,NDEX)
     X6(INDEX,NDEX)=X61(INDEX,NDEX)
     X6(INDEX2,NDEX)=X62(INDEX,NDEX)
     IF(INDEX3.GT.50)GO TO 304
```

```
      X2(INDEX3,NDEX)=X23(INDEX,NDEX)
      X4(INDEX3,NDEX)=X43(INDEX,NDEX)
      X6(INDEX3,NDEX)=X63(INDEX,NDEX)
  304 CONTINUE
  302 X3(INDEX,NDEX)=X31(INDEX,NDEX)
      X3(INDEX2,NDEX)=X32(INDEX,NDEX)
      X3(INDEX3,NDEX)=X33(INDEX,NDEX)
      X3(INDEX4,NDEX)=X34(INDEX,NDEX)
      X3(INDEX5,NDEX)=X35(INDEX,NDEX)
      X5(INDEX,NDEX)=X51(INDEX,NDEX)
      X5(INDEX2,NDEX)=X52(INDEX,NDEX)
      X5(INDEX3,NDEX)=X53(INDEX,NDEX)
      X5(INDEX4,NDEX)=X54(INDEX,NDEX)
      X5(INDEX5,NDEX)=X55(INDEX,NDEX)
  301 CONTINUE
  300 CONTINUE
C   * NDAYS                   NUMBER OF DAYS IN A CRITICAL PERIOD
                                                                       *
      READ(5,500)NR,NDAYS
  500 FORMAT(2I5)
      DO 5 I=1,NR
    5 READ(5,502)NUR(I),JR(I),(NUR(I,J),J=1,3)
  502 FORMAT(5I5)
      DO 101 K=1,NDAYS
      READ(5,501)(TIME(I),QWASTE(I),QSUM(I),XMNDO(I),XMXDEF(I),
     1     DOWST(I),DEFSUM(I),BODWST(I),BODSUM(I),XK1(I),XK2(I),
     2     XK3(I),A(I),R(I),DOS(I),I=1,NR)
  501 FORMAT(5F11.4)
C   * CALCULATION OF AMPLIFICATION AND ATTENUATION FACTORS *
      DO 100 KK=1,NR
      ARG1=-XK2(KK)*TIME(KK)
      ARG2=-(XK1(KK)+XK3(KK))*TIME(KK)
      ALPHA(KK)=EXP(ARG1)
      XLAMDA(KK)=EXP(ARG2)
      XMU(KK)=(1.-XLAMDA(KK))*(R(KK)/(XK1(KK)+XK3(KK)))*
     1        8.34
      GAMMA(KK)=(XK1(KK)*(XLAMDA(KK)-ALPHA(KK)))/(XK2(KK)-(XK1(KK
                                                                   )
     1          +XK3(KK)))
      RHO(KK)=-GAMMA(KK)*(R(KK)/(XK1(KK)+XK3(KK)))+
     1        XK1(KK)/XK2(KK)*(R(KK)/(XK1(KK)+XK3(KK))-
     2         A(KK)/XK1(KK))*(1.-ALPHA(KK))
      PSI(KK)=(DOS(KK)*(1.-ALPHA(KK))-RHO(KK))*8.34
      WRITE(6,600)KK,K,ALPHA(KK),XLAMDA(KK),XMU(KK),GAMMA(KK),
     1      RHO(KK),PSI(KK)
  600 FORMAT(1H1,'CALCULATIONS-COEF.FOR REACH ',I3,' DAY ',I4,
     1       ' ARE AS FOLLOWS'/'    ALPHA XLAMBDA      XMU   GAMMA
                                                                     RH
     10     PSI'/1H ,6F8.2)
C   * CALCULATION OF RHS VALUES
      WATIB(KK)=QWASTE(KK)
      WATIP(KK)=QSUM(KK)
      BODIB(KK)=QWASTE(KK)*8.34*BODWST(KK)
      BODIP(KK)=QSUM(KK)*8.34*BODSUM(KK)
      DIOIB(KK)=QWASTE(KK)*8.34*DOWST(KK)
C   * CALCULATION OF FLOW IN EACH HEADWATER AND INTERIOR REACH *
      QREG(KK)=QWASTE(KK)+QSUM(KK)
      WRITE(6,800)KK,QREG(KK)
  800 FORMAT(1H ,'QREG',I4,'=',F10.4)
  100 DIOIP(KK)=QSUM(KK)*8.34*DEFSUM(KK)
C   * SUMMATION OF FLOWS AND CALCULATION OF WATER QUALITY STANDAR
                                                               DS *
      DO 104 KK=1,NR
      J=1
      SUM=QREG(JR(KK))
      IF(NUR(JR(KK),J).EQ.0)GO TO 106
  103 DO 105 J=1,3
      ND=NUR(JR(KK),J)
      IF(ND.EQ.0)GO TO 106
      ID=IREACH(ND)
      WRITE(6,801)JR(KK),J,ID
  801 FORMAT(1H ,'NUR(',I5,',',I5,')=',I5)
  105 SUM=SUM+QREG(ID)
      WRITE(6,803)SUM
  803 FORMAT(1H ,'SUM= ',F10.4)
  106 QREG(JR(KK))=SUM
      QUALIP(JR(KK))=SUM*XMNDO(JR(KK))*8.34
  104 CONTINUE
C   * RHS FORMAT FOR LP MODEL - PRINT OUT - *
      NUM1=NR-1
      WRITE(6,601)(JR(I),WATIB(JR(I)),JR(I),WATIP(JR(I)),I=1,NR)
      WRITE(6,602)(JR(I),BODIB(JR(I)),JR(I),BODIP(JR(I)),I=1,NR)
      WRITE(6,603)(JR(I),DIOIB(JR(I)),JR(I),DIOIP(JR(I)),I=1,NR)
      WRITE(6,604)(JR(I),QUALIP(JR(I)),I=1,NUM1)
      I=NR
      WRITE(6,605)JR(I),QUALIP(JR(I))
      PUNCH 601,  (JR(I),WATIB(JR(I)),JR(I),WATIP(JR(I)),I=1,NR)
      PUNCH 602,  (JR(I),BODIB(JR(I)),JR(I),BODIP(JR(I)),I=1,NR)
      PUNCH 603,  (JR(I),DIOIB(JR(I)),JR(I),DIOIP(JR(I)),I=1,NR)
      PUNCH 604,  (JR(I),QUALIP(JR(I)),I=1,NUM1)
      I=NR
      PUNCH 605,  JR(I),QUALIP(JR(I))
  601 FORMAT('1',/(' ',T5, 'STP1980',T15,   'WAT',I1,'B',T25,F12.
                                                              4,
     1T40,   'WAT',I1,'P',T50,F12.4))
  602 FORMAT(' ',T5, 'STP1980',T15,   'BOD',I1,'B',T25,F12.4,T40,
                                                              'BOD
     1',I1,'P',T50,F12.4)
  603 FORMAT(' ',T5, 'STP1980',T15,   'DIO',I1,'B',T25,F12.4,T40,
                                                              'DIO
     1',I1,'P',T50,F12.4)
  604 FORMAT(' ',T5, 'STP1980',T15,  'QUAL',I1,'P',T25,F12.4,T40,
                                                                'QUAL
     1',I1,'P',T50,F12.4)
  605 FORMAT(' ',T5, 'STP1980',T15,  'QUAL',I1,'P',T25,F12.4)
C    * COLUMN AND ROW FORMAT FOR LP MODEL - PRINT OUT - *
      NUM=3*NR-1
      NUM2=2*NR
      J=1
      DO 1 I=2,NUM,3
      X1(I,5)=-XMU(J)
      X1(I,8)=-PSI(J)
      X1(I+1,5)=PSI(J)
    1 J=J+1
      J=1
      DO 2 I=2,NUM2,2
      X3(I-1,8)=-XLAMDA(J)
      X3(I,5)=GAMMA(J)
      X3(I,8)=-GAMMA(J)
    2 J=J+1
      J=1
      DO 3 I=2,NUM2,2
      X5(I-1,8)=-ALPHA(J)
      X5(I,5)=ALPHA(J)
    3 J=J+1
      NUM3=3*NR
      WRITE(6,700)((X1(I,J),J=1,8),I=1,NUM3)
      WRITE(6,700)((X2(I,J),J=1,8),I=1,NR)
      WRITE(6,700)((X3(I,J),J=1,8),I=1,NUM2)
      WRITE(6,700)((X4(I,J),J=1,8),I=1,NR)
      WRITE(6,700)((X5(I,J),J=1,8),I=1,NUM2)
      WRITE(6,700)((X6(I,J),J=1,8),I=1,NR)
      PUNCH 700,  ((X1(I,J),J=1,8),I=1,NUM3)
      PUNCH 700,  ((X2(I,J),J=1,8),I=1,NR)
      PUNCH 700,  ((X3(I,J),J=1,8),I=1,NUM2)
      PUNCH 700,  ((X4(I,J),J=1,8),I=1,NR)
      PUNCH 700,  ((X5(I,J),J=1,8),I=1,NUM2)
      PUNCH 700,  ((X6(I,J),J=1,8),I=1,NR)
  700 FORMAT(1H ,T5,2A4,T15,2A4,T25,F12.4,T40,2A4,T50,F12.4)
  101 CONTINUE
      RETURN
      END
      FUNCTION IREACH (NU)
      COMMON/FLOW1/NR,NG,NOR(50),NUR(50,3),DAU(50,3),
     1       TDA(50),NGAGE(10),NGR(10),DAG(10),IDR(50)
      COMMON/FLOW2/FL(50),ROUGH(50),SLOPE(50)
      COMMON/FLOW3/JR(50),WT(50,10)
      II = 1
    3 IF (NOR(II) - NU) 1,2,1
    1 II = II + 1
      IF (II.EQ.NR) GO TO 2
      GO TO 3
    2 IREACH = II
```

```
      RETURN
      END
/*
//GO.SYSIN DD *
0000700001
   10    1
   20    2
   30    3   10   20
   40    4
   50    5   30   40
   60    6   50
   70    7   60
       .235         5.      1355.         7.       3.20
         1.       9.50       248.       1.66        .31
       1.02        .02        .85        .15      10.20
      1.330        37.      1290.        7.5       2.45
         1.         8.       408.        .68        .41
        .60        .03        .14        .14       9.95
      1.087         8.         0.         7.       2.00
         1.         0.       240.         0.        .36
        .63        .04        .18        .14       9.00
      2.067        14.       296.         6.       3.54
         1.       9.70      1440.         1.        .35
        .09        .04        .05        .11       9.54
       .306         0.         0.        6.5       2.50
         0.         0.         0.         0.        .34
        .72        .05        .39        .11       9.00
      1.050        26.         0.         6.       2.35
         1.         0.      2180.         0.        .35
        .14        .06        .07        .13       8.35
      6.130        41.         0.         4.       4.17
         1.         0.       279.         0.        .30
        .02        .00        .00        .00       8.17
/*EOF
```

A4.8 MPS Control Program

A4.8.1 Purpose

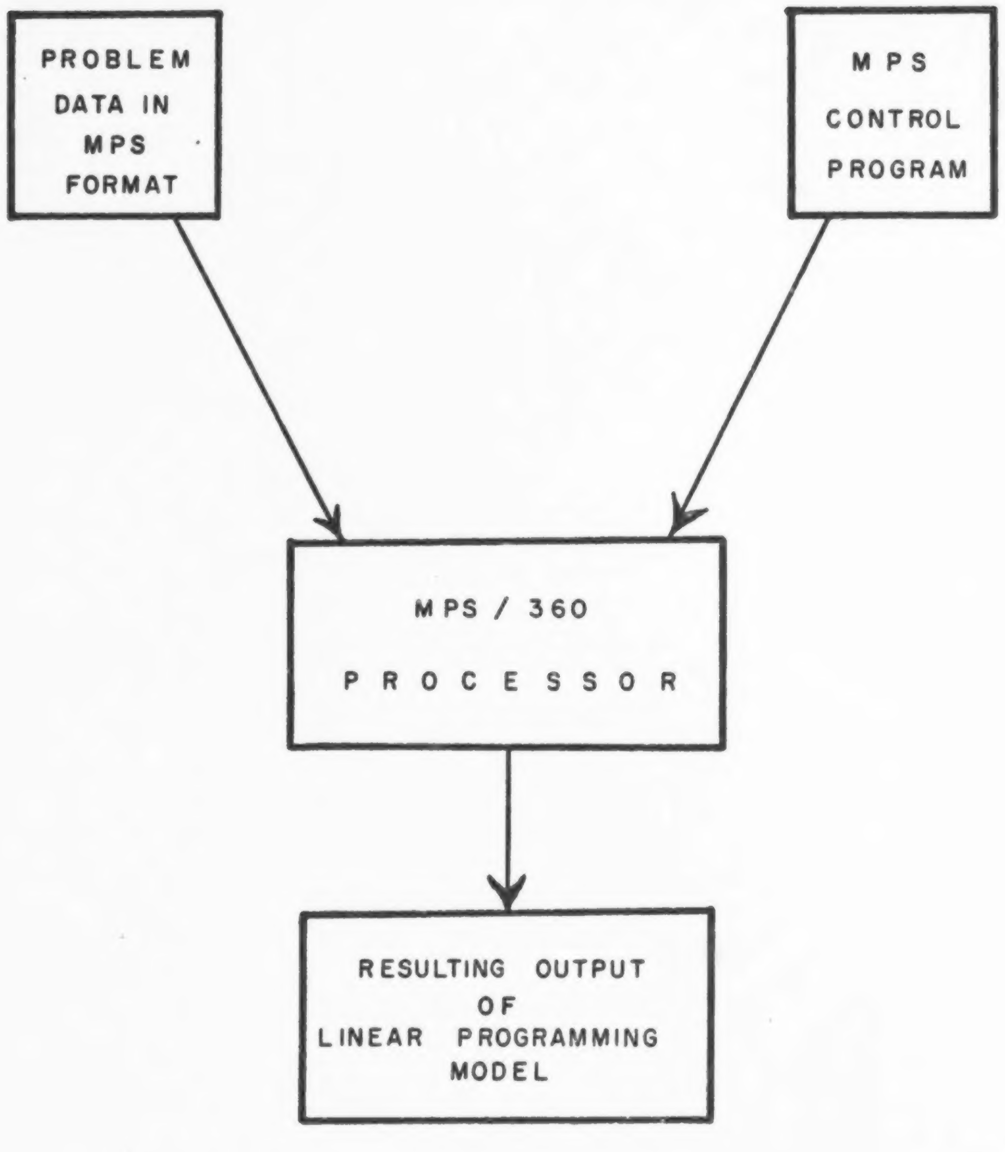
The control program used for the optimization model is the MPS Control Program. This auxiliary program has the flexibility to specify the necessary outputs needed in analysis of an optimal solution, depending only on the statements used in the program and minor changes in the LP matrix. After INTERF (Refer to A4.7) generates the linear programming tableau structured in the MPS format, the user conveys the proposed strategy via the MPS control language. Figure A4-13 shows how the problem data and the specified procedures in the control program simultaneously feed into the MPS/360 Processor. The control language statements call the linear programming procedures and transfers arguments to them. The resulting output of the linear programming model is then generated for analysis.

A4.8.2 Program Components

There are various MPS/360 LP procedures which may be specified in the control program, depending on the analysis needed. Included in these are the different optimization, post-optimal, and output schemes. The user of the MPS Control Program should first set up a basic control program. After obtaining an optimal solution to the model, then sophistication may be added in the control program for further analysis. The following is an example of a basic MPS Control Program.

```
PROGRAM
INITIALZ
MOVE(XDATA,'DATA NAME')
MOVE(XPBNAME, 'PBFILE')
CONVERT('SUMMARY')
BCDOUT
SETUP
MOVE(XOBJ,'OBJECTIVE NAME')
MOVE(XRHS,'CONSTRAINT NAME')
PRIMAL
SOLUTION
EXIT
PEND
```

FIGURE A4-13
OVERVIEW OF MPS PROCEDURE


PROBLEM DATA IN MPS FORMAT
MPS CONTROL PROGRAM
MPS / 360 PROCESSOR
RESULTING OUTPUT OF LINEAR PROGRAMMING MODEL

It is important that the first alphabetic character in each statement of the control program be in column 10 of a standard IBM punched card. Also of importance is to notice that when maximum (MAX) or minimum (MIN) is not specified in the MPS Control Program, the system assumes minimization.

A4.8.3 Remarks

To set up an MPS Control Program, the following three steps should be considered.

1. Obtain an IBM Mathematical Programming Input Form as a guide to the correct column and field format for the MPS/360.

2. Use a basic control program to correct the model and obtain the initial optimal solution.

3. Apply the various MPS Control Program procedures needed for analysis and sophistication of the model.

A4.9 LPLF - Linear Programming Model

A4.9.1 Purpose

LPLF is the mathematical programming model developed for allocating waste treatment requirements and/or low flow augmentation to meet preset water quality standards, and determine the optimal solution for a general region. The model was developed in a multi-commodity network format to provide a clearer technique needed for tracking commodities. With this format, each variable in the model may be traced through the network representation of the river basin. This provides the information needed for analysis in each reach or analysis of the entire region. The network is viewed as a unidirectional transportation network conveying water, the carrier commodity, and two water quality constituents, viz, dissolved oxygen (DO) and biochemical oxygen demand (BOD).

The model may be applied to any region in the watershed. A region is defined as a subset of the watershed, consisting of an area encompassing multiple reaches. Hence, the region may be investigated independently of the watershed. The region is then further divided into headwater and interior reaches. The flow, BOD, and DO in each reach are determined from the flow and concentration of the wastewater and tributary into the reach. From the continuity equations written around each node, representing the beginning or end of each reach, the model keeps track of the total flow, DO, and BOD in the region. Any potential sources of flow augmentation will be located at the beginning of the headwater reaches. Figure A4-14 shows six wastewater treatment facilities in a seven reach region problem for a hypothetical watershed with potential reservoir locations at either of the three headwater reaches.

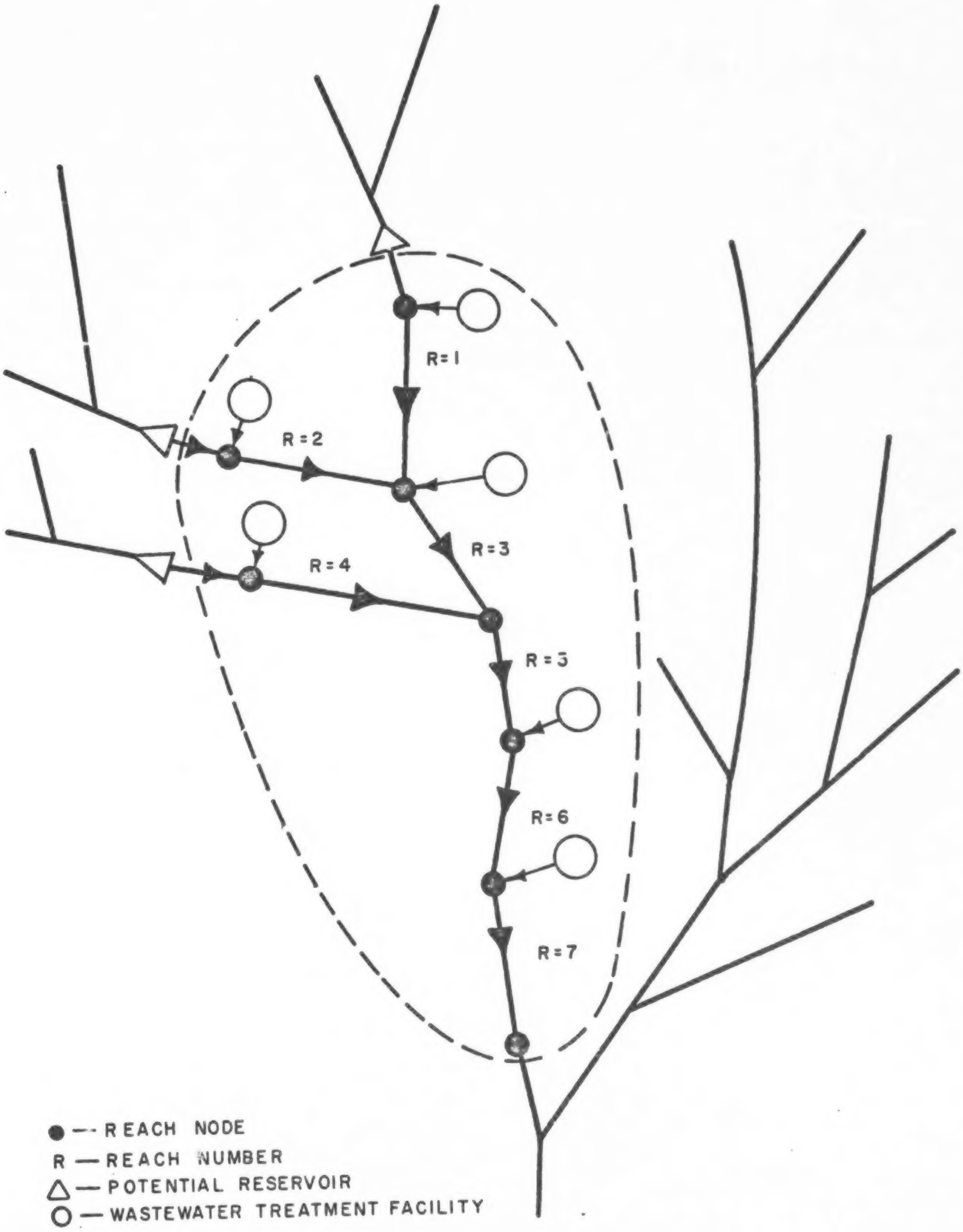
LPLF was first tested on a hypothetical river basin, a region similar to the one shown in Figure A4-14. The data obtained for this test was contained in the 1967 article of Loucks, Revelle, and Lynn (12). The BOD and DO concentrations are computed using the equations developed by Camp (10) and by Dobbins (11). These concentrations are converted to mass units for the purpose of maintaining a mass balance in the model. Natural flow conditions were analyzed first with a zero level of flow augmentation. After satisfactory results were obtained, changes made in the model included the capability of handling flow augmentation or handling the zero level DO case previously mentioned.

A4.9.2 Program Components

LPLF is made up mainly of INTERF (refer to A4.7), MPS Control Program (refer to A4.8), cost data, and L.P. bounds. The output from the interface program is the linear programming tableau structured in the MPS/360 format. With the MPS Control Program, the user specifies the procedure in which the problem data will be handled. Finally, the LPLF package includes the cost data and linear programming bounds for the objective function. Unit cost data were determined from predicted annual cost of BOD removal (in dollars). Considering that treatment plant costs are usually convex within a specified range, the cost function for each treatment facility was divided into segments representing percentage of BOD removal. Costs were assumed to be linear within each segment and the L.P. bounds were determined as the pounds of BOD removed within each segment.

The data set organization for the LPLF program consist of the Computing Center Job Stream, MPS Control Program, Rows Section, Columns Section, RHS Section, Bounds Section, and End Data.

FIGURE A4-14
OPTIMIZATION PROBLEM FOR
HYPOTHETICAL REGION



In the Mathematical Programming System/360 the optional Bounds section specifies either upper or lower bounds, or both. Without this feature the representation of the bounds would be by explicit constraints. Therefore, its use leads to a reduction in the number of constraints and in the computing time. Another optional section for the MPS/360 is the Ranges section which specifies ranges on the values of the right-hand side. This section was not used in LPLF. Figure A4-15 is an overview of the data set organization for LPLF.

A4.9.3 Program Input

The input data for LPLF are the Rows, Columns, RHS, and Bounds. To introduce this input data into the system a data set name, e.g., DAT1980 was used. Following the DAT1980 card are the Rows.

Rows is the first required section in the input data. This section distinguishes the row types for each row used in the model. Four different row types may be specified.

N Represents nonconstrained type, e.g., objective function

$$Z = \Sigma c_j \, x_j \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; \text{[Eq. A4.106]}$$

E Represents equality constraints, e.g.,

$$\Sigma a_{ij} \, x_j = b_i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; \text{[Eq. A4.107]}$$

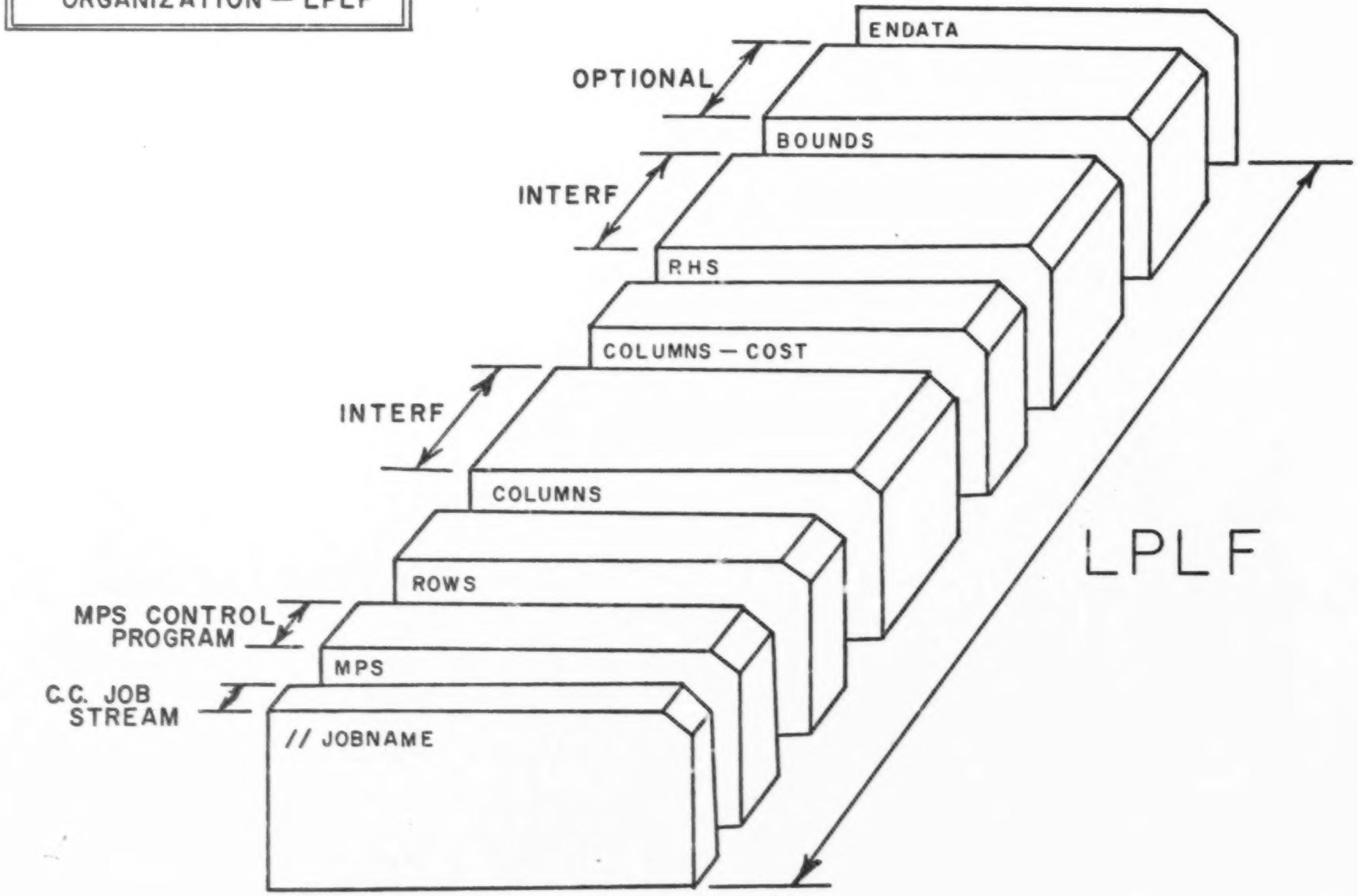
L Represents less-than-or-equal-to constraints, e.g.,

$$\Sigma a_{ij} \, x_j \leq b_i \quad . \; . \; . \; . \; . \; . \; . \; \text{[Eq. A4.108]}$$

G Represents greater-than-or-equal-to constraints, e.g.,

$$\Sigma a_{ij} \, x_j \geq b_i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; \text{[Eq. A4.109]}$$

FIGURE A4—15
OVERVIEW OF DATA SET
ORGANIZATION—LPLF



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Columns is the next required section. The columns, corresponding row names and parameter values are specified in this section. Only the nonzero matrix elements are needed by the MPS/360. All the elements for each column should appear together, but it is not mandatory that the same order be used as in the Rows section.

The RHS section is the third required section. All nonzero RHS values must be specified for as many RHS vectors as needed, differentiating each RHS vector by a name. STP 1980 was the RHS vector name used in LPLF. Each card should have the RHS name, corresponding row name, and RHS value.

The Bounds section, as previously mentioned, is optional, but conveniently used when limits on the values of the LP variables are needed. Similar to the RHS section, the Bounds section is defined by a Bound row name.

Letting R_i, $i=1,2,\ldots,m$, represent E, L, or G, for the row types, the general data format for the input data is as follows:

NAME	Data Name				
ROWS					
N	Objective Name				
R_1	Row Name$_1$				
R_2	Row Name$_2$				
.					
.					
.					
R_m	Row Name$_m$				
COLUMNS					
	Column Name$_1$	Row Name$_1$	$a_{1,1}$	Row Name$_2$	$a_{2,1}$
	.	.	.	.	.
	.	.	.	.	.
	Column Name$_1$	Row Name$_m$	$a_{m,1}$	Objective Name	c_1
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	Column Name$_n$	Row Name$_m$	$a_{m,n}$	Objective Name	c_n
RHS					
	RHS Name	Row Name$_1$	V_1		
	RHS Name	Row Name$_2$	V_2		
	.	.	.		
	.	.	.		
	RHS Name	Row Name$_m$	V_m		

BOUNDS

. . . .

. . . .

UP or LO   Bound Row Name   Column Name$_j$ L_j

. . . .

. . . .

ENDATA

Where:

$a_{i,j}$ - Value of LP parameter, $i = 1,\ldots,m$; $j = 1,\ldots,n$

c_j - Unit cost data for objective function, $j = 1,\ldots,n$

V_i - Value of RHS, $i = 1,\ldots,m$

L_j - Limiting value of variable, j is a subset of $1,\ldots,n$

Using the correct column and field format is of prime importance in this section. The user should become acquainted with the IBM Mathematical Programming Input Form before attempting to set up an MPS/360 program.

A4.9.4 Program Output

The output for LPLF consists of eight parts, each part containing different information according to the characteristics of the Mathematical Programming System/360. The following is a brief summary of the contents in the order listed.

1. The OS/360 Job Control Language (JCL) statements and their associated resource allocation.

2. The MPS/360 Control Program listing.

3. A summary of the minor and major errors for the input sections.

4. A summary of the number of elements by column order and the number of elements by row order. The row element summary excludes the RHS's but includes one slack element per row. Each row has associated with it the row type. Included here is a summary of the problem statistics, viz, the number of rows, variables, elements, and the model's density.

5. A complete listing of the LP model, including the Rows, Columns, RHS, and Bounds sections.

6. This part is produced by the Setup and Primal procedures called in the MPS Control Program. Of interest here is a summary of the statistics for the model and an iteration log relative to the stages in the problem from infeasibility to feasibility.

7. The Solution procedure called for in the MPS Control Program produces this output. It consists of three sections, the first being the heading, which states the procedure name, problem status time, and iteration number. The first section also contains the functional value and name, the restraints name (RHS), and the bounds name. The second section is the Rows section. This section contains the internal slack variable number, row name, row status, row activity, slack activity, value of lower and upper limits, and the dual activity. The third section is the Columns section. In this section is found the internal vector number, column name, column status, column activity, original input cost, lower and upper limits, and the reduced cost of the associated variable. If any A appears in the far left-hand column, this specifies an alternate optima.

8. The last of the output consist of the EXIT procedure called for in the MPS Control Program. This procedure returns control to the OS/360 and simply outputs the procedure name and total elapsed time for the run.

A4.9.5 Definition of Program Variables

The rows and columns in the LP model are representative of the three commodities being transported down the river system, viz, water, BOD and DO. In the node-branch concept of the network, the rows are the node names and the columns (variables) are the branches connecting the nodes. The rows and columns are defined as follows:

ROWS

WAT(I)B
WAT(I)P
BØD(I)B
BØD(I)P
DIØ(I)B
DIØ(I)P
QUAL(I)P

where

(I) = The reach number

B	Represents wastewater
P	Represents tributary
WAT(I)B	Wastewater node
WAT(I)P	Tributary node for water
B0D(I)B	BOD node for wastewater
B0D(I)P	BOD node for tributary
DI0(I)B	DO node for wastewater
DI0(I)P	DO node for tributary
QUAL(I)P	Water quality node

Columns

QIBIPK
QIPJPK
QI-%K

where

B	Represents wastewater
P	Represents tributary
K=1	Water
K=2	BOD
K=3	DO
QIBIP	Branch from wastewater node to tributary node in reach I
QIPJP	Branch from beginning of upstream reach I to beginning of next downstream reach J
QI-%K	Percent of BOD removal in reach I (K=2)

A4.9.6 Program Coding

```
//LPLF1    JOB (1143,47,003,20,1000),'CARTER                    ',CLASS
=M
//JOBLIB DD DSNAME=GATOR.MPS,DISP=(SHR)
// EXEC MPS
//CONTROL.SYSIN DD *
          PROGRAM
          INITIALZ
          MOVE(XDATA,'DAT1980')
          MOVE(XPBNAME,'PBFILE')
          CONVERT('SUMMARY')
          BCDOUT
          SETUP('BOUND','TRT80','MIN')
          MOVE(XOBJ,'COST')
          MOVE(XRHS,'STP1980')
          CRASH
          PRIMAL
          SOLUTION
          EXIT
          PEND
/*
//PROBLEM.SYSIN DD *
NAME          DAT1980
ROWS
 N  COST
 E  WAT1B
 E  WAT1P
 E  WAT2B
 E  WAT2P
 E  WAT3B
 E  WAT3P
 E  WAT4B
 E  WAT4P
 E  WAT5P
 E  WAT6B
 E  WAT6P
 E  WAT7B
 E  WAT7P
 E  WAT8P
 E  BOD1B
 E  BOD1P
 E  BOD2B
 E  BOD2P
 E  BOD3B
 E  BOD3P
 E  BOD4B
 E  BOD4P
 E  BOD5P
 E  BOD6B
 E  BOD6P
 E  BOD7B
 E  BOD7P
 E   BOD8P
 E   DIO1B
 E   DIO1P
 E   DIO2B
 E   DIO2P
 E   DIO3B
 E   DIO3P
 E   DIO4B
 E   DIO4P
 E   DIO5P
 E   DIO6B
 E   DIO6P
 E   DIO7B
 E   DIO7P
 E   DIO8P
 G   QUAL1P
 G   QUAL2P
 G   QUAL3P
 G   QUAL4P
 G   QUAL5P
 G   QUAL6P
 G   QUAL7P
COLUMNS
    Q1P3P1     WAT1P            1.0000   WAT3P            -1.0000
    Q1P3P1     BOD3P           -0.2829   DIO3P           -19.6027
    Q1P3P1     QUAL1P          19.6027                      0.0
    Q2P3P1     WAT2P            1.0000   WAT3P            -1.0000
    Q2P3P1     BOD3P           -1.1756   DIO3P           -46.4206
    Q2P3P1     QUAL2P          46.4206                      0.0
    Q3P5P1     WAT3P            1.0000   WAT5P            -1.0000
    Q3P5P1     BOD5P           -1.0293   DIO5P           -38.2245
    Q3P5P1     QUAL3P          38.2245                      0.0
    Q4P5P1     WAT4P            1.0000   WAT5P            -1.0000
    Q4P5P1     BOD5P           -1.3018   DIO5P           -13.7926
    Q4P5P1     QUAL4P          13.7926                      0.0
    Q5P6P1     WAT5P            1.0000   WAT6P            -1.0000
    Q5P6P1     BOD6P           -0.2646   DIO6P           -15.7224
    Q5P6P1     QUAL5P          15.7224                      0.0
    Q6P7P1     WAT6P            1.0000   WAT7P            -1.0000
    Q6P7P1     BOD7P           -0.9250   DIO7P            -9.9169
    Q6P7P1     QUAL6P           9.9169                      0.0
    Q7P8P1     WAT7P            1.0000   WAT8P            -1.0000
    Q7P8P1     BOD8P           -0.0      DIO8P            -7.8619
    Q7P8P1     QUAL7P           7.8619                      0.0
    Q1B1P1     WAT1B            1.0000   WAT1P            -1.0000
    Q2B2P1     WAT2B            1.0000   WAT2P            -1.0000
    Q3B3P1     WAT3B            1.0000   WAT3P            -1.0000
    Q4B4P1     WAT4B            1.0000   WAT4P            -1.0000
    Q6B6P1     WAT6B            1.0000   WAT6P            -1.0000
    Q7B7P1     WAT7B            1.0000   WAT7P            -1.0000
    Q8P9B1     WAT8P            1.0000   WAT9B            -1.0000
```

```
Q1P3P2     BOD1P         1.0000   BOD3P        -0.9254
Q1P3P2     DIO3P         0.0622   QUAL1P       -0.0622
Q2P3P2     BOD2P         1.0000   BOD3P        -0.5570
Q2P3P2     DIO3P         0.2736   QUAL2P       -0.2736
Q3P5P2     BOD3P         1.0000   BOD5P        -0.6474
Q3P5P2     DIO5P         0.2242   QUAL3P       -0.2242
Q4P5P2     BOD4P         1.0000   BOD5P        -0.4466
Q4P5P2     DIO5P         0.4476   QUAL4P       -0.4476
Q5P6P2     BOD5P         1.0000   BOD6P        -0.8875
Q5P6P2     DIO6P         0.0878   QUAL5P       -0.0878
Q6P7P2     BOD6P         1.0000   BOD7P        -0.6502
Q6P7P2     DIO7P         0.2763   QUAL6P       -0.2763
Q7P8P2     BOD7P         1.0000   BOD8P        -0.1590
Q7P8P2     DIO8P         0.7775   QUAL7P       -0.7775
Q1B1P2     BOD1B         1.0000   BOD1P        -1.0000
Q2B2P2     BOD2B         1.0000   BOD2P        -1.0000
Q3B3P2     BOD3B         1.0000   BOD3P        -1.0000
Q4B4P2     BOD4B         1.0000   BOD4P        -1.0000
Q6B6P2     BOD6B         1.0000   BOD6P        -1.0000
Q7B7P2     BOD7B         1.0000   BOD7P        -1.0000
Q8P9P2     BOD8P         1.0000   BOD9B        -1.0000
Q1P3P3     DIO1P         1.0000   DIO3P        -0.7869
Q1P3P3     QUAL1P        0.7869                 0.0
Q2P3P3     DIO2P         1.0000   DIO3P        -0.4502
Q2P3P3     QUAL2P        0.4502                 0.0
Q3P5P3     DIO3P         1.0000   DIO5P        -0.5042
Q3P5P3     QUAL3P        0.5042                 0.0
Q4P5P3     DIO4P         1.0000   DIO5P        -0.8302
Q4P5P3     QUAL4P        0.8302                 0.0
Q5P6P3     DIO5P         1.0000   DIO6P        -0.8023
Q5P6P3     QUAL5P        0.8023                 0.0
Q6P7P3     DIO6P         1.0000   DIO7P        -0.8633
Q6P7P3     QUAL6P        0.8633                 0.0
Q7P8P3     DIO7P         1.0000   DIO8P        -0.8846
Q7P8P3     QUAL7P        0.8846                 0.0
Q1B1P3     DIO1B         1.0000   DIO1P        -1.0000
Q2B2P3     DIO2B         1.0000   DIO2P        -1.0000
Q3B3P3     DIO3B         1.0000   DIO3P        -1.0000
Q4B4P3     DIO4B         1.0000   DIO4P        -1.0000
Q6B6P3     DIO6B         1.0000   DIO6P        -1.0000
Q7B7P3     DIO7B         1.0000   DIO7P        -1.0000
Q8P9B3     DIO8P         1.0000   DIO9B        -1.0000
Q1-672     BOD1B       1.0        COST        0.00
Q1-752     BOD1B       1.0        COST       26.70
Q1-852     BOD1B       1.0        COST       53.60
Q1-902     BOD1B       1.0        COST       83.40
Q2-102     BOD2B       1.0        COST        0.00
Q2-352     BOD2B       1.0        COST       17.40
Q2-502     BOD2B       1.0        COST        0.30
Q2-602     BOD2B       1.0        COST        6.20
Q2-752     BOD2B       1.0        COST        7.90
```

```
    Q2-852      BOD2B           1.0          COST          16.40
    Q2-902      BOD2B           1.0          COST          29.10
    Q3-262      BOD3B           1.0          COST           0.00
    Q3-352      BOD3B           1.0          COST         111.00
    Q3-502      BOD3B           1.0          COST           4.20
    Q3-602      BOD3B           1.0          COST          25.00
    Q3-752      BOD3B           1.0          COST          28.10
    Q3-852      BOD3B           1.0          COST          28.40
    Q3-902      BOD3B           1.0          COST          68.70
    Q4-242      BOD4B           1.0          COST           0.00
    Q4-352      BOD4B           1.0          COST          17.50
    Q4-502      BOD4B           1.0          COST           0.60
    Q4-602      BOD4B           1.0          COST           4.40
    Q4-752      BOD4B           1.0          COST           4.40
    Q4-852      BOD4B           1.0          COST           6.10
    Q4-902      BOD4B           1.0          COST           8.60
    Q6-122      BOD6B           1.0          COST           0.00
    Q6-352      BOD6B           1.0          COST           3.50
    Q6-502      BOD6B           1.0          COST           0.30
    Q6-602      BOD6B           1.0          COST           2.00
    Q6-752      BOD6B           1.0          COST           2.00
    Q6-852      BOD6B           1.0          COST           3.20
    Q6-902      BOD6B           1.0          COST           4.60
    Q7-262      BOD7B           1.0          COST           0.00
    Q7-352      BOD7B           1.0          COST          78.00
    Q7-502      BOD7B           1.0          COST           1.40
    Q7-602      BOD7B           1.0          COST          15.70
    Q7-752      BOD7B           1.0          COST          16.20
    Q7-852      BOD7B           1.0          COST          16.80
    Q7-902      BOD7B           1.0          COST          24.60
RHS
    STP1980     WAT1B             5.0000     WAT1P          1355.0000
    STP1980     WAT2B            37.0000     WAT2P          1290.0000
    STP1980     WAT3B             8.0000     WAT3P             0.0
    STP1980     WAT4B            14.0000     WAT4P           296.0000
    STP1980     WAT5B             0.0        WAT5P             0.0
    STP1980     WAT6B            26.0000     WAT6P             0.0
    STP1980     WAT7B            41.0000     WAT7P             0.0
    STP1980     WAT8P            0.
    STP1980     BOD1B         10341.5937     BOD1P         18759.1523
    STP1980     BOD2B        125900.5625     BOD2P          7315.8437
    STP1980     BOD3B         16012.7930     BOD3P             0.0
    STP1980     BOD4B        168134.3125     BOD4P          2468.6396
    STP1980     BOD5B             0.0        BOD5P             0.0
    STP1980     BOD6B        472711.1250     BOD6P             0.0
    STP1980     BOD7B         95401.1875     BOD7P             0.0
    STP1980     DIO1B            41.7000     DIO1P        107356.5625
    STP1980     DIO2B           308.5798     DIO2P         86068.7500
    STP1980     DIO3B            66.7200     DIO3P             0.0
    STP1980     DIO4B           116.7600     DIO4P         23945.8008
    STP1980     DIO5B             0.0        DIO5P             0.0
    STP1980    DIO6B          216.8400    DIO6P             0.0
    STP1980    DIO7B          341.9399    DIO7P             0.0
    STP1980    QUAL1P       79396.7500    QUAL2P       83003.8125
    STP1980    QUAL3P      157334.0625    QUAL4P       15512.3984
    STP1980    QUAL5P      162901.0000    QUAL6P      151671.1875
    STP1980    QUAL7P      102481.8750
BOUNDS
 FX TRT80      Q1-672      6900.
 UP TRT80      Q1-752       830.
 UP TRT80      Q1-852      1030.
 UP TRT80      Q1-902       520.
 FX TRT80      Q2-102     12600.
 FX TRT80      Q2-352     31500.
 UP TRT80      Q2-502     18900.
 UP TRT80      Q2-602     12600.
 UP TRT80      Q2-752     18900.
 UP TRT80      Q2-852     12600.
 UP TRT80      Q2-902      6300.
 FX TRT80      Q3-262      4200.
 FX TRT80      Q3-352      1400.
 UP TRT80      Q3-502      2400.
 UP TRT80      Q3-602      1600.
 UP TRT80      Q3-752      2400.
 UP TRT80      Q3-852      1600.
 UP TRT80      Q3-902       800.
 FX TRT80      Q4-242     40400.
 FX TRT80      Q4-352     18500.
 UP TRT80      Q4-502     25200.
 UP TRT80      Q4-602     16800.
 UP TRT80      Q4-752     25200.
 UP TRT80      Q4-852     16800.
 UP TRT80      Q4-902      8400.
 FX TRT80      Q6-122     56700.
 FX TRT80      Q6-352    108700.
 UP TRT80      Q6-502     70900.
 UP TRT80      Q6-602     47300.
 UP TRT80      Q6-752     70900.
 UP TRT80      Q6-852     47300.
 UP TRT80      Q6-902     23600.
 FX TRT80      Q7-262     24800.
 FX TRT80      Q7-352      8600.
 UP TRT80      Q7-502     14300.
 UP TRT80      Q7-602      9500.
 UP TRT80      Q7-752     14300.
 UP TRT80      Q7-852      9500.
 UP TRT80      Q7-902      4800.
ENDATA
/*EOF
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APPENDIX A5
SAMPLE INTERFACING OF SIMULATION AND OPTIMIZATION MODELS

A5.1 Introduction

The purpose of this analysis is to demonstrate the interfacing of the simulation and optimization components presented in the body of this report. The selected test area was the Farmington River Basin in west central Massachusetts and Connecticut (see Section 8). Results from the simulation model were used to select the region and critical period, and to provide the requisite data input to the optimization model. Illustrative output from the optimization program is presented. This sample analysis should not be construed to represent an evaluation of the actual or projected conditions in the Farmington.

A5.2 Selection of Test Conditions

The simulation model permits rapid sampling of the system's response to a wide variety of assumed conditions. The initial analyses sought to examine the system response to "worst case" conditions. Consequently, a 20 year simulation was made with regulated flows, BOD loadings for the year 2000 and 35% removal required. The test condition was selected from analysis of periods in which the DO fell below 4.0 mg/l at any of the reaches. This analysis showed that the worst condition occurred during week 33 of year 15 when the DO fell to 2.1 mg.l in the Pequabuck River reach and 3.3 mg/l in the Salmon Brook reach. The DO in all other reaches exceeded 8.0 mg/l. The water quality conditions in the reaches with unacceptable DO levels can be classed as independent cases of local competition (see Section 7) so that it was unnecessary to utilize the optimization model for more refined analysis of this case.

From these results, it appeared that regional competition would probably be restricted to the lower reaches. Consequently, a different set of assumed conditions was used in a subsequent simulation run of the following subset of reaches [1,2,3,4,5,6,7,8,9,19,20,21,22]. In this revised case, all reservoirs were removed from the watershed. BOD loadings for the year 2000 and 35% required BOD removal were retained. The results indicated a DO less than 4.0 mg/l during week 28 of the second year of the simulation. This period was selected for further analysis.

A5.3 Input Data for Optimization Model

For this sample interfacing, the input for the optimization model consist of the following:

1. the output data, shown in Table A5-1, from the simulation model; and
2. the annual wastewater treatment cost data, shown in Table A5-2, for the indicated reaches.

In the simulated output data, the original reaches 5,6, and 7 were aggregated into a single reach to simplify the analysis. The annual wastewater treatment costs are based on the results from Section 5.

TABLE A5-1
OUTPUT FROM SIMULATION MODEL FOR SELECTED PERIOD

Reach	River Flow	Waste Flow	Reach Travel Time		Wastewater Quality		Rate Constants		Quality of Reach Outflow		Minimum D.O.
			Actual	Critical	DO	BOD	K_1	K_2	BOD	DO Deficit	
	cfs	cfs	days	days	mg/1	mg/1	days^{-1}	days^{-1}	mg/1	mg/1	mg/1
1	315.	7.1	.13	.34	2.0	143.	.31	2.1	22.9	3.2	5.3
2	311.	0.	.07	.24	--	--	.31	2.1	21.2	3.0	5.5
3	310.	0.1	.03	.27	--	--	.31	2.1	21.7	2.9	5.5
4	308.	2.3	.05	.32	2.0	143.	.31	2.1	21.8	2.9	5.6
5	251.	3.9	.13	.37	2.0	143.	.31	2.0	20.2	2.8	5.7
6	241.	2.4	.12	.39	2.0	143.	.31	2.0	19.1	2.6	5.8
7	230.	6.1	.18	.50	2.0	143.	.31	2.0	18.5	2.5	6.0
8	182.	0.	.12	.05	--	--	.31	2.0	8.4	1.4	7.1
9	182.	.8	.09	.16	2.0	143.	.31	2.0	8.7	1.4	7.1
19	18.	25.6	.33	1.04	2.0	143.	.32	1.7	53.6	6.0	2.5
20	0	0.	.45	1.14	--	--	--	1.5	0.0	0.0	8.5
21	36.	5.1	.30	1.14	2.0	143.	.32	1.5	28.6	3.2	5.3
22	0.	0.	.51	.34	--	--	--	1.4	0.0	0.0	8.5

TABLE A5-2
TREATMENT COST DATA FOR INDICATED REACHES

Reach	Annual Cost For Indicated % BOD Removal : $\$x10^3$					
	35	50	60	75	85	90
2	134	140	146	155	179	200
4	62	65	67	74	86	96
19	107	112	115	124	144	161
5,6,7	198	202	208	224	258	288
21	322	337	346	368	415	463

A5.4 Discussion of Results

The optimal solution is shown in Table A5-3. Comparison of columns four and five of that table indicates headwater competition in reaches 19 and 21. Only the minimum treatment (35%) is required in interior reaches 2 and 4. The results indicate that it would be advisable to analyze reaches 5, 6, and 7 separately. Thus, subsequent investigation could deal with this modified regional configuration.

TABLE A5-3
OPTIMAL SOLUTION FOR SAMPLE PROBLEM
(35% TREATMENT REQUIRED)

Reach	% BOD Removal	Annual Cost $\$ \times 10^3$	DO in Reach	
			Actual mg/l	Allowable mg/l
2	35	134	7.10	7.0
4	35	62	7.15	7.0
19	81.4	137	7.0	7.0
5,6,7	82.4	249	7.0	7.0
21	86.1	425	6.0	6.0
--	--	1007		